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SOME NONUNIQUENESS RESULTS FOR SECOND ORDER SEMILINEAR ELLIPTIC EQUATIONS

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The present paper deals with boundary value problems for semilinear elliptic equations of the form

$$\Delta u+f(x, u)=g(x) \quad \text{in} \quad \Omega, \qquad u=0 \quad \text{on} \quad \partial\Omega \tag{1}$$

with unbounded, but growing at most linearily nonlinear term $f(x, u)$. Our goal is to establish some generic multiplicity results, similar to those obtained in the now classical work [1] in the case when the function $f(x, u)$ ultimately increases and there is some interplay with the first, but not only the first eigenvalue of the linear part. Here by generic we mean that no additional conditions are imposed on the values of the derivative of $f(x, u)$ and in fact we do not suppose that the function is differentiable. Papers treating similar situations (cf. for example [2; 3]), usually are based on sub- and supersolution approach, i. e. on the maximum principle, while in the present work its use is minimal, i. e. we use the possibility to choose positive first eigenfunction only to guarantee that certain integral has determined sign, a fact that can be inferred also by making additional assumptions on the eigenfunctions if different operators or eigenvalues are studied (as in [4], for example). We should like to note, too, that in a number of works the reverse situation, i. e. when the function $f(x, u)$ decreases, is studied.

The problem is studied by means of global Lyapunov-Schmidt method, used to reduce the infinite dimensional problem to a finite dimensional one (as for example in [5; 6], and in fact this part of our proof follows very closely the one given there) and the multiplicity is then established by means of topological argument (Lemma 4 below). Our main result, Theorem 1, concerning the range of the operator in (1) is not so full as the ones in [3], but could be completed using different technics. Then Theorem 2, which treats the simpler case of interplay with the first eigenvalue only, is similar to the results in [7] but in a slightly more general situation of lesser regularity of the function f.

Let Ω be a bounded region in R^n and H be the Hilbert space $H_0^1(\Omega)$ with scalar product $(u, v)_H=\int_\Omega \nabla u \cdot \nabla v dx$ and norm $\| u \|_H^2=(u, u)_H$. The scalar product and the norm in $L^2(\Omega)$ are denoted by (u, v) and $\| u \|$, respectively, and $\| u \|_p$ denotes the norm in $L^p(\Omega)$ for $p \neq 2$. Under appropriate growth restrictions on the function $f(x, u)$ (and the hypotheses we shall make in the sequel are even more restrictive), (1) defines a continuous mapping from $H_0^1(\Omega)$ to $H^{-1}(\Omega)$. Following standard procedure (cf. for example [5]) we shall restate the boundary value problem (1) for $g \in H^{-1}(\Omega)$ as an abstract operator equation in H.

As is well-known, in this Hilbert space context the eigenvalue problem

$$\Delta u + \lambda u = 0 \quad \text{in} \quad \Omega,$$
$$u = 0 \quad \text{on} \quad \partial\Omega$$

has infinitely many eigenvalues $0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \cdots$ and an orthonormal system of corresponding eigenfunctions $v_1, v_2, v_3, \ldots$ that have the following variational characteristics

$$\lambda_k = \min\{\| u \|_H^2 : \| u \|^2 = 1, (u, v_i) = 0, i = 1, \ldots, k-1\}. \tag{2}$$

Moreover, the first eigenvalue is simple and the corresponding eigenfunction $v_1(x)$ has constant sign and can be chosen positive in Ω.

The main hypotheses on the function $f(x, u)$ that follow shall be referred to as $H(i)$ — $H(iv)$ in the sequel.

Hypotheses. (i) *The function* $f: \Omega \times R \to R$ *satisfies the Carathéodory conditions, i. e. is measurable in* x *for all* $u \in R$ *and is continuous in* u *for almost all* x *in* Ω.

