Message-Passing Algorithms in Markov Chains

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The objective of this paper is to apply message-passing algorithms (e.g., belief propagation, sum-product algorithm) for investigating some properties of Markov chains. The message-passing algorithms are tools used for reasoning in the field of probabilistic graphical models (e.g., Bayesian Networks, factor graphs, etc.). Historically, Markov chains are not consider as a member of the family of probabilistic graphical models. Nevertheless, we argue that by applying tools from this field one can obtain some interesting results and conclusions related to the theory of Markov chains.
In the current paper, we focus our attention on the two following issues: a) an investigation of the passage time from a given initial state into a non-empty set of target states; b) an analysis of the number of visits to a given set of states. In the second point, the target set can be defined dynamically, i.e., it changes with time. This is a new problem which has not been formulated before. Traditionally, such issues are considered in the context of the ergodic Markov chains, thus, one is interested in the asymptotic behavior of the chain when time goes to infinity. In contrary, we propose to consider a finite time-window. Hence, we deal with a finite vector of random variables $(X_1, ..., X_N)$ instead of an infinite chain of variables $X_1, X_2, ...$. Since in most of the practical applications the time-horizon is finite, this assumption is not strong. Moreover, the assumption of the ergodicity is not necessary for applying the proposed algorithms. The first passage time and the number of visits are random variables. We provide new algorithm for computing K first probabilistic moments of these variables. However, the most advanced result is a method for discovering probability distributions of these variables.
The novelty of the proposed work is threefold. Firstly, the methodology is new. Best to our knowledge, the message-passing algorithms have not been used in such a context before. Secondly, the computational complexity of our algorithms is often lower (and never higher) than the complexity of other existing solutions. Thus, our method can be applicable to Markov chains with a large state space. Finally, the algorithms based on the message-passing paradigm can be easily implemented. Therefore, we argue that our work may have an impact on the practical applications of Markov chains.