Model-checking strategic ability and knowledge of the past of communicating coalitions

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Abstract. We propose a variant of alternating time temporal logic (*ATL*) with imperfect information, perfect recall, epistemic modalities for the past and strategies which are required to be uniform with respect to distributed knowledge. The model-checking problem about *ATL* with perfect recall and imperfect information is believed to be unsolvable, whereas in our setting it is solvable because of the uniformity of strategies. We propose a model-checking algorithm for that system, which exploits the interaction between the cooperation modalities and the epistemic modality for the past. This interaction allows every expressible goal φ to be treated as the epistemic goal of (eventually) establishing that φ holds and thus enables the handling of the cooperation modalities in a streamlined way.

Introduction

Alternating time temporal logic (*ATL*, [AHK97,AHK02]) was introduced as a reasoning tool for the analysis of strategic abilities of coalitions in infinite multiplayer games with temporal winning conditions. Several variants of *ATL* have been proposed in the literature. The main differences arise from various restrictions on the considered games such as the players' information on the game state, which may be either *complete* or *incomplete* (*imperfect*), and their ability to keep complete record of the past, which is known as *perfect recall* [JvdH04,Sch04]. The awareness of coalitions of the existence of winning strategies is another source of differences, which is specific to the case of incomplete information. The completeness of a proof system for *ATL* with complete information and the decidability of validity in it was demonstrated in [GvD06]. Notably, model checking is believed to be undecidable for *ATL* with imperfect information and perfect recall. The result is attributed to a personal communication of Mihalis Yannakakis to the authors of [AHK02]; this reference has been borrowed in [Sch04] too. This undecidability recall has stimulated the introduction of several systems [Sch04,vOJ05,JvdH06] with restrictions leading to more feasible model-checking. An extensive study of the complexity of the model checking problem for the variants of *ATL* which arise from allowing imperfect information and/or perfect recall was done in [DJ08].

The formal analysis of multi-agent systems has generated substantial interest in the study of combinations of *ATL* with modal logics of knowledge [vdHW03,JvdH04]. Such combinations can be viewed as related to temporal logics of knowledge (cf. e.g [HFMV95]) in the way *ATL* is related to computational tree logic *CTL*. Epistemic goals make it essential to study strategic ability with incomplete information. Variants of the cooperation modalities which correspond to different forms of coordination within coalitions were proposed in [JvdH04]. The recent work [JÅ07] proposes a combination of *ATL* with the epistemic modalities for collective knowledge. In that system formulas are interpreted at *sets* of states and the existence of strategies which are winning for all the epistemically indiscernible states can be expressed by combining epistemic and cooperation modalities. Such strategies are called *uniform* with respect to the corresponding form of collective knowledge.

Along with the alternating transition systems proposed in [AHK02], *ATL* has been given semantics on *interpreted systems*, which are known from the study of knowledge-based programs [HFMV95], and other structures, some of which have been shown to be equivalent [GJ04]. Most of the proposed extensions of *ATL* and other temporal logics by epistemic modalities include only the future temporal operators and the indiscernibility relations which are needed for the semantics of the $S5$ epistemic modalities are either defined as the equality of current local states of the corresponding agents or assumed to be given explicitly in the respective structures and required to respect equality of local state [LR06a,LR06b]. The axiomatisation of knowledge in the presence of past temporal operators has been studied in [FvdMR05], where indiscernibility is defined as equality of local state again. *ATL* with complete information can be regarded as an extension of *computation tree logic CTL*. CTL^* with past modalities was given a complete axiomatisation in [Rey05]. Extensions of *CTL* by modalities to reason about indiscernibility with respect to path observations in the past have been proposed in [AČC07]. The model-checking problem for corresponding more expressive system of μ-calculus has been found to be undecidable. A system of *CTL* with past whose set of temporal modalities is closest to that of the system which we propose in this paper was introduced and studied in [LS95,LS00].

In this paper we propose a variant of *ATL* with epistemic modalities which can be applied to past formulas to allow the formulation of epistemic goals. We assume incomplete information and perfect recall and choose a variant of the meaning of the cooperation modalities of *ATL* in a way which renders the model-checking problem decidable. We achieve this by requiring strategies to be uniform with respect to the distributed knowledge of the coalition and assuming that strategies are functions on the combined local state of all the members of the coalition. This corresponds to the unrestricted sharing of information within the coalition while implementing the coalition strategy. Requiring uniform strategies can spoil the *determinacy* of games: it is possible that neither side knows how to win without taking chances. However, the impossibility to prevent

one's opponent from achieving something using a uniform strategy still means that the opponent has a possibly non-uniform strategy. Notably, some games with imperfect information for just two players are solvable, and our technique for model-checking uniform strategies bears similarities with that from [CDHR07]. That is why the reason for the undecidability under the standard (non-uniform) interpretation of *ATL* with imperfect information and perfect recall appears to be the fact that, even though the members of a coalition are assumed to be working towards a common goal, each agent's strategy is supposed to use only its own observations on the evolution of the system. With strategies that are uniform with respect to distributed knowledge and allow the agents to act using their combined knowledge a coalition can be viewed as a single player whose abilities and information are a combination of those of all the members. We allow past formulas which are interpreted on finite histories in the scope of epistemic modalities. The corresponding indiscernibility relations are defined as equality of local state *throughout the past* and not just of current local state, which is the most marked difference from the majority of the systems known from the literature. Our model-checking algorithm exploits the interaction between uniform strategies and knowledge, which can be formulated in the logic thanks to the presence of both epistemic and strategic modalities: in our setting any strategic goal φ can be formulated as the goal of *(eventually) establishing that* φ holds, which is an epistemic goal. The respective strategies, of course, incorporate the effort of *making* the original goal φ hold.
Structure of the paper After brief preliminaries on *ATL* and its semantics on interpreted systems we introduce our extension of *ATL* by a modality for distributed knowledge of the past and our variant of the cooperation modalities. Then we propose a transformation of interpreted systems which enables the elimination of any given finite set of formulas built using the epistemic modality, and establish some properties of the new variants of the cooperation modalities which, in turn, allow the formulation of all strategic goals as corresponding epistemic goals. Finally we combine these technical results into a model checking algorithm for the proposed system of *ATL*.

