Numerical Investigation of Self-Similar Solutions of a Reaction Diffusion Equation in a Vicinity of Critical Parameters

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The self-similar problem considered is a boundary value problem for nonlinear elliptic equation. The problem has not unique solution. We focus our study on the numerical computing of the so called "spiral wave solutions".
The proposed approach is based on the continuous analog of the Newton's method and finite element methods. To reveal solutions of spiral structure appropriate initial approximations are used. The last ones are solutions of a linearized equation and are expressed by the confluent hypergeometric function ${}_1F_1(a, b; z)$. So one needs an accurate, fast and reliable computation of its values for different parameter regimes within the complex plane for the parameters a and b, as well as for different regimes of the variable z. In order to achieve this we worked out algorithms based on various methods and appropriate for different parameter ranges: Taylor expansions, asymptotic series computations, an expansion in ascending series of Chebyshev polynomials.
Another crucial point of the numerical realization is to derive a suitable boundary condition. It should be obtained from the asymptotic of the nonlinear equation. To do that one needs the asymptotic of the solution of the linearized equation, i.e. of the confluent hypergeometric function.
A detailed numerical analysis of the evolution of the "spiral wave solutions" for various medium parameters including critical values is carried out. The accuracy of this method is experimentally analyzed using embedded grids.