SM Stability for Time-dependent Problems

P. N. Vabishchevich

When time-dependent problems of mathematical physics are solved numerically, much emphasis is placed on computational algorithms of higher orders of accuracy. Along with improving the approximation accuracy with respect to space, improving the approximation accuracy with respect to time is also of interest. In this respect, the results concerning the numerical methods for ordinary differential equations (ODEs) provide an example. Taking into account the specific features of time-dependent problems for PDEs, we are interested in numerical methods for solving the Cauchy problem in the case of stiff equations.

When time-dependent problems are solved approximately, the accuracy can be improved in various ways. In the case of two-level schemes (the solution at two adjacent time levels is involved), polynomial approximations of the schemes operators on the solutions to the equation are used explicitly or implicitly. The most popular representatives of such schemes are RungeKutta methods, which are widely used in modern computations. The main feature of the multilevel schemes (multistep methods) manifests itself in the approximation of time derivatives with a higher accuracy on a multipoint stencil. A characteristic example is provided by multistep methods based on backward numerical differentiation.

Various classes of stable finite difference schemes can be constructed to obtain a numerical solution. It is important to select among all the stable schemes a scheme that is optimal in terms of certain additional criteria. In this study, we use a simple boundary value problem for a one-dimensional parabolic equation to discuss the selection of an approximation with respect to time. We consider the pure diffusion equation, the pure transport equation and combined convection-diffusion. Requirements for the unconditionally stable finite difference schemes are formulated that are related to retaining the main features of the differential problem. The concept of SM stable finite difference scheme is introduced. The starting point are difference schemes constructed on the basis of various Padé approximations.