1 Preliminaries

1.1 Interpreted systems

Definition 1 (Interpreted systems). *Given a set of agents* $\Sigma = \{1, \ldots, n\}$ *and a set of atomic propositions* AP*, an interpreted system is a tuple of the form* $\langle\langle L_i, Act_i, P_i, t_i\rangle : i \in \Sigma \cup \{e\}, I, h\rangle$ *where*

- L_i *is the set of the* local states *of agent* $i \in \Sigma \cup \{e\}$*;* $e \notin \Sigma$ *stands for the* environment*;*
- Act_i *is the set of* actions *available to agent* i*;*
- $P_i : L_i \to 2^{Act_i}$ *is the* protocol *of agent* i*;*
- $t_i : L_i \times L_e \times Act_1 \times \ldots \times Act_n \times Act_e \to L_i$ *is the* local transition function *of agent* i*;*

– $I \subseteq L$ *is the set of* initial *global states, where* $L = L_1 \times \ldots \times L_n \times L_e$ *is the set of all* global states*;*
– $h : AP \to 2^L$ *is the valuation function.*

Informally, $P_i(l)$ is the set of actions which are available to agent i when its local state is l and $t_i(l, l_e, a_1, \ldots, a_n, a_e)$ is the local state of i after a transition in which the actions chosen by the agents and the environment are $a_1, \ldots, a_n, a_e$, respectively.

A *run* of an interpreted system $IS = \langle\langle L_i, Act_i, P_i, t_i\rangle : i \in \Sigma \cup \{e\}, I, h\rangle$ is an infinite sequence $r = r^0 r^1 \ldots r^k \ldots \in L^\omega$ such that $r^0 \in I$ and for every $k < \omega$, if $r^k = \langle l_1, \ldots, l_n, l_e\rangle$ and $r^{k+1} = \langle l'_1, \ldots, l'_n, l'_e\rangle$, then there exist some $a_1 \in P_1(l_1), \ldots, a_n \in P_n(l_n), a_e \in P_e(l_e)$ such that $l'_i = t_i(l_i, l_e, a_1, \ldots, a_n, a_e)$, for $i \in \Sigma \cup \{1, \ldots, n, e\}$. In words, an interpreted system evolves by each agent choosing an available action at every step and the successor local states of the agents being determined by their respective current local states and all the actions which were simultaneously chosen by the agents and the environment. The environment behaves as an agent, except that, along with its actions, its local state influences the evolution of the local states of all the agents.

The local state of agent i within global state $l \in L$ is denoted by l_i. Furthermore, $l' = t(l, a_1, \ldots, a_n, a_e)$ denotes $l'_i = t_i(l_i, l_e, a_1, \ldots, a_n, a_e)$ for all $i \in \Sigma \cup \{e\}$.

1.2 *ATL* on interpreted systems

Syntax *ATL* formulas are built from propositional variables p from some given *vocabulary* AP and sets of agents Γ within some given set of agents Σ using the following syntax:

$$\varphi ::= \bot \mid p \mid \varphi \Rightarrow \varphi \mid \langle\!\langle \Gamma \rangle\!\rangle \circ \varphi \mid \langle\!\langle \Gamma \rangle\!\rangle(\varphi \mathsf{U} \varphi) \mid [\![\Gamma]\!](\varphi \mathsf{U} \varphi) \mid \langle\!\langle \Gamma \rangle\!\rangle \Box \varphi$$

Semantics The semantics of the $\langle\!\langle \Gamma \rangle\!\rangle$-modalities of *ATL* involves strategies for sets of agents. A *strategy* for agent i is a function f of type $L_i^+ \to Act_i$ such that $f(l_0 \ldots l_k) \in P_i(l_k)$ for all $l \in L_i^{k+1}$. Given a set of agents Γ, a global state $l \in L$, and a system of strategies $F = \langle f_i : i \in \Gamma\rangle$, $\mathrm{out}(l, F)$ denotes the set of infinite sequences $r \in L^\omega$ such that $r^0 = l$ and r^{k+1} is reached from r^k by a transition in which the action of each agent $i \in \Gamma$ is $f_i(r_i^0 \ldots r_i^k)$ for all $k < \omega$. Informally, $\mathrm{out}(l, F)$ consists of the behaviours starting at l in which the agents from Γ stick to their respective strategies in F.

Given an interpreted system $IS = \langle\langle L_i, Act_i, P_i, t_i\rangle : i \in \Sigma \cup \{e\}, I, h\rangle$, the modeling relation $\models$ is defined between global states $l \in L$ and *ATL* formulas as follows:

$l \not\models \bot$

$l \models p$ iff $l \in h(p)$

$l \models \varphi \Rightarrow \psi$ iff $l \models \psi$ or $l \not\models \varphi$

$l \models \langle\langle \Gamma \rangle\rangle \circ \varphi$ iff there exists a system of strategies F for Γ such that $r^1 \models \varphi$ for all $r \in \mathrm{out}(l, F)$

$l \models \langle\langle \Gamma \rangle\rangle (\varphi \mathsf{U} \psi)$ iff there exists a system of strategies F for Γ such that for every $r \in \mathrm{out}(l, F)$ there exists an $m < \omega$ such that $r^k \models \varphi$ for all $k < m$ and $r^m \models \psi$

$l \models [\![\Gamma]\!] (\varphi \mathsf{U} \psi)$ iff for every system of strategies F for Γ there exists an $r \in \mathrm{out}(l, F)$ and an $m < \omega$ such that $r^k \models \varphi$ for all $k < m$ and $r^m \models \psi$

The other combinations between $\langle\langle \Gamma \rangle\rangle$ and $[\![\Gamma]\!]$ and the *LTL* modalities $\circ$, $\Diamond$ and $\Box$ are introduced as abbreviations:

$$\langle\langle \Gamma \rangle\rangle \Diamond \varphi \rightleftharpoons \langle\langle \Gamma \rangle\rangle (\top \mathsf{U} \varphi),\ [\![\Gamma]\!] \Diamond \varphi \rightleftharpoons [\![\Gamma]\!] (\top \mathsf{U} \varphi),\ \ [\![\Gamma]\!] \circ \varphi \rightleftharpoons \neg \langle\langle \Gamma \rangle\rangle \circ \neg \varphi,$$
$$\langle\langle \Gamma \rangle\rangle \Box \varphi \rightleftharpoons \neg [\![\Gamma]\!] \Diamond \neg \varphi,\quad [\![\Gamma]\!] \Box \varphi \rightleftharpoons \neg \langle\langle \Gamma \rangle\rangle \Diamond \neg \varphi.$$

2 *ATL* with knowledge of the past and communicating coalitions

According to the definition of $\models$ for formulas built using a cooperation modality, despite pursuing a common goal, the members of a coalition are supposed to follow strategies which are based on each individual member's observation of the behaviour of the system as represented by its sequence of local states. This appears to render the model-checking problem for *ATL* with incomplete information undecidable. A proof of the undecidability is attributed to a private communication of Mihalis Yannakakis to the authors of [AHK02]; the reference to that communication has been borrowed also in [Sch04]. The reason for losing the decidability of model-checking appears to be the impossibility to treat a coalition as an individual player with its abilities being an appropriate combination of the abilities of the coalition members. In this section we introduce a semantics for the cooperation modalities which allows this to be done by assuming that the agents exchange information on their local state while acting as a coalition. We believe that both this assumption and the assumption of each coalition member having to cope using just its own local state are realistic. We also include knowledge modalities, which can be applied to past *LTL* formulas. Making reference to the past requires the satisfaction relation $\models$ to be defined with respect to global histories r instead of just a current state. Unlike runs, histories include the actions of the agents because the knowledge of the coalition's actions contributes restrictions on the global behaviours which correspond to the coalition's observation.

