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MICROLOCAL PARAMETRIX FOR THE CAUCHY PROBLEM FOR HYPERBOLIC SYMMETRIC SYSTEMS WITH SYMPLECTIC MULTIPLE CHARACTERISTICS, II

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A hyperbolic system L with a principal symbol of a diagonal form and a determinant $(\tau-\lambda_1)^{n_1}(\tau-\lambda_2)^{n_2}$ is considered. A microlocal parametrix for L near a point where the characteristics are of variable multiplicity and $\{\tau-\lambda_1, \tau-\lambda_2\}\neq 0$ is constructed and its singularities are studied,

1. Introduction. In our previous paper [1] we have constructed a parametrix for the Cauchy problem for the $(n_1+n_2)\times(n_1+n_2)$ system of the form

$$P(x, t, D_x, D_t)=\begin{pmatrix}(D_t-tD_{x_1})I_{n_1} & 0\\ 0 & (D_t+tD_{x_1})I_{n_2}\end{pmatrix}+\begin{pmatrix}0 & B_{12}\\ B_{21} & 0\end{pmatrix}(x, t, D_x).$$

In the present paper we consider the following $(n_1+n_2)\times(n_1+n_2)$ system

$$L(x, t, D_x, D_t)=\begin{pmatrix}(D_t-\lambda_1(x,t\ D_x))I_{n_1} & 0\\ 0 & (D_t-\lambda_2(x, t, D_x))I_{n_2}\end{pmatrix}+A(x, t, D_x, D_t),$$

where $(x, t)=(x_1, \dots, x_n, t)$ denotes a point in R^{n+1}. Here $I_{n_\varkappa}$ is the identity $(n_\varkappa\times n_\varkappa)$ matrix, $\varkappa=1, 2$, $\lambda_1(x, t, D_x)$, $\lambda_2(x, t, D_x)$ are pseudo-differential operators (ψDO) in the tangential variables depending smoothly on t near $t=0$ with real and homogeneous of degree 1 in ξ symbols $\lambda_\varkappa(x, t, \xi)$. $A(x, t, D_x, D_t)$ is a classical ψDO of order 0. This will be denoted further as $A\in CL^0(\mathsf{R}^{n+1})$.

Let $(x^0, \xi^0)\in T^*(\mathsf{R}^n)\setminus 0$ be a fixed point. Assume that for t near 0 and (x, ξ) in a small conic neighbourhood of (x^0, ξ^0) we have $\lambda_\varkappa(x, t, \xi)=\lambda(x, \xi)+tr_\varkappa(x, t, \xi)$, $\varkappa=1, 2$, with $r_1(x^0, 0, \xi^0)\neq r_2(x^0, 0, \xi^0)$. Thus we obtain $\{\tau-\lambda_1, \tau-\lambda_2\}(x^0, 0, \xi^0, \tau^0)\neq 0$, where $\tau^0=\lambda(x^0, \xi^0)$ and $\{\ ,\ \}$ denotes the Poisson bracket.

Remark 1. In [2] Alinhac has considered the system

$$L(x, t, D_x, D_t)=\begin{pmatrix}D_t-\lambda_1(x, t, D_x) & \nu(x, t, D_x)\\ 0 & D_t-\lambda_2(x, t, D_x)\end{pmatrix}+A(x, t, D_x)$$

with $\nu(x, t, \xi)$ homogeneous of degree 1 in ξ and $\nu(x^\circ, 0, \xi^\circ)\neq 0$. This system has the same characteristic manifold but is not symmetric.

Let Γ be a closed cone in $T^*(\mathsf{R}^n)\setminus 0$. Denote by $\mathcal{D}'_\Gamma(\mathsf{R}^n)$ the space of distributions with wave front set contained in Γ. Our main result is the following:

Theorem 1. *There exists a conic neighbourhood Γ of (x^0, ξ^0) and an operator $E:\mathcal{D}'_\Gamma(\mathsf{R}^n)\cap\mathcal{E}'(\mathsf{R}^n)\to\mathcal{D}'(\mathsf{R}^{n+1})$ such that for every $f\in\mathcal{D}'_\Gamma(\mathsf{R}^n)\cap\mathcal{E}'(\mathsf{R}^n)$*

we have $LEf \in C^\infty(\mathbf{R}^{n+1})$, $\gamma_0 Ef - f \in C^\infty(\mathbf{R}^n)$, *where* γ_0 *denotes the trace on* $t=0$. *Moreover, E can be represented as a sum of two Fourier integral operators* (FI0) *of order* $-1/4$ *associated with the canonical relations* $C_1(0)$ *and* $C_2(0)$ *which have the form*

$$C_\varkappa(0) = \{(x, t, \xi, \tau; y, \eta) : (x, t, \xi, \tau) \text{ lies on the null bicharacteristic of } D_t - \lambda_\varkappa(x, t, D_x) \text{ issued from } (y, 0, \eta, \lambda(y, \eta))\}, \ \varkappa = 1, 2.$$

Remark 2. The amplitudes of the constructed FI0 belong to the class $S_{\cdot 2}(X \times I) \times \Gamma_1)$ with $X \ni x^0$, $\Gamma_1 \ni \xi^0$, $\Gamma = X \times \Gamma_1$, I an open interval containing the origin. This class is not covered by the calculus of Hörmander [6] for FI0. However, as was mentioned by Melin and Sjöstrand [3], the space $I^{m}_{\varrho}(X, \Lambda)$, of Lagrangean distributions can be defined for $\varrho = 1/2$. Moreover, the calculus of FI0 with real phase functions can be extended to the case, when $\varrho = 1/2$ and the results concerning the composition of FI0 remain true in this case.

The plan of this paper is as follows. In section 2 we reduce the system L to the simple microlocal form, mentioned above and investigated in [1]. n section 3, using the results of [1], we construct a parametrix for the Cauchy problem for L. Making use of the classical Duhamel's principle, in section 4, we construct microlocal parametrices for P and L. In section 5 we study the singularities of the constructed parametrices. Finally, in section 6 we reduce the case, when the characteristics coincide in some rather complicated way to the one under constideration. This enables us to construct a microlocal parametrix in that case as well.

