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APPROXIMATION OF A CONTINUOUS SYSTEM BY POINT SYSTEMS

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Several scientific and technical problems can be described by partial differential equations of the type

$$\frac{\partial v}{\partial t}=\frac{1}{2}\Delta v+f. \tag{1}$$

A more precise model is to take into account stochastic disturbances, i. e. to the right hand of (1) there have to be added corresponding source terms. Often this disturbance occurs as the so-called white noise. So we regard the stochastic partial differential equation

$$\frac{\partial v}{\partial t}(x,\ t)=\frac{1}{2}\Delta v(x,\ t)+f(x,\ t)+\sigma(x,\ t)\xi(x,\ t). \tag{2}$$

Note that (2) is connected with the idea of diffusion and generation of particles in random spartial-temporal points. Therefore the solution of (2) could be considered as the limit of a suitable discrete particle model.

In this paper we prove the existence of such a solution. The proof is based on an approximation of decomposable Gaussian processes on Polish spaces by Poisson processes.

1. Introduction.

1.1. The Poisson process. Following [3], we introduce the notions and notations in this section. Let $[A,\ d_A]$ be a Polish space, $\mathfrak{A}$ the σ-algebra of the Borel sets in $[A,\ d_A]$, $\mathfrak{L}$ the ring of the bounded Borel sets. Further, M denotes the set of all integer-valued measures on $\mathfrak{A}$ being finite on $\mathfrak{L}$. Let $\mathfrak{M}$ be the smallest σ-algebra of M-subsets which makes the function

$$\Phi \frown \Phi(X), \quad \Phi \in M$$

measurable for each X from $\mathfrak{L}$. A measurable mapping from a probability space $[\Omega,\ \mathfrak{F},\ P]$ into $[M,\ \mathfrak{M}]$ is called a random point system in $[A,\ d_A]$, the distribution on $[M,\ \mathfrak{M}]$ generated by such a random point system is said to be a point process with phase space $[A,\ d_A]$.

To each measure H on $[M,\ \mathfrak{M}[$, a measure ρ_H on $[A,\ \mathfrak{A}]$, called the intensity measure of H, is assigned by

$$\rho_H(X):=\int\Phi(X)H(d\Phi), \quad X\in\mathfrak{A}.$$

Let $X_1, \ldots, X_m$ be elements of $\mathfrak{A}$. The measurable mapping

$$\Phi \frown [\Phi(X_1), \ldots, \Phi(X_m)]$$

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from M into N^m transforms each measure H on $[M, \mathfrak{M}]$ with

$$H(\{\Phi: \Phi(X_i)=\infty\})=0, \quad i=1, \dots, m$$

into a measure $H_{X_1,\dots,X_m}$ on $[\mathsf{N}^m, \gamma(\mathsf{N}^m)]$. If H is a point process, the measures $H_{X_1,\dots,X_m}$ are called the finite dimensional distributions of H.

To any $\mathfrak{L}$-finite measure μ on $[A, \mathfrak{U}]$ we can associate the Poisson process $P_{(\mu)}$ with intensity measure μ characterized by

$$P_{(\mu)X_1,\dots,X_m}=\mathop{\times}_{i=1}^{m} \pi_\mu(X_i)$$

for all finite sequences $(X_i)_{i=1}^m$ of pairwise disjoint sets from $\mathfrak{L}$. Here π_λ, $\lambda>0$ denotes the Poisson distribution with expectation λ. For all $\mathfrak{L}$-finite measures μ on $[A, \mathfrak{A}]$ it holds

$$(P_{(\mu)})^{n*}=P_{(n\mu)} \tag{1.1.1.}$$

where $(P_{(\mu)})^{n*}$ is the n-th convolution-power of $P_{(\mu)}$ (cf. [3]).