The proposed logic involves two types of formulas denoted by φ and ψ in the BNFs for their syntax:

$$\varphi ::= \bot \mid p \mid \varphi \Rightarrow \varphi \mid \mathsf{D}_\Gamma \psi \mid \langle\langle \Gamma \rangle\rangle^{\mathsf{D}} \circ \varphi \mid \langle\langle \Gamma \rangle\rangle^{\mathsf{D}} (\varphi \mathsf{U} \varphi) \mid [\![\Gamma]\!]^{\mathsf{D}} (\varphi \mathsf{U} \varphi) \mid \langle\langle \Gamma \rangle\rangle^{\mathsf{D}} \Box \varphi$$
$$\psi ::= \bot \mid \varphi \mid \psi \Rightarrow \psi \mid \bar{\circ}\psi \mid (\psi \mathsf{S} \psi) \mid \mathsf{D}_\Gamma \psi$$

Γ denotes a subset of Σ in $\langle\langle \Gamma \rangle\rangle^{\mathsf{D}}$, $[\![\Gamma]\!]^{\mathsf{D}}$ and D_Γ. We use D to denote the epistemic modality of *distributed knowledge*, which is usually introduced in systems with K_i as the basic modalities about the knowledge of individual agents i. Since K_i is equivalent to $\mathsf{D}_{\{i\}}$, and we have little technical need to treat the case of singleton coalitions separately, we use only D in our syntax. The superscript $^{\mathsf{D}}$ to the cooperation modalities is meant to emphasize the uniformity with respect to distributed knowledge within coalitions in their semantics. The "main" type of formulas are those defined by the BNF for φ. The past modalities $\bar{\circ}$ and $(.\mathsf{S}.)$ can appear only in the scope of D_Γ, without intermediate occurrences of $\langle\langle . \rangle\rangle^{\mathsf{D}}$ or $[\![.]\!]^{\mathsf{D}}$.

2.1 Cooperation modalities for communicating coalitions

Let $IS = \langle \langle L_i, Act_i, P_i, t_i \rangle : i \in \Sigma \cup \{e\}, I, h \rangle$ be an interpreted system IS with set of global states L. Let $Act = \prod\limits_{i \in \Sigma \cup \{e\}} Act_i$. Then $r = l^0 a^0 l^1 \dots l^{m-1} a^{m-1} l^m \in L(Act\, L)^*$ is a *global history* of IS, if $l^0 \in I$ and $a_i^{k+1} \in P_i(l_i^k)$ for all $i \in \Sigma \cup \{e\}$ and $l^{k+1} = t(l^k, a_1^k, \dots, a_n^k, a_e^k)$ for $k = 0, \dots, m$. The *length* $|r|$ of r is m. We denote the set of the global histories of IS of length m by $R(IS, m)$ and write $R(IS)$ for $\bigcup\limits_{m=0}^{\infty} R(IS, m)$. Given $r = l^0 a^0 l^1 \dots l^{m-1} a^{m-1} l^m \in R(IS, m)$ and a coalition Γ, the corresponding *local* history r_Γ of Γ is the sequence $l_\Gamma^0 a_\Gamma^0 l_\Gamma^1 \dots l_\Gamma^{m-1} a_\Gamma^{m-1} l_\Gamma^m \in L_\Gamma (Act_\Gamma L_\Gamma)^m$ where l_Γ stands for $\langle l_i : i \in \Gamma \rangle$, a_Γ stands for $\langle a_i : i \in \Gamma \rangle$, and L_Γ and Act_Γ are $\prod\limits_{i \in \Gamma} L_i$ and $\prod\limits_{i \in \Gamma} Act_i$, respectively. In case $\Gamma = \emptyset$, l_Γ and a_Γ are the empty tuple $\langle \rangle$. The local histories of the empty coalition are sequences of $\langle \rangle$s. Two histories r, r' are *indistinguishable* to coalition Γ, written $r \sim_\Gamma r'$, if $r_\Gamma = r'_\Gamma$. The definition of $\models$ for formulas built using the cooperation modalities $\langle\langle . \rangle\rangle^{\mathsf{D}}$ and $[\![.]\!]^{\mathsf{D}}$involves a notion of joint strategies for coalitions which can have internal communication.

Definition 2. *Let IS be as above and $\Gamma \subseteq \Sigma$. A (communicating) strategy for a coalition Γ in IS is a mapping $s : L_\Gamma (Act_\Gamma L_\Gamma)^* \to Act_\Gamma$ such that if the last member of r is l, then $s(r) \in \prod\limits_{i \in \Gamma} P_i(l_i)$.*

The set of outcomes $\mathrm{out}(r, s)$ of a given strategy s for Γ starting from a given global history $r \in R(IS)$ consists of the infinite extensions of r which can be obtained if Γ sticks to the strategy s from the end of r on. The clauses for $\models$ on formulas built using cooperation modalities are as follows:

$$\begin{array}{ll}
r \models \langle\langle\Gamma\rangle\rangle^{\mathsf{D}} \circ \varphi & \text{iff there exists a strategy } s \text{ for } \Gamma \text{ such that for all } r' \in [r]_{\sim_\Gamma} \\
 & \text{and all } l^0a^0l^1 \ldots l^ka^k \ldots \in \mathrm{out}(r', s) \\
 & l^0a^0l^1 \ldots l^{|r|}a^{|r|}l^{|r|+1} \models \varphi \\
r \models \langle\langle\Gamma\rangle\rangle^{\mathsf{D}}(\varphi \mathsf{U} \psi) & \text{iff there exists a strategy } s \text{ for } \Gamma \text{ such that for all } r' \in [r]_{\sim_\Gamma} \\
 & \text{and all } l^0a^0l^1 \ldots l^ka^k \ldots \in \mathrm{out}(r', s) \text{ there exists an} \\
 & m \geq |r| \text{ such that } l^0a^0l^1 \ldots a^{k-1}l^k \models \varphi \\
 & \text{for all } k \in \{|r|, \ldots, m-1\} \text{ and } l^0a^0l^1 \ldots a^{m-1}l^m \models \psi \\
r \models [\![\Gamma]\!]^{\mathsf{D}}(\varphi \mathsf{U} \psi) & \text{iff for all strategies } s \text{ for } \Gamma \text{ there exists an } r' \in [r]_{\sim_\Gamma}, \\
 & \text{an } l^0a^0l^1 \ldots l^ka^k \ldots \in \mathrm{out}(r', s) \text{ and an } m \geq |r| \\
 & \text{such that } l^0a^0l^1 \ldots a^{k-1}l^k \models \varphi \text{ for all } k \in \{|r|, \ldots, m-1\} \\
 & \text{and } l^0a^0l^1 \ldots a^{m-1}l^m \models \psi
\end{array}$$

