Numerical Upscaling of Flows in Highly Heterogeneous Porous Media

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The generalized Stokes equations (called also Brinkman equations),

$$-\mu \Delta u + \nabla p + \mu \kappa^{-1} u = f, \ \nabla \cdot u = 0 \ \text{ in } \Omega,$$

where μ is the viscosity and κ is the permeability, are used for modeling flows in highly porous media. Examples of such media are industrial open foams, filters, and insulation materials.Motivated by industrial applications of such materials we have developed a numerical method for comuting flows in heterogeneous highly porous media with complicated internal structure of the permeability.
We will present a two-scale finite element approximation of Brinkman equations. The method uses two main ingredients: (I) discontinuous Galerkin finite element method for Stokes equations, proposed and studied by J. Wang and X. Ye (2007, SINUM, v. 45) and (II) subgrid approximation developed by T. Arbogast for Darcy equations (2004, SINUM, v. 42).
There are two different applications of the proposed method: (1) numerical upscaling of Brinkman equations on coarse-grid that incorporates fine-grid features, and (2) an alternating Schwarz iteration that uses the coarse-grid in domain decomposition setting. In order to reduce coarse-grid boundary layer errors and to ensure convergence to the global fine solution, the algorithm uses overlapping subdomains around the coarse-grid interfaces. A number of numerical examples will be presented to demonstrate the performance of both the subgrid method and the iterative procedure.