2. Reduction of L to the simple microlocal form of P. We begin with a symmetrization of the characteristics. We have assumed that $\lambda_\varkappa(x, t, \xi) = \lambda(x, \xi) + tr_\varkappa(x, t, \xi)$, $\varkappa = 1, 2$. Define $r(x, t, \xi) = (\lambda_1 - \lambda_2)(x, t, \xi)/2t = (r_1 - r_2)(x, t, \xi)/2$, $\theta(x, t, \xi) = -(\lambda_1 + \lambda_2)(x, t, \xi)/2 = -\lambda(x, \xi) - t(r_1 + r_2)(x, t, \xi)/2$. Then we have $-\lambda_1 = \theta - tr$, $-\lambda_2 = \theta + tr$ and $r(x^0, 0, \xi^0) \neq 0$. With these notations

$$L(x, t, D_x, D_t) = \begin{pmatrix} (D_t + \theta(x, t, D_x) - tr(x, t, D_x)) I_{n_1} & 0 \\ 0 & (D_t + \theta(x, t, D_x) + tr(x, t, D_x)) I_{n_2} \end{pmatrix}$$

$$+ A(x, t, D_x, D_t).$$

Now we shall use the following

Proposition 1 (see [2]). *There exists a tangential, i. e. in the tangential variables x* FI0 $T(t)$, *depending smoothly on t, such that* $T(0) = \mathrm{Id}$ *and for t near* 0 *the relation* $T^{-1}(t)(D_t + \theta(x, t, D_x))T(t) \equiv D_t (\mathrm{mod}\ L^0)$ *holds.*

Introduce the operator

$$L' = T^{-1} L T$$

$$= \begin{pmatrix} (D_t - tr'(x, t, D_x)) I_{n_1} & 0 \\ 0 & (D_t + tr'(x, t, D_x)) I_{n_2} \end{pmatrix} + A'(x, t, D_x, D_t),$$

where $r'(x, t, \xi)$ is homogeneous in ξ of degree 1, $r'(x^0, 0, \xi^0) \neq 0$, $A'(x, t, D_x, D_t) \in CL^0(\mathbf{R}^{n+1})$.

Next we shall use the following reduction of the principal symbol.

P r o p o s i t i o n 2 (see [2]). *Assume that* $\xi_1^0 \neq 0$ *and moreover* $\xi_1^0 r'(x^0, 0, \xi^0) > 0$. *Then there exists a classical symbol* $\tilde{h}(x, t, \xi, \tau)$ *of order* 0 *and a) homogeneous canonical transformation* $\chi:(x, t, \xi, \tau) \to (\tilde{x}, \tilde{t}, \tilde{\xi}, \tilde{\tau})$, *defined in a conic neighbourhood* $\tilde{\Gamma}$ *of* $(x^0, 0, \xi^0, 0)$, *such that for* $t=0$ *we have* $\tilde{t}=0$, $\tilde{x}=x$, $\tilde{\xi}=\xi$, *the symbol* $\tilde{h}$ *is elliptic in* $\tilde{\Gamma}$ *and the symbols* $\tilde{h}(x, t, \xi, \tau)(\tau \mp t r'(x, t, \xi))$ *in the new coordinates become* $\tilde{\tau} \mp \tilde{t}\,\tilde{\xi}_1$.

Let $\tilde{H}(x, t, D_x, D_t)$ be a ψDO with a symbol $\tilde{h}(x, t, \xi, \tau)$, suitably prolonged. Consider a Fourier integral operator F of order 0 whose canonical relation is a closed conic subset of the graph of χ. Suppose F elliptic at $(x^0, 0, \xi^0, 0)$, $F_{u|\,t=0} = u_{|\,t=0}$. Then the operator $L'' = F\tilde{H}L'F^{-1}$ obtains the form

$$L'' = \begin{pmatrix} (D_t - tD_{x_1})I_{n_1} & 0 \\ 0 & (D_t + tD_{x_1})I_{n_2} \end{pmatrix} + B(x, t, D_x, D_t)$$

with $B \in CL^0(\mathbf{R}^{n+1})$, where we write (x, t) instead of $(\tilde{x}, \tilde{t})$.

Now we only need to reduce the lower order terms of L'' in order to obtain a system of the form of P. We shall carry out the reduction in two steps.

F i r s t s t e p. Write $B(x, t, D_x, D_t) = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$, where B_{11}, B_{12}, B_{21}, B_{22} are ψDO whose symbols are respectively $(n_1 \times n_1)$, $(n_1 \times n_2)$, $(n_2 \times n_1)$, $(n_2 \times n_2)$ matrices. Assume that the symbols $B_{kl}(x, t, \xi, \tau)$ have the following asymptotical expansions

$$B_{kl}(x, t, \xi, \tau) \sim \sum_{j=0}^{\infty} B_{kl}^{-j}(x, t, \xi, \tau), \quad k, l = 1, 2.$$

We shall determine an operator $H^0 = \begin{pmatrix} H_{11}^0 & 0 \\ 0 & H_{22}^0 \end{pmatrix}$, where $H_{\varkappa\varkappa}^0$ is a $(n_\varkappa \times n_\varkappa)$ matrix, $\varkappa = 1, 2$, so that

$$L''H^0 \equiv H^0 \begin{pmatrix} (D_t - tD_{x_1})I_{n_1} & \tilde{B}_{12}^0(x, t, D_x, D_t) \\ \tilde{B}_{21}^0(x, t, D_x, D_t) & (D_t + tD_{x_1})I_{n_2} \end{pmatrix} (\mathrm{mod}\, L^{-1})$$

and $H^0 = \mathrm{Id}$ for $t=0$.

For the symbols $H_{\varkappa\varkappa}^0(x, t, \xi, \tau)$ we obtain the following matrix equations

$$\left(\frac{\partial}{\partial t} + (-1)^\varkappa t \frac{\partial}{\partial x_1} + (-1)^{\varkappa+1}\xi_1 \frac{\partial}{\partial \tau} + iB_{\varkappa\varkappa}^0\right) H_{\varkappa\varkappa}^0 = 0$$

with initial conditions $H_{\varkappa\varkappa}^0(x, 0, \xi, \tau) = I_{n_\varkappa}$, $\varkappa = 1, 2$. The solutions H_{11}^0 and H_{22}^0 of these equations are homogeneous of degree 0 in (ξ, τ).