Formulas of the forms $\langle\langle\Gamma\rangle\rangle^{\mathsf{D}}\Diamond\varphi$, $[\![\Gamma]\!]^{\mathsf{D}}\Diamond\varphi$, $[\![\Gamma]\!]^{\mathsf{D}} \circ \varphi$, $\langle\langle\Gamma\rangle\rangle^{\mathsf{D}}\Box\varphi$ and $[\![\Gamma]\!]^{\mathsf{D}}\Box\varphi$ are introduced as abbreviations just like in *ATL* with standard semantics.

Since the same strategy is supposed to work for all the histories which the considered coalition cannot distinguish from the actual one, a coalition can achieve something in the sense of $\langle\langle.\rangle\rangle^{\mathsf{D}}$ iff it knows that it can achieve it. As it becomes clear below, it is also true that a coalition can achieve something iff it can eventually establish that it has achieved it, or that it keeps achieving it, in the case of $\Box$-goals.

2.2 Knowledge of the past

Past *LTL* modalities $\bar{\circ}$ and $(.\mathsf{S}.)$ in the scope of D are used to express properties of the history of behaviour of the considered interpreted system. We call formulas built using just these modalities *past LTL formulas.* The semantics of D is defined in terms of the indistinguishability of global histories to coalitions. The clause for $\models$ for knowledge formulas is as follows:

$$r \models \mathsf{D}_\Gamma \psi \text{ iff } r' \models \psi \text{ for all } r' \in [r]_{\sim_\Gamma}$$

Formulas built using past temporal modalities and propositional connectives have their usual meaning:

$$\begin{array}{ll}
l^0a^0l^1 \ldots a^{k-1}l^k \not\models \bot & \\
l^0a^0l^1 \ldots a^{k-1}l^k \models p & \text{iff } l^k \in h(p) \\
l^0a^0l^1 \ldots a^{k-1}l^k \models \varphi \Rightarrow \psi & \text{iff either } l^0a^0l^1 \ldots a^{k-1}l^k \not\models \varphi \\
 & \quad \text{or } l^0a^0l^1 \ldots a^{k-1}l^k \models \psi \\
l^0a^0l^1 \ldots a^{k-1}l^k \models \bar{\circ}\varphi & \text{iff } k \geq 1 \text{ and } l^0a^0l^1 \ldots a^{k-2}l^{k-1} \models \varphi \\
l^0a^0l^1 \ldots a^{k-1}l^k \models (\varphi \mathsf{S} \psi) & \text{iff there exists an } m \leq k \\
 & \quad \text{such that } l^0a^0l^1 \ldots a^{m-1}l^m \models \psi \\
 & \quad \text{and } l^0a^0l^1 \ldots a^{j-1}l^j \models \varphi \text{ for } j = m+1, \ldots, k
\end{array}$$

We denote the dual of D by P and use $\bar{\Diamond}$, $\bar{\Box}$ and I as abbreviations in the usual way:

$$\mathsf{P}_\Gamma \psi \rightleftharpoons \neg \mathsf{D}_\Gamma \neg \psi, \qquad \bar{\Diamond}\psi \rightleftharpoons (\top \mathsf{S} \psi), \qquad \bar{\Box}\psi \rightleftharpoons \neg \bar{\Diamond} \neg \psi, \qquad \mathsf{I} \rightleftharpoons \neg \bar{\circ} \top.$$

This completes the definition of our variant of ATL, which we denote by ATL^P_{D}.

The definition of D_Γ affects the way local state contributes to the agents' knowledge. Instead of an explicit encoding of *the entire memory* of agent i as originally proposed [HFMV95], l_i becomes just the projection of the global state which is *visible* to i. Now agents' understanding of the overall structure of the given interpreted system, including their knowledge of its set of initial states I and the effect of actions as described by the functions t_i are involved in calculating the global histories r' which are indistinguishable from the actual one, and $\mathsf{D}_\Gamma\psi$ holds if ψ holds at all these runs. This means that, e.g., $\langle\langle\Gamma\rangle\rangle^{\mathsf{D}}\circ(\mathsf{D}_i\psi\vee\mathsf{D}_i\neg\psi)$ is an expression for *Γ can enforce a transition after which i's local history is sufficient to determine whether the global history satisfies ψ or not.* Facts about the past include (possibly missed) opportunities to enforce certain properties of behaviours; such facts are expressible by writing $\langle\langle\Gamma\rangle\rangle^{\mathsf{D}}$-formulas in the scope of past modalities. Our model-checking algorithm below involves restoring to local state the role of explicitly storing all the information *which is relevant to a fixed set of epistemic goals about the past.*

3 Encoding knowledge of the past in the local state

In this section we show how, given a finite interpreted system

$$IS = \langle\langle L_i, Act_i, P_i, t_i\rangle : i \in \Sigma \cup \{e\}, I, h\rangle$$

and a finite set Φ^Γ of past LTL formulas for each coalition Γ, one can construct a corresponding (bigger) finite interpreted system $IS_{\langle\Phi^\Gamma:\Gamma\subseteq\Sigma\rangle}$ with its states encoding whatever knowledge Γ can extract on the satisfaction of the formulas from Φ_Γ by observing the evolution of its local state in IS. The transitions of $IS_{\langle\Phi^\Gamma:\Gamma\subseteq\Sigma\rangle}$ correspond to the transitions of IS, but connect states of $IS_{\langle\Phi^i:i\in\Sigma\rangle}$ with appropriately related accounts on the satisfaction of the formulas from Φ^Γ for each coalition Γ in them.

To encode knowledge of the past in the local state we use a guarded normal form for the past formulas from the sets Φ_Γ. A finite set of formulas A is said to be a *full system*, if the formulas from A are pairwise inconsistent and their disjunction $\bigvee A$ is valid.