(ii) *The function* $f(x, u)$ *grows at most linearily with respect to* u, *i. e. there exists a function* $k \in L^2(\Omega)$ *and a constant* $\alpha > 0$ *such that*

$$| f(x, u) | \leq k(x) + \alpha | u |, \quad u \in R. \tag{3}$$

(iii) *There exists a constant* M, *such that the function*

$$f(x, u) - Mu \tag{4}$$

is monotone nonincreasing for almost all $x \in \Omega$.

Remark. It is evident that only the case $M > 0$ even $M > \lambda_1$ is of interest. The condition (4) is satisfied for instance when the function is differentiable with respect to $u \in R$ and $f'_u(x, u) \leq M$.

(iv) The limits

$$f^{\pm}(x) = \lim_{u \to \pm\infty} f(x, u)/u \tag{5}$$

exist uniformly with respect to $x \in \Omega$ *and moreover*

$$f^{-}(x) < \lambda_1 < f^{+}(x). \tag{6}$$

Now we are in a position to formulate the abstract operator equation in H mentioned above. We define a continuous operator $F: H \to H$ implicitely by

$$(Fu, v)_H = \int_{\Omega} f(x, u)\, v dx, \tag{7}$$

the right-hand side being well-defined by virture of the properties of Niemitzkii's operators (cf. [8]) and $H(i)$, $H(ii)$. We can also establish one-to-one correspondence between the elements of H and those of $H^{-1}(\Omega)$ as in [7] for example. Now we have to solve for $g \in H$ the equation

$$-u + Fu = g. \tag{8}$$

To this end we use global Lyapunov-Schmidt method as usual for similar cases, following [5] or [7] for instance. In what follows we divide the study of the problem in a sequence of lemmas. First let k be the smallest nonnegative integer such that $\lambda_{k+1}>M$ and let V be the finite dimensional space spanned by the first k eigenfunctions $v_1,\ldots,v_k$, and let W be the $L^2(\Omega)$ orthogonal complement of V in H, i. e. if we denote by Q the $L^2(\Omega)$ orthogonal projection of H onto V, then for $P=I-Q$ we have $W=P(H)$.

Let us note for further use that from the variational characteristic (2) follow

$$(u, v_i)_H=\lambda_i (u, v_i) \quad i=1, 2, \ldots \tag{9}$$

and hence for $u\in W$, $(u, v)_H=0$ for every $v\in V$ and $(Pu, w)_H=(u, w)_H$ for $w\in W$.

If now $u=v+w$ with $v\in V$, $w\in W$, the problem (8) is equivalent to

$$A(v+w)\equiv w-PF(v+w)=-Pg, \tag{10}$$

$$B(v+w)\equiv v-QF(v+w)=-Qg. \tag{11}$$

Lemma 1. *Under the hypotheses H(i)—H(iii), the operator $A(v+w)$ is continuous, monotone and coercitive on W for every $v\in V$ fixed and hence there exists unique solution $w=w(v, y)$ of the equation*

$$A(v+w)=y \tag{12}$$

for every $y\in W$. Moreover, the function $w=w(v, y)$ is continuous with respect to v and y.

Proof. The operator A is continuous, because such are the projection P and the mapping F. For the rest of the proof we follow with minor variations the proof in [5], using (4) instead of the boundedness of the derivatives of f. First (2) implies $\|w\|_H^2\geq\lambda_{k+1}\|w\|^2$ for $w\in W$. Then

$$(A(v+w_1)-A(v+w_2),\ w_1-w_2)_H=\|w_1-w_2\|_H^2$$
$$-\int_\Omega\{f(x, v+w_1)-M(v+w_1)-f(x, v+w_2)+M(v+w_2)\}(w_1-w_2)dx$$
$$-M\|w_1-w_2\|^2\geq\delta\|w_1-w_2\|_H^2+((1-\delta)\lambda_{k+1}-M)\|w_1-w_2\|^2$$

and the monotonicity follows for $\delta>0$ small enough, since the integrand is nonpositive. Since now

