First Instances of Euler Beta-function B-splines and Simplicial Finite Elements on Triangulations

L. T. Dechevsky, B. Bang, A. R. Kristoffersen,A. Lakså, P. Zanaty

In the univariate case, the derivative of an expo-rational B-spline (ERBS) between the consecutive knots t_k, $t_{k+1} : t_k < t_{k+1}$ of a strictly increasing knot-vector is either identically zero or it is an expo-rational function (i.e., a function which is the exponent of a rational function taking negative values for $t : t_k < t < t_{k+1}$ and having poles at t_k and t_{k+1}). The computation of the integral in the definition of ERBS is by fast-converging numerical quadratures. Euler Beta-function B-splines (BFBS) are an instance of generalized ERBS (GERBS) where some tradeoff has been made between the properties and the ease of computation. In the case of BFBS, the expo-rational bell-shaped function in the definition of the derivative of ERBS is replaced by a Bernstein polynomial rescaled to the interval $[t_k, t_{k+1}]$. While multiplying ERBS with a Taylor series in powers of $t - t_k$ of an analytic function makes sense and has the effect of transfinite Hermite interpolation, the same can be done with the BFBS only with a Taylor polynomial of degree not exceeding the multiplicity of t_k as a zero of the respective Bernstein polynomial. What is lost in the range of this important property is compensated in the ease of computation of the BFBS which is piecewise polynomial. This advantage considerably increases in the multivariate case, and especially on triangulated domains, since BFBS continues to be piecewise polynomial on every triangle of its support.

The purpose of this communication is to provide first instances of smooth BFBS associated with the vertices of a triangulation and supported (like the respective ERBS) on the star-1 neighbourhood of 'its'vertex (the same support as that of the respective piecewise linear/affine B-spline), as well as first instances of the corresponding BFBS simplicial finite elements associated the triangles in the triangulation, with Hermite interpolation on their boundary.