Lemma 1. *Let π be a past LTL formula. Then there exists a finite set of formulas Φ_π of the form*

$$\theta \wedge \mathsf{I} \vee \bigvee_i \alpha_i \wedge \bar{\circ}\beta_i$$

where θ and the α_is are purely propositional, and the α_is form a full system, such that

- *π has an equivalent in Φ_π;*
- *if $\theta \wedge \mathsf{I} \vee \bigvee\limits_i \alpha_i \wedge \bar{\circ}\beta_i \in \Phi_\pi$, then all the β_is have equivalents in Φ_π.*

Proof. An induction on the construction of π shows that it has an equivalent of the above form with the *modal height* of the β_is in it being no greater than that of π itself. The latter implies that a closure of $\{\pi\}$ under taking the β_is from guarded forms would contain only finitely many pairwise non-equivalent formulas.

Note that no syntactical restriction is imposed on the β_is in the normal form above. For example, one normal form for $(p\mathsf{S}q)$ is

$$q \wedge \mathsf{I} \vee q \wedge \bar{\circ}\top \vee (p \wedge \neg q) \wedge \bar{\circ}(p\mathsf{S}q) \vee (\neg p \wedge \neg q) \wedge \bar{\circ}\bot$$

and $\Phi_{(p\mathsf{S}q)}$ can be chosen to consist of the latter formula and the formulas $\bot \wedge \mathsf{I} \vee \top \wedge \bar{\circ}\bot$ and $\top \wedge \mathsf{I} \vee \top \wedge \bar{\circ}\top$, which are normal forms for $\bot$ and $\top$, respectively.

A more accurate estimate of the size of Φ_π can be obtained by taking a deterministic finite state machine $\langle Q, q_0, \delta, F\rangle$ which recognizes the language $\{r^k \dots r^0 \in L^+ : r^0 \dots r^k \models \pi\}$ and taking Φ_π to consist of the formulas π_q which define in the same way the languages accepted by the finite state machines $\langle Q, q, \delta, F\rangle$ for each $q \in Q$.

In the sequel we assume a fixed Φ_π for every given formula π and, without loss of generality, we assume that $\Phi^\Gamma = \bigcup_{\pi\in\Phi^\Gamma} \Phi_\pi$ for each $\Gamma \subseteq \Sigma$.

Next we show that at each step of the evolution of IS the knowledge of coalition Γ on the satisfaction of the formulas from $\Phi = \Phi^\Gamma$ can be represented as a collection of facts of the form:

> *The current global state of IS is one of the states in X and, for each $l \in X$, if the global state of IS is actually l, then the past satisfies the formulas from Φ which are in Ψ_l.*

That is, the knowledge of Γ can be encoded as the tuple $\langle X, \Psi_l : l \in X\rangle$, where $X \subseteq L$ and $\Psi_l \subseteq \Phi$ for every $l \in X$.

Consider a local history $v^0b^0v^1 \dots v^{m-1}b^{m-1}v_m$. The possible corresponding global histories $l^0a^0l^1 \dots l^{m-1}a^{m-1}l^m$ satisfy the conditions

$$l^k_\Gamma = v^k \text{ for } k = 0, \dots, m, \text{ and } a^k_\Gamma = b^k \text{ for } k = 0, \dots, m-1.$$

The initial state of the global history can be any of the states from $X^0 = \{l \in I : l_\Gamma = v^0\}$, and if the actual initial state is l, then Ψ^0_l consists of those $\pi \in \Phi$ which have a disjunctive member $\theta \wedge \mathsf{I}$ such that $l \models \theta$. Given $v = v^0$, in the sequel we denote the corresponding $H_0 = \langle X^0, \Psi^0_l : l \in X\rangle$ defined above by $I_{\Gamma,\Phi}(v)$. Let $k < m$ and the knowledge of Γ on Φ corresponding to $v^0b^0v^1 \dots v^{k-1}b^{k-1}v^k$ be $H_k = \langle X^k, \Psi^k_l : l \in X^k\rangle$. Then Γ's knowledge $H_{k+1} = \langle X^{k+1}, \Psi^{k+1}_{l'} : l' \in X^{k+1}\rangle$ at $v^0b^0v^1 \dots v^kb^kv^{k+1}$ can be derived as follows. The set X^{k+1} of the possible global states is

$$\{l' \in L : l'_\Gamma = v^{k+1}, (\exists l \in X^k)(\exists a \in \prod_{i\in\Sigma\cup\{e\}} P_i(l_i))(a_\Gamma = b^k \text{ and } l' = t(l, a_1, \dots, a_n, a_e)\}$$

To determine $\Psi_{l'}^{k+1}$, $l' \in X^{k+1}$, observe that $\theta \wedge \mathsf{I} \vee \bigvee_s \alpha_s \wedge \bar{\circ}\beta_s \in \Psi_{l'}^{k+1}$ for $l' \in X^{k+1}$ iff $\beta_s \in \Psi_l$ for the only s such that $l' \models \alpha_s$ and all $l \in X^k$ such that $l' = t(l, a_1, \dots, a_n, a_e)$ for some $a \in \prod\limits_{i \in \Sigma \cup \{e\}} P_i(l_i)$ such that $a_\Gamma = b^k$.

In the sequel, given $H = H_k$, $v = v_k$ and $b = b_k$, we denote H_{k+1} by $T_{\Gamma,\Phi}(H, b, v)$. Given that the current knowledge of Γ on the satisfaction of the formulas from Φ is encoded by H, $T_{\Gamma,\Phi}(H, b, v)$ encodes Γ's knowledge on the satisfaction of the formulas from Φ after a transition by action b which leads to local state v for Γ. Since $X \subseteq \{l \in L : l_\Gamma = v\}$, the local state v can always be determined from $H = \langle X, \Psi_l : l \in X\rangle$. Given H, we denote the corresponding local state by $v_\Gamma(H)$. Now we are ready to define

$$IS_{\langle \Phi^\Gamma : \Gamma \subseteq \Sigma\rangle} = \langle \tilde{L}_\Gamma, \tilde{Act}_\Gamma, \tilde{P}_\Gamma, \tilde{t}_\Gamma : \Gamma \in 2^\Sigma \cup \{e\}, \tilde{I}, \tilde{h}\rangle,$$

in which each coalition from $\Gamma \subseteq \Sigma$ is represented as an agent. We put:

$\tilde{L}_e = L_e$, $\tilde{L}_\Gamma = \{\langle X, \Psi_l : l \in X\rangle : X \subseteq \{l \in L : l_\Gamma = v\}$ for some $v \in L_\Gamma, \Psi_l \subseteq \Phi^\Gamma$ for each $l \in X\}$;