$$(A(v+w), w)_H\geq\|w\|_H^2-|(A(v), w)_H| \tag{13}$$

we estimate the second term as follows

$$|(A(v), w)_H|=|(F(v), w)_H|\leq\int_\Omega|f(x, v)|\,|w|dx$$

$$\leq\int_\Omega k(x)|w(x)|dx+\alpha\int_\Omega|v(x)|\,|w(x)|dx\leq C(1+\|v\|)\|w\|_H,$$

where in the first inequality a well-known Poincaré inequality for functions in $H_0^1(\Omega)$ is used; now

$$\frac{(A(v+w), w)_H}{\|w\|_H}\geq\delta\|w\|_H-C(1+\|v\|), \tag{14}$$

which gives the coercivity. The existence and the uniqueness of solution is now standard from the theory of monotone operators (cf. for example [9] or [10]).

If we now have w_i, $i=1, 2$ solutions of $A(w_i+v_i)=y_i$ similar computations give, by adding and subtracting appropriate terms, that

$$\| y_1-y_2 \|_H \| w_1-w_2 \|_H \geqq (y_1-y_2, w_1-w_2)_H$$
$$\geqq \delta \| w_1-w_2 \|_H^2 - C \| f(v_1+w_1)-f(v_2+w_1) \| \; \| w_1-w_2 \|_H,$$

i. e. $$\| w_1-w_2 \|_H \leqq \delta^{-1} C (\| y_1-y_2 \|_H + \| f(v_1+w_1)-f(v_2+w_1) \|),$$

and the continuity follows from the well-known results about Niemitzki's operators in the spaces $L^p(\Omega)$. Furthermore, if $w(y, v)$ is a solution of (12), then (14) implies

(15) $$\| w(y, v) \|_H \leqq \delta^{-1} C(1+\| y \|_H + \| v \|).$$

Now we define a continuous application from V in V and we shall study its properties. Let $B: V \to V$ be defined by

(16) $$B(v)=B(v+w(y, v))$$

for fixed $y \in W$. It is clear that the mapping thus defined is continuous in V (even in both variables). For the sequel we shall need the following definition.

Definition 1. *A continuous mapping from a topological space X to a topological space Y is called "proper" iff the invers image of every compact in Y is compact in X.*

The next lemma deals with this notion, but for the sake of further needs we formulate it in a slightly different manner.

Lemma 2. *Let y_n and z_n be bounded sequences in W and V, respectively, and let v_n satisfy the equality*

(17) $$B(v_n+w(y_n, v_n))=z_n.$$

Then the sequence v_n is also bounded.

Proof. Let us suppose that on the contrary, $t_n=\| v_n \| \to +\infty$. For $w_n=w(y_n, v_n)$, (17) implies that $A(v_n+w_n)=y_n$. Then for t_n above, (15) implies

$$\| t_n^{-1} w_n \|_H \leqq \delta^{-1} C t_n^{-1} (1+\| y_n \|_H + \| v_n \|) \leqq C'.$$

Since V is finite dimensional and $t_n^{-1} \| v_n \| = 1$, Sobolev's imbedding theorem implies that there exists a subsequence, which we again index by n, such that

$$t_n^{-1} w_n \to w_0 \text{ strongly in } L^2(\Omega),$$
$$t_n^{-1} w_n(x) \to w_0(x) \text{ a. e. in } \Omega,$$
$$t_n^{-1} v_n \to v_0 \text{ in } V,$$
$$t_n^{-1} v_n(x) \to v_0(x),$$

the last even uniformly. For $\alpha_n(x)=t_n^{-1}(v_n(x)+w_n(x))$, we have $\alpha_n \to \alpha_0$ in $L^2(\Omega)$, $\alpha_n(x) \to \alpha_0(x)$ a. e. in Ω and $\alpha_0(x) \neq 0$, since $\| v_0 \| = 1$. From (3) it follows that the functions