Suppose that we have reduced L'' to the form

$$\begin{pmatrix} (D_t - tD_{x_1})I_{n_1} & 0 \\ 0 & (D_t + tD_{x_1})I_{n_2} \end{pmatrix} + \sum_{j=0}^{m-1} \begin{pmatrix} 0 & B_{12}^{-j} \\ B_{21}^{-j} & 0 \end{pmatrix}(x, t, D_x, D_t) + B^{-m}(x, t, D_x, D_t)$$

with

$$B^{-m}(x, t, \xi, \tau) \sim \sum_{j=m}^{\infty} \begin{pmatrix} B_{11}^{-j} & B_{12}^{-j} \\ B_{12}^{-j} & B_{22}^{-j} \end{pmatrix} (x, t, \xi, \tau).$$

Next we shall find an operator $H^{-m} = \begin{pmatrix} H_{11}^{-m} & 0 \\ 0 & H_{22}^{-m} \end{pmatrix} \in CL^{-m}(\mathsf{R}^{n+1})$ such that $H^{-m} = 0$ for $t=0$ and

$$\left\{ \begin{pmatrix} (D_t - tD_{x_1})I_{n_1} & 0 \\ 0 & (D_t + tD_{x_1})I_{n_2} \end{pmatrix} + \sum_{j=0}^{m-1} \begin{pmatrix} 0 & B_{12}^{-j} \\ B_{21}^{-j} & 0 \end{pmatrix} + \begin{pmatrix} B_{11}^{-m} & B_{12}^{-m} \\ B_{21}^{-m} & B_{22}^{-m} \end{pmatrix} \right\} (\mathrm{Id} + H^{-m})$$

$$\equiv (\mathrm{Id} + H^{-m}) \left\{ \begin{pmatrix} (D_t - tD_{x_1})I_{n_1} & 0 \\ 0 & (D_t + tD_{x_1})I_{n_2} \end{pmatrix} + \sum_{j=0}^{m} \begin{pmatrix} 0 & B_{12}^{-j} \\ B_{21}^{-j} & 0 \end{pmatrix} \right\} \pmod{L^{-m-1}}.$$

For the symbols $H_{\varkappa\varkappa}^{-m}(x, t, \xi, \tau)$ we get the equations

$$\left(\frac{\partial}{\partial t} + (-1)^{\varkappa} t \frac{\partial}{\partial x_1} + (-1)^{\varkappa+1} \xi_1 \frac{\partial}{\partial \tau} \right) H_{\varkappa\varkappa}^{-m} + iB_{\varkappa\varkappa}^{-m} = 0$$

with initial conditions $H_{\varkappa\varkappa}^{-m}(x, 0, \xi, \tau) = 0$, $\varkappa = 1, 2$. The solutions of these equations are homogeneous of degree $-m$ in (ξ, τ).

Let $H \in CL^0(\mathsf{R}^{n+1})$ be an elliptic ψDO with symbol

$$H(x, t, \xi, \tau) \sim \begin{pmatrix} H_{11}^0 & 0 \\ 0 & H_{22}^0 \end{pmatrix} \prod_{j=1}^{\infty} \begin{pmatrix} I_{n_1} + H_{11}^{-j} & 0 \\ 0 & I_{n_2} + H_{22}^{-j} \end{pmatrix},$$

$H = \mathrm{Id}$ for $t=0$ and H^{-1} be a parametrix for H, i. e. $H^{-1}H \equiv \mathrm{Id}$, $HH^{-1} \equiv \mathrm{Id} \pmod{L^{-\infty}}$. Denoting $L''' = H^{-1}L''H$, we obtain

$$L''' \equiv \begin{pmatrix} (D_t - tD_{x_1})I_{n_1} & B_{12}(x, t, D_x, D_t) \\ B_{21}(x, t, D_x, D_t) & (D_t + tD_{x_1})I_{n_2} \end{pmatrix} \pmod{L^{-\infty}}.$$

Second step. We shall determine an operator $K \in CL^{-1}(\mathsf{R}^{n+1})$ such that

$$L''' \equiv \begin{pmatrix} (D_t - tD_{x_1})I_{n_1} & \tilde{B}_{12}(x, t, D_x) \\ \tilde{B}_{21}(x, t, D_x) & (D_t + tD_{x_1})I_{n_2} \end{pmatrix} (\mathrm{Id} + K) \pmod{L^{-\infty}},$$

where $\tilde{B}_{12}, \tilde{B}_{21} \in CL^0(\mathsf{R}^n)$ depend smoothly on t. Let

$$K(x, t, \xi, \tau) \sim \sum_{j=1}^{\infty} \begin{pmatrix} 0 & K_{12}^{-j} \\ K_{21}^{-j} & 0 \end{pmatrix} (x, t, \xi, \tau).$$

Then the symbols K_{ml}^{-1}, $m=1, 2$, $l=3-m$ must satisfy the equalities $B_{ml}^0(x, t, \xi, \tau) = \tilde{B}_{ml}^0(x, t, \xi) + (\tau + (-1)^m t\xi_1) K_{ml}^{-1}(x, t, \xi, \tau)$. Taylor's formula yields

$$B_{12}^0(x, t, \xi, \tau) - B_{12}^0(x, t, \xi, t\xi_1) = \int_0^1 \frac{\partial}{\partial s} [B_{12}^0(x, t, \xi, s\tau + (1-s)t\xi_1)] ds$$

$$= (\tau - t\xi_1) \int_0^1 \left(\frac{\partial B_{12}^0}{\partial \tau} \right) (x, t, \xi, s\tau + (1-s)t\xi_1) ds.$$

We choose $\tilde{B}_{12}^0(x, t, \xi) = B_{12}^0(x, t, \xi, t\xi_1)$.

$$K_{12}^{-1}(x,\ t,\ \xi,\ \tau)=\int_0^1\left(\frac{\partial B_{12}^0}{\partial\tau}\right)(x,\ t,\ \xi,\ s\tau+(1-s)t\xi_1)ds$$

and similarly we define $\tilde{B}_{21}^0$ and K_{21}^{-1}.