$\tilde{Act}_{\{i\}} = Act_i$, $\tilde{P}_{\{i\}}(H) = P_i(v(H))$ for singleton coalitions $\{i\}$; $\tilde{Act}_\Gamma = \{*\}$, $\tilde{P}_\Gamma(H) = \{*\}$ for non-singleton coalitions Γ;

$\tilde{t}_\Gamma(H, l_e, a_1, \dots, a_n, a_e) = T_{\Phi^\Gamma,\Gamma}(H, a_\Gamma, \langle t_i(v_i(H), l_e, a_1, \dots, a_n, a_e) : i \in \Gamma\rangle)$;

$\tilde{I} = \{\langle I_{\Gamma,\Phi^\Gamma}(l_\Gamma) : \Gamma \subseteq \Sigma, l_e\rangle : l \in I\}$;

The set of atomic propositions $\tilde{AP}$ for $IS_{\langle \Phi^i : \Gamma \subseteq \Sigma\rangle}$ extends AP by the fresh propositions $p_{\mathsf{D}_\Gamma \pi}$, $\pi \in \Phi^\Gamma$, $\Gamma \subseteq \Sigma$. For $p \in AP$, $\tilde{h}$ is defined by the equality

$$\tilde{h}(p) = \{\langle H_\Gamma : \Gamma \subseteq \Sigma, l_e\rangle : \langle v_{\{1\}}(H_{\{1\}}), \dots, v_{\{n\}}(H_{\{n\}}), l_e\rangle \in h(p)\}.$$

For the new propositions we put

$$\tilde{h}(p_{\mathsf{D}_\Gamma \varphi}) = \{\langle\langle X^\Gamma, \Psi_l^\Gamma : l \in X^\Gamma\rangle : \Gamma \subseteq \Sigma, l_e\rangle : \varphi \in \Psi_l^\Gamma \text{ for all } l \in X^\Gamma\}.$$

According to this definition, only agents in $IS_{\langle \Phi^\Gamma : \Gamma \subseteq \Sigma\rangle}$ who correspond to singleton coalitions in IS have proper choice of actions, which is the same as that of the respective individual agents in IS; all other coalitions have just singleton action sets, but have the combined ability of observation of their member agents.

Proposition 1. *Let $r = l^0 a^0 l^1 \dots l^{k-1} a^{k-1} l^k \dots$ be a run of IS. Let*

$$\tilde{l}^0 \tilde{a}^0 \dots \tilde{l}^{k-1} \tilde{a}^{k-1} \tilde{l}^k \dots$$

be a run of $IS_{\langle \Phi^\Gamma : \Gamma \subseteq \Sigma\rangle}$ with the actions of the singleton coalitions $\{i\}$, $i \in \Sigma$ in $\tilde{a}^k$ being those from a^k and let

$$\tilde{l}^0 = \langle I_{\Gamma,\Phi^\Gamma}(l_\Gamma^0) : \Gamma \in \Sigma, l_e^0\rangle.$$

Then for all $k < \omega$ we have

$$l^k = \langle v_{\{i\},\Phi^{\{i\}}}(\tilde{l}^k) : i \in \Sigma, \tilde{l}_e^k\rangle$$

and

$$l^0 a^0 l^1 \dots l^{k-1} a^{k-1} l^k \models \mathsf{D}_\Gamma \pi \text{ iff } \tilde{l}^0 \tilde{a}^0 \dots \tilde{l}^{k-1} \tilde{a}^{k-1} \tilde{l}^k \models p_{\mathsf{D}_\Gamma \pi}.$$

This can be established by a straightforward argument using induction on j. The proposition holds with $\tilde{L}_{\{i\}}$ being just L_i and $\tilde{t}_i$ being defined as t_i on the appropriate arguments in case $\Phi^{\{i\}}$ is empty; agents Γ can be omitted altogether for non-singleton Γ with empty Φ^Γ.

Corollary 1. *Let φ be an ATL^P_{D} formula written in the vocabulary $\tilde{AP}$, then IS satisfies the result $[\mathsf{D}_\Gamma\pi/p_{\mathsf{D}_\Gamma\pi} : \pi \in \Phi^\Gamma, \Gamma \subseteq \Sigma]\varphi$ of substituting the propositional variables $p_{\mathsf{D}_\Gamma\pi}$ by the respective formulas $\mathsf{D}_\Gamma\pi$ in φ iff $IS_{\langle \Phi^\Gamma : \Gamma\subseteq\Sigma\rangle}$ satisfies φ itself.*

Remark 1. The crucial property of LTL which enables the technique from this section is Lemma 1. Similar statements apply to quantified propositional LTL, the (linear time) modal μ-calculus [Koz83], regular expressions, propositional interval-temporal logic [Mos85], etc. All these systems have the expressive power of (weak) monadic second order logic on the natural numbers, which is greater than that of just LTL's past modalities, and can be used to define the formulas allowed in the scope of D without any substantial change to the model-checking algorithm described in this paper.

4 Some properties of the ATL^P_{D} cooperation modalities

The previous section describes a method for the elimination of D-formulas by replacing them with dedicated propositional variables in appropriately extended interpreted systems. In this section we describe a similar method for formulas built using the cooperation modalities. As it becomes clear in the next section, this allows our model-checking algorithm to work bottom-up by replacing modal formulas with dedicated propositional variables and moving to corresponding extensions of the given interpreted system. Only formulas with cooperation modality-free and D-free arguments need to be considered at each step. In this section we establish the properties of the ATL^P_{D} cooperation modalities which we need for the handling of modal formulas built with them.

Let us fix an interpreted system IS with its components named as previously.

Proposition 2. *Let φ and ψ be boolean combinations of propositional variables. Let $r \in R(IS)$ and $X = \{l \in L : (\exists r' \in R(IS))r' \sim_\Gamma r$ and $r'_{|r|} = l\}$. Then*

$$r \models \langle\langle\Gamma\rangle\rangle^{\mathsf{D}}(\varphi \mathsf{U} \psi) \text{ is equivalent to } l \models \langle\langle\Gamma\rangle\rangle^{\mathsf{D}}\Diamond\mathsf{D}_\Gamma\overline{\Diamond}(\psi \wedge \neg\overline{\circ}\neg\overline{\Box}\varphi)$$

for all the 0-length histories l consisting of an initial state $l \in X$ of the interpreted system $IS_X = \langle\langle L_i, Act_i, P_i, t_i\rangle : i \in \Sigma\cup\{e\}, X, h\rangle$. Similarly

$$r \models [\![\Gamma]\!]^{\mathsf{D}}(\varphi \mathsf{U} \psi) \text{ is equivalent to } l \models [\![\Gamma]\!]^{\mathsf{D}}\Diamond\mathsf{P}_\Gamma\overline{\Diamond}(\psi \wedge \neg\overline{\circ}\neg\overline{\Box}\varphi)$$

for all $l \in X$ in IS_X.