1.2. σ-additive processes

1.2.1. Definition. *A random process* $\eta=(\eta(B), B\in\mathfrak{L})$ *on* $[\Omega, \mathfrak{F}, \boldsymbol{P}]$ *is called σ-additive if for all sequences* $(B_i)_{i=1}^\infty$ *of pairwise disjoint elements from* $\mathfrak{L}$ *such that* $\bigcup_{i=1}^\infty B_i\in\mathfrak{L}$, *we have*

$$\boldsymbol{P}(\sum_{i=1}^{\infty} \eta(B_i)=\eta(\bigcup_{i=1}^{\infty} B_i))=1.$$

1.2.2. Definition. *A σ-additive process* $\eta=(\eta(B), B\in\mathfrak{L})$ *is called decomposable if for all finite sequences* $(B_i)_{i=1}^m$ *of pairwise disjoint bounded measurable sets the random variables* $\eta(B_1), \dots, \eta(B_m)$ *are independent (cf. Feldman [2]).*

If ψ is a random point system in $[A, d_A]$ *then the family* $\bar\psi:=(\psi(B), B\in\mathfrak{L})$ *represents a σ-additive process. Moreover, if the distribution of ψ is a Poisson process then $\bar\psi$ is a decomposable process. The white noise based on the Lebesgue measure on* $[\mathsf{R}^d, \mathfrak{A}^d]$ *represents a decomposable process in this sense (cf. [4]).*

1.2.3. Definition. For each $n\in\mathsf{N}$ let $\eta^{(n)}$ and η be σ-additive processes on $[\Omega, \mathfrak{F}, \boldsymbol{P}]$. The sequence $\eta^{(n)}$ is said to converge toward η if for every finite sequence $(B_i)_{i=1}^m$ of elements from $\mathfrak{L}$ the random vector $[\eta^{(n)}(B_1), \dots, \eta^{(n)}(B_m)]$ converges weakly to the random vector $[\eta(B_1), \dots, \eta(B_m)]$. Definition 1. 2. 3. can be generalized as follows:

1. 2. 4. Definition. *Let* $\bar\eta^{(n)}=(\eta_t^{(n)})_{t\ge0}$ and, $\bar\eta=(\eta_t)_{t\ge0}$ *be families of σ-additive processes on* $[\Omega, \mathfrak{F}, \boldsymbol{P}]$. *Then the sequence* $\bar\eta^{(n)}$ *is said to converge toward* $\bar\eta$ *if for each finite sequence* $(t_i)_{i=1}^m$ *of non-negative real numbers and for every finite sequence* $(B_i)_{i=1}^m$ *of elements from* $\mathfrak{L}$ *the random vector* $[\eta_{t_1}^{(n)}(B_1), \dots, \eta_{t_m}^{(n)}(B_m)]$ *converges weakly to the random vector* $[\eta_{t_1}(B_1), \dots, \eta_{t_m}(B_m)]$.

1.3. A special type of a stochastic partial differential equation. We consider the stochastic partial differential equation

$$\frac{\partial v}{\partial t}(x, t)=\frac{1}{2}\Delta v(x, t)+f(x, t)+\sigma(x, t)\xi(x, t) \tag{1.3.1.}$$

with initial value $v(x, 0)=\varphi(x)$, $x\in\mathsf{R}^1$, $t\in\mathsf{R}^+$.

The equation (1.3.1.) can be interpreted in the following way: v describes the charge density of a continuous medium consisting of "very small particles". The term

$1/2\,\Delta v(x, t)$ in (1.3.1.) corresponds to a diffusion of the particles. Furthermore, the source term $f(x, t)$ describes the intensity with which positive ($f(x, t)>0$) or negative ($f(x, t)<0$) charged particles are added. Finally, a stochastic source term describing the stochastic disturbance is added. $\sigma^2(x, t)$ is the intensity of the noise. It must be noted that in the higher dimensional case ($x \in \mathsf{R}^d$, $d>1$) the mathematical concept symbolized by (1.3.1.) is not quite clear. A treatment of this case seems to be possible by using the convergence theorem 1.3.6. given below. Firstly let us explain the main idea. Let φ be a bounded measurable function defined on R^d and $K_\lambda(\cdot, \cdot)$, $\lambda>0$, be the stochastic kernel on $[\mathsf{R}^d, \mathfrak{R}^d]$ given by

$$K_\lambda(q, \cdot) = N(q, \lambda\mathscr{E}, \cdot)$$

Here $N(q, \lambda\mathscr{E}, \cdot)$ denotes the normal distribution with the expectation vector q and the diagonal matrix $\lambda\mathscr{E}$ as covariance matrix. Then

$$(1.3.2.) \qquad A_t(\cdot) := \int K_t(y, \cdot)\varphi(y)dy$$

describes how the charge present at initial time zero diffuses until the time t.