Following the same way we define the symbols K_{ml}^{-j}, $\tilde{B}_{ml}^{-j+1}$ for $j=2,\ 3,\ldots$ Finally, we obtain $L'''\equiv P(\mathrm{Id}+K)$ with P in the form

$$P=\begin{pmatrix}(D_t-tD_{x_1})I_{n_1} & 0\\ 0 & (D_t+tD_{x_1})I_{n_2}\end{pmatrix}+\begin{pmatrix}0 & B_{12}\\ B_{21} & 0\end{pmatrix}(x,\ t,\ D_x)$$

in a small conic neighbourhood $\tilde{\Gamma}$ of $(x^0,\ 0,\ \xi^0,\ 0)$, $\xi_1^0\neq 0$, with $B_{ml}\in CL^0(\mathsf{R}^n)$, depending smoothly on t.

3. Construction of a parametrix for the Cauchy problem for L. Introduce notation $f\equiv g(\mathrm{mod}\ C^\infty(\mathsf{R}^n))$ which means that for $f,\ g\in\mathscr{D}'(\mathsf{R}^n)$ we have $-g\in C^\infty(\mathsf{R}^n)$. Suppose that $\xi_1^0>0$. Then (see [1] there exists a conic neighbourhood Γ of $(x^0,\ \xi^0)$ and an operator $E_1:\mathscr{D}'_{\Gamma}(\mathsf{R}^n)\cap\mathscr{E}'(\mathsf{R}^n)\to\mathscr{D}'(\mathsf{R}^{n+1})$ such that for $f\in\mathscr{D}'_{\Gamma}(\mathsf{R}^n)\cap\mathscr{E}'(\mathsf{R}^n)$ we have $PE_1f\in C^\infty(\mathsf{R}^{n+1})$, $\gamma_0E_1f\equiv f\ (\mathrm{mod}\ C^\infty(\mathsf{R}^n))$. For $\xi_1^0<0$ we can obtain the same result with slight modifications of the arguments- in [1].

Since $HP\equiv L''H(\mathrm{Id}+K)^{-1}(\mathrm{mod}\ L^{-\infty})$ we have $L''H(\mathrm{Id}+K)^{-1}E_1f\equiv HPE_1f\equiv 0\ (\mathrm{mod}\ (C^\infty(\mathsf{R}^{n+1}))$. Putting $E_2=H(\mathrm{Id}+K)^{-1}E_1$, we obtain $L''E_2f\in C^\infty(\mathsf{R}^{n+1})$, $\gamma_0E_2f=\gamma_0H(\mathrm{Id}+K)^{-1}E_1f=\gamma_0(\mathrm{Id}+K)^{-1}E_1f$. The parametrix $(\mathrm{Id}+K)^{-1}$ has the form $\mathrm{Id}+G$, $G\in CL^{-1}(\mathsf{R}^{n+1})$. Let

$$G(x,\ t,\ \xi,\ \tau)\sim\sum_{j=1}^{\infty}g^{-j}(x,\ t,\ \xi,\ \tau),$$

g^{-j} being homogeneous of degree $-j$ in $(\xi,\ \tau)$ matrix symbols. By Taylor's a formula we get $g^{-1}(x,\ t,\ \xi,\ \tau)=g^{-1}(x,\ t,\ \xi,\ 0)+\tau\tilde{g}^{-2}(x,\ t,\ \xi,\ \tau)$, where $\tilde{g}^{-2}(x,\ t,\ \xi,\ \tau)=\int_0^1\left(\frac{\partial\tilde{g}^{-1}}{\partial\tau}\right)(x,\ t,\ \xi,\ s\tau)ds$ is homogeneous of degree -2 in $(\xi,\ \tau)$. This implies $\gamma_0(g^{-1}(x,\ t,\ D_x,\ D_t)E_1f)\equiv g^{-1}(x,\ 0,\ D_x,\ 0)\ f+\gamma_0(\tilde{g}^{-2}(x,\ t,\ D_x,\ D_t)D_tE_1f)$ $(\mathrm{mod}\ C^\infty(\mathsf{R}^n))$.
Since

$$0\equiv PE_1f=\left\{\begin{pmatrix}(D_t-tD_{x_1})I_{n_1} & 0\\ 0 & (D_t+tD_{x_1})I_{n_2}\end{pmatrix}+B(x,\ t,\ D_x)\right\}E_1f$$

with

$$B(x,\ t,\ D_x)=\begin{pmatrix}0 & B_{12}(x,\ t,\ D_x)\\ B_{21}(x,\ t,\ D_x) & 0\end{pmatrix}$$

we have

$$D_tE_1f\equiv\begin{pmatrix}I_{n_1} & 0\\ 0 & -I_{n_2}\end{pmatrix}tD_{x_1}E_1f-B(x,\ t,\ D_x)E_1f\ (\mathrm{mod}\ C^\infty(\mathsf{R}^{n+1})).$$

Thus we conclude that

$$\gamma_0(\tilde{g}^{-2}(x,\ t,\ D_x,\ D_t)D_t\ E_1f)$$

$$\equiv \gamma_0(\tilde{g}^{-2}(x,\ t,\ D_x,\ D_t)\begin{pmatrix} I_{n_1} & 0 \\ 0 & -I_{n_2}\end{pmatrix} tD_{x_1}E_1 f - \tilde{g}^{-2}(x, t, D_x, Dt)B(x, t, D_x)E_1 f) \pmod{C^\infty(\mathbf{R}^n)}.$$

The commutator $[\tilde{g}^{-2}(x,\ t,\ D_x,\ D_t), \begin{pmatrix} I_{n_1} & 0 \\ 0 & -I_{n_2}\end{pmatrix} tD_{x_1}]$ belongs to $CL^{-2}(\mathbf{R}^{n+1})$. Denoting

$$\tilde{\tilde{g}}^{-2}(x,\ t,\ D_x, D_t) = [\tilde{g}^{-2}(x,\ t,\ D_x,\ D_t), \begin{pmatrix} I_{n_1} & 0 \\ 0 & -I_{n_2}\end{pmatrix} tD_{x_1}]$$
$$-g^{-2}(x,\ t,\ D_x,\ D_t)\ B(x, t,\ D_x) \in CL^{-2}(\mathbf{R}^{n+1}),$$

we find

$$\gamma_0(\tilde{g}^{-2}(x,\ t,\ D_x,\ D_t)D_tE_1 f)$$
$$\equiv \gamma_0 \left\{\begin{pmatrix} I_{n_1} & 0 \\ 0 & -I_{n_2}\end{pmatrix} tD_{x_1}\ \tilde{g}^{-2}(x,\ t,\ D_x)E_1 f + \tilde{\tilde{g}}^{-2}(x,\ t,\ D_x,\ D_t)E_1 f\right\}$$
$$= \gamma_0(\tilde{\tilde{g}}^{-2}(x,\ t,\ D_x,\ D_t)E_1 f) \pmod{C^\infty(\mathbf{R}^n)}.$$