Informally, this means that a strategy which enforces $(\varphi\mathsf{U}\psi)$ also enables the coalition to eventually learn that $(\varphi\mathsf{U}\psi)$ was enforced. Learning this can as well be postponed by some steps from the time point at which ψ first happens to be satisfied, and it is indeed possible that the coalition "overlook" several events of satisfying ψ, before being able to deduce that such an event took place.

Proof. The backward implication is obvious. For the forward implication, let s be a strategy for Γ which enforces $(\varphi \mathsf{U} \psi)$ starting from all $r' \in [r]_{\sim_\Gamma}$ in IS. Let r_Γ be the local history for Γ which corresponds to r. Let s' be a strategy for Γ which is defined by the equality $s'(v^0 b^0 v^1 \dots v^k) = s(r_\Gamma \cdot b^0 v^1 \dots v^k)$ for all local histories $v^0 b^0 v^1 \dots v^k$ in IS_X. Note that, by the definition of X, all the local histories in IS_X start at the last local state of r_Γ. Assume that $l \not\models \langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Diamond \mathsf{D}_\Gamma \overline{\Diamond}(\psi \wedge \neg\overline{\circ}\neg\overline{\Box}\varphi)$ for some $l \in X$ for the sake of contradiction. Then this formula is satisfied for no $l \in X$, since, by the definition of X, the local states l_Γ are the same for all $l \in X$ and, consequently, the 0-length histories $l \in R(IS_X, 0)$ are all indistinguishable to Γ. This means that there are arbitrarily big numbers m and $r = l^0 a^0 \dots l^k a^k \dots \in \mathrm{out}(l, s')$ such that $l^0 a^0 \dots a^{m-1} l^m \not\models \mathsf{D}_\Gamma \overline{\Diamond}(\psi \wedge \neg\overline{\circ}\neg\overline{\Box}\varphi)$. By the definition of D_Γ, for such m and r there exist histories $r' \in IR(IS_X, m)$ such that $r' \sim_\Gamma l^0 a^0 \dots a^{m-1} l^m$ and $r' \not\models \overline{\Diamond}(\psi \wedge \neg\overline{\circ}\neg\overline{\Box}\varphi)$. Since s' is observation-based, the r's with the above property are in prefixes of infinite runs in $\mathrm{out}(l', s')$ for some $l' \in X$ as well. Since the finite prefixes of the behaviours from $\mathrm{out}(l', s')$ form trees of finite width, König's Lemma entails that for some $l' \in X$ there is an infinite run $l^{\infty,0} a^{\infty,0} \dots l^{\infty,k} a^{\infty,k} \dots \in \mathrm{out}(l', s')$ such that $l^{\infty,0} a^{\infty,0} \dots a^{\infty,m-1} l^{\infty,m} \not\models \overline{\Diamond}(\psi \wedge \neg\overline{\circ}\neg\overline{\Box}\varphi)$ for all m. By the definition of s', $r' \cdot a^{\infty,0} l^{\infty,1} \dots l^{\infty,k} a^{\infty,k} \dots \in out(r', s)$ for some $r' \in [r]_{\sim_\Gamma r}$ such that r' has l' as its last state. Hence s does not enforce $(\varphi \mathsf{U} \psi)$ starting from r', which is a contradiction. The proof about $[\![\Gamma]\!]^{\mathsf{D}}(.\mathsf{U}.)$-formulas is similar.

Similar statements apply to formulas built using $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \circ$ and $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Box$:

Proposition 3. *The following equivalences are valid in* ATL^P_{D}*:*

$$\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \circ \varphi \Leftrightarrow \langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \circ \mathsf{D}_\Gamma \varphi \quad (1)$$

$$\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Box \varphi \Leftrightarrow \langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Box \mathsf{D}_\Gamma \varphi \quad (2)$$

The proofs are similar, though simpler, because $\Box$ and $\circ$ express safety properties and therefore it is not necessary to use König's Lemma. Cutting of the proper past part of the satisfying behaviour, which is done by moving from IS to IS_X, and is required in the case of $(.\mathsf{U}.)$ because of the need to satisfy it from the end of that behaviour on, is not needed either. This makes it possible to express the connection between the satisfaction of $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \circ \varphi$, $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Box \varphi$ and the corresponding formulas with D_Γ applied to their designated arguments as a straightforward equivalence.

5 Model-checking ATL^P_{D}

Now we are ready to describe a model-checking algorithm for ATL^P_{D}. The algorithm works by a series of transformations of the given interpreted system. The transformations have the forms described in Section 3 and Proposition 2. The number of transformations is equal to the *modal depth* $d(\varphi)$ of the given formula

φ, which is defined by the clauses

$$\begin{array}{l} d(\bot) = d(p) = 0; \\ d(\varphi \Rightarrow \psi) = d((\varphi \mathsf{S} \psi)) = \max\{d(\varphi), d(\psi)\}; \\ d(\bar{\circ}\varphi) = d(\varphi); \\ d(\mathsf{D}_\Gamma \varphi) = d(\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \circ \varphi) = d(\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Box \varphi) = d(\varphi) + 1; \\ d(\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} (\varphi \mathsf{U} \psi)) = d([\![\Gamma]\!]^{\mathsf{D}} (\varphi \mathsf{U} \psi)) = \max\{d(\varphi), d(\psi)\} + 1. \end{array}$$

Note that the past modalities $\bar{\circ}$ and $(.\mathsf{S}.)$ have no effect on d; the reasons for this become clear below.

Unless the given formula φ is modality-free, which renders the model-checking problem trivial, φ has either

(i) a subformula $\mathsf{D}_\Gamma \psi$ with no occurrences of the cooperation modalities, or

(ii) a subformula of one of the forms $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \circ \varphi$, $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} (\varphi \mathsf{U} \psi)$, $[\![\Gamma]\!]^{\mathsf{D}} (\varphi \mathsf{U} \psi)$ and $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Box \varphi$ in which φ and ψ are modality-free.

Despite that $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Box \varphi$ is just an abbreviation for $\neg [\![\Gamma]\!]^{\mathsf{D}} \Diamond \neg \varphi$, we consider it separately in this case distinction because, as it becomes clear below, it can be handled more efficiently. The better efficiency justifies treating $[\![\Gamma]\!]^{\mathsf{D}} \Diamond \varphi$ as its equivalent $\neg \langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Box \neg \varphi$ as well.