(18) $$g_n(x)=t_n^{-1} f(x, t_n \alpha_n(x))$$

form a bounded sequence in $L^2(\Omega)$. Let $x \in \Omega$ be a point in the complement of a set of measure zero, such that $\alpha_0(x) > 0$, $\alpha_n(x) \to \alpha_0(x)$. Then for n sufficiently big we have $\alpha_n(x) \geqq 0$ and $t_n \alpha_n(x) \to +\infty$. Hence

$$g_n(x)=\frac{f(x,\, t_n\alpha_n(x))}{t_n\alpha_n(x)}\,\alpha_n(x)\to f^+(x)\alpha_0(x).$$

In the same way, if $\alpha_0(x)<0$, $g_n(x)\to f^-(x)\alpha_0(x)$ and finally $g_n(x)\to 0$ if $\alpha_0(x)=0$, the last case being a little more delicate. In brief we can write

$$g_n(x)\to f^+(x)\max\{\alpha_0(x),\,0\}+f^-(x)\min\{\alpha_0(x),\,0\}\equiv g_0(x).$$

Since (3) implies that $f^{\pm}(x)\in L^\infty(\Omega)$, $g_0(x)\in L^2(\Omega)$. Thus we have a sequence $g_n(x)\in L^2(\Omega)$, with $g_n(x)\to g_0(x)$ a. e. in Ω and $\|g_n\|\leqq$const. This is sufficient to deduce that $g_n\to g_0$ weakly in $L^2(\Omega)$ (cf. [10], Lemma 1.3). From the definition (11) of $B(v+w)$ and those of Q and F, it follows

$$t_n^{-1}B(v_n+w_n)=t_n^{-1}v_n-\sum_{i=1}^{k}\frac{1}{\lambda_i}\int_\Omega t_n^{-1}f(x,\, t_n\frac{v_n+w_n}{t_n})v_i(x)dx\,.\,v_i$$

and the convergence considerations above imply that

$$\lim_{n\to\infty} t_n^{-1}B(v_n+w_n)=v_0(x)-\sum_{i=1}^{k}\frac{1}{\lambda_i}\int_\Omega g_0(x)v_i(x)dx\,.\,v_i.$$

On the other hand, $\lim t_n^{-1}\|z_n\|=0$, so we obtain

$$v_0(x)-\sum_{i=1}^{k}\frac{1}{\lambda_i}\int_\Omega g_0(x)v_i(x)dx\,.\,v_i=0. \tag{19}$$

The function $g_0(x)$ can be writen also as

$$\frac{1}{2}[f^+(x)+f^-(x)]\alpha_0(x)+\frac{1}{2}[f^+(x)-f^-(x)]\,|\,\alpha_0(x)\,|.$$

By multiplying now (19) scalarly by $v_1(x)$, since $\alpha_0(x)=v_0(x)+w_0(x)$ and $w_0,\, v_1)=0$, one gets

$$\int_\Omega\{[\frac{1}{2}(f^++f^-)-\lambda_1](v_0+w_0)+\frac{1}{2}(f^+-f^-)\,|\,v_0+w_0\,|\}v_1dx=0. \tag{20}$$

But now (6) implies that $\frac{1}{2}(f^+(x)-f^-(x))>0$ in Ω and that

$$\left|\frac{1}{2}(f^+(x)+f^-(x))-\lambda_1\right|\leqq\frac{1}{2}(f^+(x)-f^-(x)),$$

i. e. the first term in (20) is nonnegative since $v_0\not\equiv 0$, $w_0\perp v_0$ and then $v_0+w\not\equiv 0$. Since $v_1(x)$ is positive in Ω, this provides the desired contradiction.

Corollary. *For any $y\in W$ fixed, the mapping $B(v)$ is proper.*

Proof. Obvious.