The discretization of the process corresponding to the source term occurs as follows: Let f be a bounded measurable function defined on $\mathsf{R}^d \times \mathsf{R}^+$. Furthermore, let T^+ and T^- be the supports of the functions $f^+ := \max\{0, f\}$, $f^- := \max\{0, -f\}$. The non-negative function $|f|$ is the density of a measure μ_1 on $[\mathsf{R}^{d+1}, \mathfrak{R}^{d+1}]$. $\tilde{\Phi}^n$ denotes a random point system in R^{d+1} with distribution $P_{(n\mu_1)}$. Thus $\tilde{\Phi}^n$ describes the configuration of charge points which are added spatially-temporally with mean intensity $n|f|$. If $\tilde{\Phi}^n\{(x, \tau)\}=1$ then a charge unit of the amount $1/n$ is added; the charge is either positive or negative depending on the position of (x, τ). Now the contribution of this source process to the charge density until time $t>0$ is given by

$$(1.3.3.) \qquad D_t^{(n)}(\cdot) = \frac{1}{n}\int (K_{t-\tau}(y, \cdot)\, I_{(0,t)}(\tau) + \delta_y(\cdot) I_{\{t\}}(\tau))(I_{T^+}(y, \tau) - I_{T^-}(y, \tau))\tilde{\Phi}^n(d[y, \tau]).$$

The disturbance term is transformed analagously. Let σ be a bounded measurable function defined on $\mathsf{R}^d \times \mathsf{R}^+$. σ^2 is considered as the density of a measure μ_2 on $[\mathsf{R}^{d+1}, \mathfrak{R}^{d+1}]$. We define a measure μ on $[\mathsf{R}^{d+2}, \mathfrak{R}^{d+2}]$ by

$$\mu := \mu_2 \times \frac{1}{2}(\delta_1 + \delta_{-1}).$$

Let Φ^n be a random point system in R^{d+2} with distribution $P_{(n\mu)}$. Analogously to $\tilde{\Phi}^n$ the random point system Φ^n describes the configuration of charge points added with mean intensity $n\sigma^2$ including the "sign" of the unit charges. Differently to the effect of the first source process individual charges of the value $n^{-1/2}$ are generated. The contribution of this second source process to the charge density until time $t>0$ is given by

$$(1.3.4.) \qquad C_t^{(n)}(\cdot) = n^{-1/2}\int z(K_{t-\tau}(y, \cdot) I_{(0,\tau)}(\tau) + \delta_y(\cdot) I_{\{t\}}(\tau))\Phi^n(d[y, \tau\ z]).$$

Each particle should give a contribution to the entire charge and evaluate independently on the other ones. These requirements do not follow from the properties of the Poisson process. Therefore, we assume that $\tilde{\Phi}^n$ and Φ are independent. Consequently the entire process can be defined as the sum of independent random variables

$$(1.3.5.) \qquad u_t^{(n)}(\cdot) = A_t(\cdot) + C_t^{(n)}(\cdot) + D_t^{(n)}(\cdot).$$

By the normalization $1/n$, respectively $n^{-1/2}$, the "potential" of one generated particle decreases when n increases, whereas the average number of the particles increases, i. e. the produced charge is "smeared over" in a certain manner.

