New Conjectures in the Hausdorff Geometry of Polynomials

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Let $D(C(p); R(p))$ be the smallest disk containing all zeros of the polynomial $p(z) = (z - z_1)(z - z_2) \cdots (z - z_n)$. Half a century ago, we conjectured that for every zero z_k of $p(z)$, the disk $D(z_k; R(p))$ contains at least one zero of the derivative $p'(z)$. In this paper a stronger conjecture is announced and proved for polynomials of degree $n = 3$. A number of other conjectures are announced, including a variation of the Smale's mean value conjecture.