Hence $\gamma_0(\mathrm{Id}+K)^{-1}E_1 f \equiv (\mathrm{Id}+g^{-1}(x,\ 0,\ D_x,\ 0)) f + \gamma_0\tilde{G} E_1\ f \pmod{C^\infty(\mathbf{R}^n)}$ with $\tilde{G} \in CL^{-2}(\mathbf{R}^{n+1})$. The principal symbol of $\tilde{G}$ we treat in the same way, etc. Finally, we obtain $\gamma_0(\mathrm{Id}+K)^{-1}E_1 f \equiv (\mathrm{Id}+M) f \pmod{C^\infty(\mathbf{R}^n)}$ with $M \in CL^{-1}(\mathbf{R}^n)$. Let $(\mathrm{Id}+M)^{-1}$ be a properly supported parametrix for $\mathrm{Id}+M$. Setting $E_3 = E_2(\mathrm{Id}+M)^{-1} = H(\mathrm{Id}+K)^{-1}E_1(\mathrm{Id}+M)^{-1}$, it follows that $L''E_3 f \in C^\infty(\mathbf{R}^{n+1})$, $\gamma_0 E_3 f \equiv f \pmod{C^\infty(\mathbf{R}^n)}$. On the other hand, $L'' = F\tilde{H}L'F^{-1}$, where F and $\tilde{H}$ are elliptic, hence $L'F^{-1}E_3 f \in C^\infty(\mathbf{R}^{n+1})$ and $\gamma_0 F^{-1}E_3 f \equiv f \pmod{C^\infty(\mathbf{R}^n)}$. Next $L' = T^{-1}LT$. Therefore $LTF^{-1}E_3 f \in C^\infty(\mathbf{R}^{n+1})$ and $\gamma_0 TF^{-1}E_3 f \equiv f \pmod{C^\infty(\mathbf{R}^n)}$, i e. $E = TF^{-1}E_3$ is a parametrix for the Cauchy problem for L.

4. Construction of a microlocal parametrix. Let E_1 be the parametrix for the Cauchy problem for P, that is, for $f \in \mathscr{D}'_{\Gamma}(\mathbf{R}^n) \cap \mathscr{E}(\mathbf{R}^n)$ we have

$$PE_1 f \in C^\infty(\mathbf{R}^{n+1})$$
$$\gamma_0 E_1 f - f = g \in C^\infty(\mathbf{R}^n).$$

Define $\bar{E}$ by $\bar{E} f(x,\ t) = E_1 f(x,\ t) - g(x)$. Then we have $P\bar{E} f \in C^\infty(\mathbf{R}^{n+1})$, $\gamma_0\bar{E} f = f$ In order to construct a microlocal parametrix for P we shall use the classica Duhamel's principle (see [4]). We construct a family of operators $\bar{E}_s$ depending smoothly on $s \in \mathbf{R}^1$ so that for $f \in \mathscr{D}'_{\Gamma}(\mathbf{R}^n) \cap \mathscr{E}'(\mathbf{R}^n)$ we have $P\bar{E}_s f \in C^\infty(\mathbf{R}^{n+1})$ $\gamma_s \bar{E}_s f = f$. Here γ_s denotes the trace on the plane $t=s$. This can be done in the following way. Repeat the construction in [1] choosing the phase functions

$$\varphi_1 = \frac{t^2-s^2}{2}\xi_1 + \langle x,\ \xi\rangle, \quad \varphi_2 = \frac{s^2-t^2}{2}\xi_1 + \langle x,\ \xi\rangle$$

which satisfy the initial conditions $\varphi_1 = \varphi_2 = \langle x,\ \xi\rangle$ for $t=s$,

$$\varphi_1^{(1)} = \left(\frac{t^2+s^2}{2} - \tau_1^2\right)\xi_1 + \langle x,\ \xi\rangle,\ \varphi_2^{(1)} = \left(\tau_1^2 - \frac{t^2+s^2}{2}\right)\xi_1 + \langle x,\ \xi\rangle, \ldots$$

Next instead of $\int_0^t e^{\pm i\tau_1^2\xi_1} d\tau_1$ we take $\int_s^t e^{\pm i\tau_1^2\xi_1} d\tau_1$, etc.

Define $E_4 : \mathscr{D}'_{\tilde{\Gamma}}(\mathsf{R}^{n+1}) \cap \mathscr{E}'(\mathsf{R}^{n+1}) \to \mathscr{D}'(\mathsf{R}^{n+1})$, where $\tilde{\Gamma}$ is a conic neighbourhood of $(x^0, 0, \xi^0, 0)$, $\Gamma \subset \pi_{x,\xi} \tilde{\Gamma}$, as follows

$$E_4 f(x, t) = i \int_0^t (\bar{E}_s \gamma_s f)(x, t)\, ds.$$

Then

$$PE_4 f(x, t) = (\bar{E}_s \gamma_s f)(x, t)|_{s=t} + i \int_0^t (P\bar{E}_s f)(x, t)\, ds = f(x, t) + \tilde{R} f(x, t),$$

where $\tilde{R} f(x, t) = \int_0^t \int_{\mathsf{R}^n} \tilde{r}(x, t, y, s)\, f(y, s) dy ds$, $\tilde{r} \in C^\infty(\mathsf{R}^{2n+2})$.

Next we shall use the following

L e m m a. $WF'(\tilde{R}) \subset \{(x, t, \xi, \tau; y, s, \eta, \sigma) : \xi = \eta = 0\}$.