The result just obtained shows that if we take the one-point compactification of R^k, that is $S^k\subset R^{k+1}$, we can extend B to a continuous mapping $B_\infty: S^k\to S^k$ by putting $B_\infty(\infty)=\infty$, i. e. to obtain a continuous mapping of the unit sphere into itself. As is well-known, for such mappings the topological degree is defined, which we denote by deg B (cf. [12]).

Lemma 3. *$\deg B_\infty=0$.*

Proof. $H(iv)$, (4)—(6) imply that there exists a function $\gamma\in L^2(\Omega)$, such that for all u in R

$$f(x,u)-\lambda_1 u\geqq\gamma(x).$$

Now the definition of B, (9), $w(y, v) \perp v_1$ and $v_1(x) > 0$ imply

$$-(B(v+w(y, v)), v_1)_H = -\lambda_1(v, v_1) + \int_\Omega f(x, v+w(y, v))v_1(x)dx \geqq (\gamma, v_1).$$

The last inequality shows that the mapping B is not onto, its image being always "above" a certain hyperplane. Nor then is the mapping B_∞ onto, which allows to conclude that deg $B_\infty = 0$. (The fact that B_∞ is not surjective implies that it is homotopic to the constant map of S^k, and the last has degree zero).

Before proceeding further on, we need the following

Definition. *Let $f: X \to Y$ be a continuous mapping between two topological spaces. A point $x \in X$ is called "regular" for f iff there exist neighbourhoods $x \in U \subset X$ and $f(x) \in V \subset Y$, such that $f: U \to V$ is a homeomorphism. A point $y \in Y$ is called "regular value" for f iff all the points in $\{f^{-1}(y)\}$ are regular* (cf. [12]).

Lemma 4. *Let $f: S^k \to S^k$ be continuous, deg $f = 0$ and let v be a regular point for f. Then there exists at least one more point z, such that $f(z) = f(v)$.*

Proof. If $y = f(v)$ is not a regular value, then by definition there exists a point z with $f(z) = y$, which is not regular for f. Thus we can limit ourselves to the case when y is a regular value and suppose the contrary, i. e. that $f(z) \neq y$ for $z \neq v$. Then there exist neighbourhoods U and V of v and y, respectively, such that $f(U) = V$, $f_{|U}$ is a homeomorphism and $f(S^k \setminus U) \subset S^k \setminus V$. Furthermore U and V can be chosen homeomorphic to the upper hemisphere S^k_+. Indeed, let U_1 and V_1 be neighbourhoods of v and y, respectively, such that f maps U_1 onto V_1 homeomorphically. Such neighbourhoods exist according to the definition of regular point. Moreover, since $S^k \setminus U_1$ is compact the same is true for $f(S^k \setminus U_1)$ and since $y \bar{\in} f(S^k \setminus U_1)$, there exists a neighbourhood V of y, homeomorphic to S^k_+ and such that $\bar{V} \cap f(S^k \setminus U_1) = \emptyset$. For U we take $f^{-1}(V)$. Since $V \subset V_1$ and f is a homeomorphism onto V, U is also homeomorphic to S^k_+. Now we have $f(S^k \setminus U_1) \subset S^k \setminus V$. On the other hand, $f(U_1 \setminus U) = V_1 \setminus V$, i. e. we also have $f(S^k \setminus U) \subset S^k \setminus V$. Contracting now $S^k \setminus U$ and $S^k \setminus V$ to points we obtain a map which is a global homeomorphism of S^k and is homotopic to f by means of the above contraction. As is well-known, the global homeomorphisms of S^k have degree ± 1, which contradicts Lemma 3. (cf. for example [11] or [13] for similar topological arguments.)