Assume that (i) holds. Let

$$\Phi^\Gamma = \{\psi : \mathsf{D}_\Gamma \psi \in \mathrm{Subf}(\varphi) \text{ and } \psi \text{ is cooperation modality-free}\}$$

for every $\Gamma \subseteq \Sigma$. Then φ can be written as $[\mathsf{D}_\Gamma \psi / p_{\mathsf{D}_\Gamma \psi} : \psi \in \Phi^\Gamma, \Gamma \subseteq \Sigma]\varphi'$ for some appropriate φ' which has no D-subformulas without a cooperation modality in their scope. Then by Corollary 1 IS satisfies φ iff $IS_{\langle \Phi^\Gamma : \Gamma \subseteq \Sigma \rangle}$, which is defined as in Section 3, satisfies φ'. Clearly $d(\varphi') = d(\varphi) - 1$.

Now assume that (ii) holds. For the sake of simplicity, we assume that there is just one subformula of the considered form. If there are more, then the transformations below can be done for all of them simultaneously.

Let the subformula in question be $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} (\varphi \mathsf{U} \psi)$. Note that, since φ and ψ make no reference to the past, the satisfaction of $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} (\varphi \mathsf{U} \psi)$ depends just on the set of states which are the ends of histories r' that Γ cannot distinguish from the actual reference history r. Consider $IS' = IS_{\langle \Phi^\Delta : \Delta \subseteq \Sigma \rangle}$ where $\Phi^\Delta = \emptyset$ for all $\Delta \subseteq \Sigma$. Moving from IS to IS' with Φ^Δs is equivalent to a subset construction for IS. The states of IS' are the sets of indistinguishable states for all the possible coalitions as the states of the new system. The satisfiability of $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} (\varphi \mathsf{U} \psi)$ is preserved in IS' according to Corollary 1. Moreover, we can define $l \models \langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} (\varphi \mathsf{U} \psi)$ for every individual global state l of IS', and, if $l = \langle \langle X_\Delta \rangle : \Delta \in 2^\Sigma \cup \{e\} \rangle$, that would be equivalent to the existence of a strategy for Γ to enforce $(\varphi \mathsf{U} \psi)$ for all the states of IS in $X = X_\Gamma$. According to Proposition 2, this is equivalent to the satisfaction of $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Diamond \mathsf{D}_\Gamma \overline{\Diamond} (\psi \wedge \neg \bar{\circ} \neg \overline{\Box} \varphi)$ at the system IS'_Y which is obtained by replacing the set of the initial states of IS' with the set $Y = \{ \langle \langle X_\Delta \rangle : \Delta \in 2^\Sigma \cup \{e\} \rangle : X_\Gamma = X \}$, which consists of the states of IS' which are the ends of 0-length histories in IS' that Γ cannot tell apart from the 0-length history l. To model-check $\langle\!\langle \Gamma \rangle\!\rangle^{\mathsf{D}} \Diamond \mathsf{D}_\Gamma \overline{\Diamond} (\psi \wedge \neg \bar{\circ} \neg \overline{\Box} \varphi)$ in IS'_Y, we construct $IS'' = (IS'_Y)_{\langle \Psi^\Delta : \Delta \subseteq \Sigma \rangle}$ where $\Psi^\Delta = \emptyset$ for $\Delta \neq \Gamma$ again, and $\Psi^\Gamma = \{\overline{\Diamond}(\psi \wedge \neg \overline{\circ} \neg \overline{\Box} \varphi)\}$. According to Corollary 1, what remains to be done is to model-check $\langle\langle \Gamma \rangle\rangle^{\mathsf{D}} \Diamond p_{\mathsf{D}_\Gamma \overline{\Diamond}(\psi \wedge \neg \overline{\circ} \neg \overline{\Box} \varphi)}$ in IS'', which can be done by calculating the appropriate fixpoint just like for $\langle\langle \Gamma \rangle\rangle \Diamond p_{\mathsf{D}_\Gamma \overline{\Diamond}(\psi \wedge \neg \overline{\circ} \neg \overline{\Box} \varphi)}$ with respect to the standard semantics of $\langle\langle \Gamma \rangle\rangle(.\mathsf{U}.)$ in *ATL*.

The steps for formulas of the forms $\langle\langle \Gamma \rangle\rangle^{\mathsf{D}} \circ \varphi$ and $\langle\langle \Gamma \rangle\rangle^{\mathsf{D}} \Box \varphi$ are similar, with the role of Proposition 2 being played by the equivalences (1) and (2), respectively. Since there is no need to replace the set of the initial states as done upon moving from IS' to IS'_Y in the case of $\langle\langle \Gamma \rangle\rangle^{\mathsf{D}}(.\mathsf{U}.)$, a single extension of the form from Proposition 1 with Φ^Γ being $\{\varphi\}$ is sufficient.

If the formula in question is $[\![\Gamma]\!]^{\mathsf{D}}(\varphi \mathsf{U} \psi)$, then we use the equivalence between $[\![\Gamma]\!]^{\mathsf{D}} \Diamond \mathsf{P}_\Gamma \overline{\Diamond}(\psi \wedge \neg \overline{\circ} \neg \overline{\Box} \varphi)$ and $\neg \langle\langle \Gamma \rangle\rangle^{\mathsf{D}} \Box \mathsf{D}_\Gamma \overline{\Box} \neg (\psi \wedge \neg \overline{\circ} \neg \overline{\Box} \varphi)$ and solve the case by model-checking the latter formula at IS'_Y, which is defined as in the case of $\langle\langle \Gamma \rangle\rangle^{\mathsf{D}}(\varphi \mathsf{U} \psi)$.

This concludes the description of our model-checking procedure for ATL^P_{D}.

Conclusion

We have proposed a system of *ATL* with imperfect information, perfect recall, an epistemic modality for past *LTL* formulas and cooperation modalities which are interpreted over strategies that are uniform with respect to the distributed knowledge of the respective coalition on the past. The proposed system has decidable model-checking problem and can be used to specify goals which combine enforcing conditions on the future behaviour of the given system with the acquisition of knowledge or the prevention of acquisition of knowlegde on its past behaviour. Our model-checking algorithm exploits the interaction between the epistemic modality and the cooperation modalities in order to encode all strategy goals as epistemic goals and works by transforming the model-checked system in a way which allows the relevant knowledge of the past to be encoded in the local states of the respective agents and coalitions and thus eliminate the explicit occurrences of the epistemic modality from the model-checked formula.

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