P r o o f. Let $\varphi, \psi \in C_0^\infty(\mathsf{R}^{n+1})$. Denote by $K_{\tilde{R}}$ the kernel of $\tilde{R}$. For $\alpha > 1$ we have

$$\langle K_{\tilde{R}}, \varphi(x, t) e^{-i\alpha(\langle x, \xi\rangle + t\tau)} \otimes \psi(y, s) e^{-i\alpha(\langle y, \eta\rangle + s\sigma)} \rangle$$

$$= \int_{\mathsf{R}^{n+1}} \int_0^t \int_{\mathsf{R}^n} e^{-i\alpha(\langle x, \xi\rangle + \langle y, \eta\rangle + t\tau + s\tau)} \varphi(x, t) \psi(v, s) \tilde{r}(x, t, y, s)\, dy ds dx dt.$$

If $(\xi, \eta) \neq (0, 0)$, an integration by parts yields a rapidly decreasing function of α. For $\varrho = (x, t, \xi, \tau) \in \tilde{\Gamma}$ we have $\xi \neq 0$. Therefore, if $(\varrho, \varrho_1) \in \tilde{\Gamma} \times \tilde{\Gamma}$, we obtain $(\varrho, \varrho_1) \notin WF'(PE_4 - \mathrm{Id})$. In particular, $(x^0, 0, \xi^0, 0; x^0, 0, \xi^0, 0) \notin WF'(PE_4 - \mathrm{Id})$, i. e. E_4 is a microlocal parametrix for P at the point $(x^0, 0, \xi^0, 0)$.

Moreover, $(\varrho, \varrho_1) \notin WF'(HPE_4 - H)$. On the other hand, $HPE_4 - H \equiv L''H(\mathrm{Id} + K)^{-1} E_4 - H \equiv (L''H(\mathrm{Id} + K)^{-1} E_4 H^{-1} - \mathrm{Id}) H \pmod{L^{-\infty}}$. Setting $E_5 = H(\mathrm{Id} + K)^{-1} E_4 H^{-1}$ it follows that $(\varrho, \varrho_1) \notin WF'(L''E_5 - \mathrm{Id})$. Next we have $L''E_5 - \mathrm{Id} = F\tilde{H}L'F^{-1}E_5 - \mathrm{Id} \equiv F\tilde{H}(L'F^{-1}E_5 - \tilde{H}^{-1}F^{-1}) \pmod{L^{-\infty}}$. Suppose, now that for some $\varrho_2 \in \chi^{-1}(\tilde{\Gamma})$ and some $\varrho_1 \in \tilde{\Gamma}$ we have $(\varrho_2, \varrho_1) \in WF'(\tilde{H}(L'F^{-1}E_5 - \tilde{H}^{-1}F^{-1}))$ which implies $(\varrho_2, \varrho_1) \in WF'\{F^{-1}[F\tilde{H}(L'F^{-1}E_5 - \tilde{H}^{-1}F^{-1})]\}$. This shows that there exists $\varrho \in T^*(\mathsf{R}^{n+1}) \setminus 0$ such that $(\varrho_2, \varrho) \in WF'(F^{-1})$, $(\varrho, \varrho_1) \in WF'(L''E_5 - \mathrm{Id})$. The first inclusion leads to $\varrho \in \tilde{\Gamma}$ which contradicts the second one. Therefore we obtain $(\varrho_2, \varrho_1) \notin WF'(\tilde{H}(L'F^{-1}E_5 - \tilde{H}^{-1}F^{-1}))$. Since $\tilde{H}$ is elliptic, we have $(\varrho_2, \varrho_1) \notin WF'(L'F^{-1}E_5 - \tilde{H}^{-1}F^{-1})$. On the other hand, $L'F^{-1}E_5 - \tilde{H}^{-1}F^{-1} \equiv (L'F^{-1}E_5F\tilde{H} - \mathrm{Id})\tilde{H}^{-1}F^{-1} \pmod{L^{-\infty}}$. As before, we get that for $E_6 = F^{-1}E_5F\tilde{H}$ and $\varrho_3 \in \chi^{-1}(\tilde{\Gamma})$ we have $(\varrho_2, \varrho_3) \notin WF'(L'E_6 - \mathrm{Id})$. Hence E_6 is a microlocal parametrix for L' at $(x^0, 0, \xi^0, 0)$.

Finally, since $L'E_6 - \mathrm{Id} = T^{-1}LTE_6 - \mathrm{Id} \equiv T^{-1}(LTE_6T^{-1} - \mathrm{Id})T \pmod{L^{-\infty}}$ the operator $\mathscr{E} = TE_6T^{-1}$ is a microlocal parametrix for L at $(x^0, 0, \xi^0, \tau^0)$ and the construction is complete.

5. Singularities of the constructed parametrices. The parametrix for the Cauchy problem for P (see [1]) has the form

$$E_1 f(x, t) = (2\pi)^{-n} \int\int e^{i\Phi_1(x, t, y, \theta)} e_1(x, t, \theta) f(y) dy d\theta$$

$$+ (2\pi)^{-n} \int\int e^{i\Phi_2(x, t, y, \theta)} e_2(x, t, \theta) f(y)\, dy\, d\theta$$

with $\Phi_\varkappa(x, t, y, \theta) = (-1)^{\varkappa+1} \frac{t^2}{2} \theta_1 + \langle x - v, \theta\rangle$ and $e_\varkappa(x, t, \theta) \in S^0_{1/2}((X \times I) \times \Gamma_1)$, $\varkappa = 1, 2$. The notation that we use here was introduced in section 1.

We have $WF'(E_1)\subset\tilde{C}_1(0)\cup\tilde{C}_2(0)$, where

(1)
$$\tilde{C}_\varkappa(0)=\{(x,\ t,\ \xi,\ \tau\ ;\ y,\ \eta)\in(T^*(\mathbb{R}^{n+1})\setminus 0)\times(T^*(\mathbb{R}^n)\setminus 0):\ (x,\ t,\ \xi,\ \tau)\text{ lies on the null bicharacteristic of } D_t+(-1)^\varkappa tD_{x_1},\text{ issued from }(y,\ 0,\ \eta,\ 0)\},\ \varkappa=1,\ 2.$$

Since E_3 is obtained from E_1 by a composition with ψDO, it follows that $WF'(E_3)\subset\tilde{C}_1(0)\cup\tilde{C}_2(0)$. Next $WF'(F^{-1}E_3)\subset C_1'(0)\cup C_2'(0)$, where $C_\varkappa'(0)=WF'(F^{-1})_0C_\varkappa(0)=\{(x,\ t,\ \xi,\ \tau)$ lies on the null bicharacteristic of $D_t+(-1)^\varkappa tr'(x,\ t,\ D_x)$ issued from $(y,\ 0,\ \eta,\ 0)\}$, $\varkappa=1,\ 2$, and similarly we find $WF'(E)\subset C_1(0)\cup C_2(0)$, where

(2)
$$C_\varkappa(0)=\{(x,\ t,\ \xi,\ \tau\ ;\ y,\ \eta):(x,\ t,\ \xi,\ \tau)\text{ lies on the null bicharacteristic of } D_t-\lambda_\varkappa(x,\ t,\ D_x)\text{ issued from }(y,\ 0,\ \eta,\ \lambda(y,\ \eta))\},\ \varkappa=1,\ 2.$$

Thus the proof of theorem 1 is completed.