Now we can sum up and state the following qualitative result:

Theorem 1. *Under the hypotheses $H(i)$ — $H(iv)$, the boundary value problem* (1) (*or what is the same, the equation* (8) *in H*) *either has no solution, or has at least two solutions with possible exception at the images of some singular points, where there may be uniqueness. In particular, there is no uniqueness at regular points. Moreover, the mapping $I-F$ is proper and hence its range is closed.*

Proof. The only thing there is to prove is that $I-F$ is proper, everything else following from lemmas 1-4. To this end let us first consider the mapping $G: H \to H$ defined as follows: $G(y, v) = (y, B(v+w(y, v)))$. If $K \subset H$ is compact and (y_n, v_n) is a sequence in $G^{-1}(K)$, for $z_n = B(v_n + w(y_n, v_n))$ we have $(z_n, y_n) \in K$ and hence there exists a constant C, such that $\|y_n\|_H \leqq C$, $\|z_n\| \leqq C$. Lemma 2 implies that $\|v_n\|$ is also bounded. Since V is finite-dimensional, we can choose a convergent subsequence, which we again index by n. The fact that we can choose similar subsequence from y_n is obvious from the definition of G. If we now have $(v_n + w_n) - F(v_n + w_n) = z_n + y_n \in K$, then in fact

$w_n = w(y_n, v_n)$ and we can extract a convergent subsequence from w_n, too, since the function $w: H \to W$ is continuous. The fact that the range is closed is a trivial consequence from this.

Example. We give the following obvious one. Let the function $f: R \to R$ be differentiable and satisfy $H(i)$—$H(iv)$. If moreover $f(0)=0$ and $f'(0) \neq \lambda_k$, then the equation

$$\Delta u + f(u) = 0 \quad \text{in} \quad \Omega,$$

$$u = 0 \quad \text{on} \quad \partial\Omega$$

has at least one nontrivial solution, since now the origin is a regular point by the invers function theorem.

The question of more detailed description of the range remains open as far as the methods we had used till now are concerned. However, in the case when interplay with the first eigenvalue only is allowed, we can go a little further and obtain the following precisation of Theorem 1.

Theorem 2. *Under the hypotheses of Theorem 1 and the additional assumption that $M < \lambda_2$, we have that the only points at which the uniqueness is possible are on the boundary of the range $R(-I+F)$ of $-I+F$. Moreover, there exists a continuous function $\tau(y)$ defined for $y \in W$, such that the boundary of $R(-I+F)$ is given by $y + \tau(y)v_1$, while $R(-I+F)$ consists of the points of the form $y + sv_1$ for which $s \geqq \tau(y)$. In particular int $R(-I+F) = \{y + sv_1 : s > \tau(y)\}$ and at any point g in int $R(-I+F)$ the equation* (8) *has at least two solutions.*

Proof. In this simpler situation $v = tv_1$ and Lemma 2 implies that $|B(tv_1)| \to \infty$ for $|t| \to \infty$, while the proof of Lemma 3 contains the fact that the function $-B(tv_1)$ is bounded from below. Let $\tau(y) = \min -B(tv_1)$ for $t \in R$. It is obvious that a point $y + sv_1$ is in the range of $I-F$ iff $s \geqq \tau(y)$ and also that for $s > \tau(y)$ the solutions are at least two, one on each side of any point at which the above minimum is attained. It remains to prove that $\tau(y)$ is continuous with respect to y. But writing with more detail

$$B(tv_1) = B(tv_1 + w(y, tv_1))$$

we see, that considering t as parameter, the function τ is the minimum of a family of continuous functions, so it is upper-semicontinuous. On the other hand, the range of $-I+F$ can be identified with the epigraph of the function $\tau(y)$ (making the obvious identification $W \times V \approx W \times R^1$) and Theorem 1 says that this epigraph is closed. It is well-known that a function is lower-semicontinuous if and only if its epigraph is closed, so τ is continuous being both upper- and lower-semicontinuous.

Remark. It is not hard to see that now making additional hypotheses on the differentiability of the function $f(x, u)$ with respect to u, or supposing it monotone or convex one can repeat the considerations in [3] and obtain results concerning the uniqueness at all boundary points of $R(I-F)$ or the additional properties of the function τ in the particular situation of Theorem 2.

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