Explicit Runge–Kutta Schemes and Finite Elements with Symmetric Stabilization for First-Order Linear PDE Systems

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We analyze explicit Runge–Kutta schemes in time combined with stabilized finite elements in space to approximate evolution problems with a first-order linear differential operator in space of Friedrichs-type. For the time discretization, we consider explicit second- and third-order Runge–Kutta schemes. We identify a general set of properties on the space stabilization, encompassing continuous and discontinuous finite elements, under which we prove stability estimates using energy arguments. Then, we establish L^2-norm error estimates with quasi-optimal convergence rates for smooth solutions in space and time. These results hold under the usual CFL condition for third-order Runge–Kutta schemes and any polynomial degree in space and for second-order Runge–Kutta schemes and first-order polynomials in space. For second-order Runge–Kutta schemes and higher polynomial degrees in space, a tightened 4/3-CFL condition is required. Numerical results are presented for smooth and rough solutions. The case of finite volumes is briefly discussed.