Now we shall investigate the wave front set of the microlocal parametrix constructed in section 4. We have

$$E_4f(x,\ t)=i\int_0^t(\bar{E}_r\gamma_r\ t)(x,\ t)dr.$$

We shall use the following representation of the kernel of E_4:

$$K_{E_4}(x,\ t,\ y,\ s)=i\int_{-\infty}^{\infty}H(t-r)H(r)K_{E'}(r,\ x,\ t,\ y,\ s)dr,$$

where $K_{E'}$ is the kernel of the operator $E':f(y,\ s)\rightarrow(E_r\gamma_r f)(x,\ t)$, $H(r)$ being the Heaviside function. As it was discussed in [4], applying the inclusions for the wave front sets of a product of distributions as well as of the map π_*, where π is the projection $(\cdot,\ x,\ t,\ y,\ s)\rightarrow(x,t,y,s)$ (see [4] for a more precise definition) we find that

$$WF'(E_4)\subset\Delta_{T^*(\mathbb{R}^{n+1})\setminus 0}\cup\tilde{C}_1\cup\tilde{C}_1'(0)\cup\tilde{C}_2\cup\tilde{C}_2'(0),$$

where $\Delta_{T^*(\mathbb{R}^{n+1})\setminus 0}$ denotes the diagonal in $(T^*(\mathbb{R}^{n+1})\setminus 0)\times(T^*(\mathbb{R}^{n+1})\setminus 0)$. $\tilde{C}_\varkappa=\{(x,\ t,\ \xi,\ \tau\ ;\ y,\ s,\ \eta,\ \sigma):(x,\ t,\ \xi,\ \tau)$ lies on the bicharacteristic of $D_t+(-1)^\varkappa tD_{x_1}$ issued from $(y,\ s,\ \eta,\ \sigma)\}$, $\varkappa=1,\ 2$, $\tilde{C}_\varkappa'(0)=\tilde{C}_\varkappa(0)_0R(0)$, where $R(0)$ is the canonical relation associated with the operator γ_0 and the canonical relations $\tilde{C}_\varkappa(0)$ were defined in (1). Following the same arguments as before we obtain

(3)
$$WF'(\mathcal{E})\subset\Delta_{T^*(\mathbb{R}^{n+1})\setminus 0}\cup C_1\cup C_1'(0)\cup C_2\cup C_2'(0),$$

where $C_\varkappa=\{(x,\ t,\ \xi,\tau\ ;\ y,\ s,\ \eta,\ \sigma):(x,\ t,\ \xi,\ \tau)$ lies on the bicharacteristic of $D^t-\lambda_\varkappa(x,\ t,\ D_x)$ issued from $(y,\ s,\ \eta,\ \sigma)\}$, $C_\varkappa'(0)=C_\varkappa(0)_0R(0)$, $\varkappa=1,\ 2$. $C_\varkappa(0)$ were defined in (2). This completes the proof of the following

Theorem 2. *There exists a conic neighbourhood* $\tilde{\Gamma}$ *of* $(x^0, 0, \xi^0, \tau^0)$ *and an operator* $\mathscr{E}: \mathscr{D}'_{\tilde{\Gamma}}(\mathbf{R}^{n+1}) \cap \mathscr{E}'(\mathbf{R}^{n+1}) \to \mathscr{D}'(\mathbf{R}^{n+1})$ *such that* $(x^0, 0, \xi^0, \tau^0, x^0, 0, \xi^0, \tau^0) \notin WF(L\mathscr{E}-\mathrm{Id})$. *Moreover, the wave front set of* $\mathscr{E}$ *satisfies* (3).

6. Construction of a microlocal parametrix in the case when the characteristics coincide in a point. In this section we shall make the following assumptions. Let X' be a paracompact C^∞ manifold with dimension n, $X=\mathbf{R}\times X'$. We denote the points of X by $x=(x_0, x')=(x_0, x_1, \ldots, x_n)$. Our purpose is to study the $(n_1+n_2)\times(n_1+n_2)$ system

$$P(x, D_x)=\begin{pmatrix}(D_{x_0}-\lambda_1(x, D_{x'}))I_{n_1} & 0\\ 0 & (D_{x_0}-\lambda_2(x, D_{x'}))I_{n_2}\end{pmatrix}+P_0(x, D_x).$$

Let $(x^0, \xi'^0)\in\mathbf{R}\times(T^*(X')\setminus 0)$. Assume that λ_1, λ_2 are properly supported ψDO in the x' variables, depending smoothly on x_0 near x_0^0 with real and homogeneous of degree 1 in ξ' symbols. Moreover, $P_0(x, D_x)$ is a properly supported classical ψDO of order 0.

Let $\lambda_1(x^0, \xi'^0)=\lambda_2(x^0, \xi'^0)$, $\frac{\partial\lambda_1}{\partial x_0}(x^0, \xi'^0)\neq\frac{\partial\lambda_2}{\partial x_0}(x^0, \xi'^0)$. Setting $\xi_0^0=\lambda_1(x^0, \xi'^0)=\lambda_2(x^0, \xi'^0)$, $\xi^0=(\xi_0^0, \xi'^0)$, the last inequality means precisely that $\{\xi_0-\lambda_1(x, \xi'), \xi_0-\lambda_2(x, \xi')\}(x^0, \xi^0)\neq 0$.

