A Multiscale Stochastic Framework for Stokes-Darcy Flow and Transport

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We discuss a multiscale stochastic framework for uncertainty quantification in modeling flow and transport in surface-subsurface hydrological systems. The governing flow equations are the Stokes-Darcy system with Beavers-Joseph-Saffman interface conditions. The permeability in the Darcy region is stochastic and it is represented with a Karhunen-Loève (KL) expansion. The porous media can be statistically non-stationary, which is modeled by different KL expansions in different regions. Statistical moments of the solution are computed via sparse grid stochastic collocation. The spatial domain is decomposed into a series of small subdomains (coarse grid) of either Stokes or Darcy type. The flow solution is resolved locally (on each coarse element) on a fine grid, allowing for non-matching grids across subdomain interfaces. The subdomain discretizations utilize stable Stokes or Darcy elements. Coarse scale mortar finite elements are introduced on the interfaces to approximate the normal stress and impose weakly continuity of flux. The transport equation is discretized via a local discontinuous Galerkin method. We precompute a multiscale basis, which involves solving subdomain problems with for each realization of the local KL expansion. The basis is then used to solve the coarse scale mortar interface problem in parallel for each global KL realization. The resulting algorithm is orders of magnitude faster than a global stochastic collocation approach. Error analysis for the statistical moments of the pressure, velocity, and tracer concentration is performed. Computational experiments are presented.