We shall reduce this case to the one discussed in the previous sections (see [5]). First we find a homogenous canonical transformation χ_1 which maps a conic neighbourhood of (x^0, ξ^0) into a conic neighbourhood of $(y^0, \eta^0)\in T^*(\mathbf{R}^{n+1})\setminus 0$ in such manner that $(\xi_0-\lambda_1)(\chi_1^{-1}(y, \eta))=\eta_0$. For this purpose we shall use the generating function $\Phi(x, \eta)=\varphi(x, \eta')+x_0\eta_0$, where the phase function φ is determined as a solution of the Cauchy problem

$$\partial\varphi/\partial x_0=\lambda_1(x, d_{x'}\varphi),\quad \varphi_{|x_0=0}=\langle x', \eta'\rangle.$$

Then χ_1 is defined by the equalities

$$\xi=d_x\Phi=(\eta_0+\lambda_1, d_{x'}\varphi),\quad y=d_\eta\Phi=(x_0, d_{\eta'}\varphi).$$

Obviously, $\eta_0^0=0$, $(\xi_0-\lambda_2)(x_1^{-1}(y, \eta))=\eta_0+\alpha(y, \eta')$, where $\alpha(y, \eta')$ is homogenous of degree 1 in η', $\alpha(y^0, \eta'^0)=0$ and $\{\eta_0, \alpha(y, \eta')\}=\partial\alpha/\partial y_0\neq 0$ in a conic neighbourhood of (y^0, η^0). According to the implicite function theorem there exists a function $Y_0(y', \eta')$ homogenous of degree 0 in η', such that $\alpha(Y_0(y', \eta'), y', \eta')=0$ and $Y_0(y'^0, \eta'^0)=y_0^0$. Then in a conic neighbourhood of (y^0, η^0) we have

$$\alpha(y, \eta')=(y_0-Y_0(y', \eta'))\gamma(y, \eta'),\quad \gamma(y, \eta')\neq 0$$

and $\{\eta_0, y_0-Y_0(y', \eta')\}=1$ Put $z_0=y_0-Y_0(y', \eta')$, $\zeta_0=\eta_0$. Using a standard argument we can determine the functions $\zeta_j(y', \eta), z_j(y, \eta)$ $j=1, \ldots, n$, homogeneous respectively of degree 1 and 0 in η, such that $\{\zeta_i, z_j\}=\delta_{ij}$, $\{\zeta_i, \zeta_j\}=0$, $\{z_i, z_j\}=0$, $i, j=0, 1, \ldots, n$. This imples that there exists a canonical transformation χ_2 which maps a conic neighbourhood of (y^0, η^0) into a conic neighbourhood of $(0, z'^0, 0, \zeta'^0)$ so that $\eta_0(\chi_2^{-1}(z, \zeta))=\zeta_0$, $(y_0-Y_0)(\chi_2^{-1}(z, \zeta))=z_0$. Denoting $\chi=\chi_2\circ\chi_1$, we find two scalar properly supported Fourier integral operators A and A^{-1}, associated with the graph k of χ and such that $A\in I^0(\mathbf{R}^{n+1}, X; K')$, $A^{-1}\in I^0(X, \mathbf{R}^{n+1}; (K^{-1})')$, $(x^0, \xi^0 x^0, \xi^0)$, $\notin WF'(\mathrm{Id}_x-A^{-1}A)$, $(z^0, \zeta^0, z^0, \zeta^0)\notin WF'(\mathrm{Id}_{\mathbf{R}^{n+1}}-AA^{-1})$.

Then the principal symbol of APA^{-1} becomes

$$\begin{pmatrix} \zeta_0 I_{n_1} & 0 \\ 0 & (\zeta_0+z_0\theta(z,\zeta))I_{n_2} \end{pmatrix}$$

with $\theta(z,\zeta)\neq 0$. Moreover, $\zeta_0+z_0\theta(z,\zeta)=0$ for $z_0=0, \zeta_0=0$ while $\frac{\partial}{\partial\zeta_0}(\zeta_0+z_0\theta(z,\zeta))=1+z_0\frac{\partial\theta}{\partial\zeta_0}(z,\zeta)\neq 0$ in a sufficiently small conic neighbourhood of $(0, z'^0, 0, \zeta'^0)$. Applying again the implicite function theorem we find that there exists a function $\zeta^0(z,\zeta')$, homogeneous of degree 1 in ζ' and such that $\zeta_0(z,\zeta')+z_0\theta(z,\zeta_0(z,\zeta'),\zeta')=0$. We denote $\tilde{\theta}(z,\zeta')=\theta(z,\zeta_0(z,\zeta'),\zeta')$ and obtain $\zeta_0+z_0\theta(z,\zeta)=(\zeta_0-\zeta_0(z,\zeta'))\beta(z,\zeta)=(\zeta_0+z_0\tilde{\theta}(z,\zeta'))\beta(z,\zeta)$, $\beta(z,\zeta)\neq 0$ being homogeneous of degree 0. Let $B(z, D_z)$ be ψDO with a symbol

$$\begin{pmatrix} I_{n_1} & 0 \\ 0 & \beta(z,\zeta)I_{n_2} \end{pmatrix}$$

and $B^{-1}(z, D_z)$ be a parametrix for B. Then the principal symbol of $APA^{-1}B^{-1}$ has the form

$$\begin{pmatrix} \zeta_0 I_{n_1} & 0 \\ 0 & (\zeta_0+z_0\tilde{\theta}(z,\zeta'))I_{n_2} \end{pmatrix}$$

with $\tilde{\theta}(z,\zeta')\neq 0$. We see that the operator $APA^{-1}B^{-1}$ is a particular case of the operator L considered in section 1 with $\lambda_1\equiv 0$, $\lambda_2=-z_0\tilde{\theta}(z,\zeta')$. Let $\mathscr{E}$ be the corresponding microlocal parametrix and $(0, z'^0, 0, \zeta'^0; 0, z'^0, 0, \zeta'^0)\notin WF(APA^{-1}B^{-1}\mathscr{E}-\mathrm{Id})$. Next we find $(x^0,\xi^0,x^0,\xi^0)\notin WF'(PA^{-1}B^{-1}\mathscr{E}A-\mathrm{Id})$. Therefore, $A^{-1}B^{-1}\mathscr{E}A$ will be a microlocal parametrix for P at the point (x^0,ξ^0)

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