1.3.6. Theorem. a) *The sequence of the σ-additive processes* $u^{(n)}=(u_t^{(n)}(B), B\in\mathfrak{L}^d, t\geq 0)$ *converges (in the sense of definition 1.2.4.) to a σ-additive process* $u=(u_t(B), B\in\mathfrak{L}^d, t\geq 0)$.

b) *For each finite sequence* $(t_i)_{i=1}^m$ *of non-negative real numbers and for every finite sequence* $(B_i)_{i=1}^m$ *from* $\mathfrak{L}^d$ *the random vector* $[u_{t_1}(B_1), \ldots, u_{t_m}(B_m)]$ *is normally distributed. The distribution is characterized by the expectation values*

$$A_{t_i}(B_i)+\int K_{t_i-\tau}(y, B_i)f(y, \tau)I_{(0,t_i)}(\tau)d[y, \tau], \quad 1\leq i\leq m,$$

and by the covariances

$$\int K_{t_i-\tau}(y, B_i)K_{t_j-\tau}(y, B_j)\sigma^2(y, \tau)\, I_{(0, \min\{t_i, t_j\})}(\tau)d[y, \tau], \quad 1\leq i, j\leq m.$$

Let us note that the process u can be interpreted as the solution of the integral equation corresponding to (1.3.1.). Theorem 1.3.6. can be proved by limit assertions specified in the next section.

2. A limit theorem. Let ν be an $\mathfrak{L}$-finite measure on $[A, \mathfrak{A}]$ and Φ^n denote a random point system in $A\times\{-1, 1\}$ distributed according to $P_{(h\nu\times\frac{1}{2}(\delta_1+\delta_{-1}))}$. Thus by

$$\psi^{(n)}(\cdot): =\frac{1}{\sqrt{n}}\int zI_{(\cdot)}(x)\Phi^n(d[x, z])$$

we define a sequence of random signed measures $\psi^{(n)}$ on $[A, \mathfrak{A}]$. The sequence $\psi^{(n)}$ converges in the sense of definition 1.2.3. to a decomposable process ψ. If ν is the Lebesgue measure on $[\mathsf{R}^d, \mathfrak{A}^d]$ then ψ is the white noise process; if ν is the counting measure on a countable set A, then $\psi(\{x\})$, $x\in A$ forms a sequence of independent standard normally distributed random variables. Moreover, the following statement holds.

2.1. Theorem. *Let* $g_1, \ldots, g_m$ *be functions which are integrable and square integrable with respect to* ν. *Then the distribution of the random vector*

$$\frac{1}{\sqrt{n}}[\int zg_1(x)\Phi^n(d[x, z]), \ \ldots, \int zg_m(x)\Phi^n(d[x, z])]$$

converges weakly to the normal distribution $N(0, \Sigma, \cdot)$ *where the covariance matrix* $\Sigma=(\Sigma_{ij})$ *is given by*

$$\Sigma_{ij}=\int g_i(x)g_j(x)\nu(dx), \quad 1\leq i, j\leq m.$$

We will prove theorem 2.1. in section 3.

If Φ^n is a random point system in A distributed according to $P_{(n\nu)}$ then by

$$\tilde{\psi}^{(n)}(\cdot): = \frac{1}{n}\Phi^n(\cdot)$$

we define a sequence of random measures $\tilde{\psi}^{(n)}$ on $[A, \mathfrak{A}]$. $\tilde{\psi}^{(n)}$ converges in the sense of 1.2.3. to the trivial decomposable process. This follows from the law of large numbers and the convolution property (1.1.1.).

2.2. *If* $g_1, \ldots, g_m$ *are* ν*-integrable functions then the random vector*

$$\frac{1}{n}[\int g_1(x)\Phi^n(dx), \ldots, \int g_m(x)\Phi^n(dx)]$$

converges weakly to the Dirac measure corresponding to the vector

$$[\int g_1(x)\nu(dx), \ldots, \int g_m(x)\nu(dx)].$$

3. Proof of theorem 2. 1. The theorem will be proved by using the central limit theorem for random vectors. We apply the latter one for a special case.

3.1. Let $(\eta_i)_{i=1}^{\infty}$ be a sequence of independent and identically distributed random vectors on a probability space $[\Omega, \mathfrak{F}, \boldsymbol{P}]$.

Assume that the mean vector of η_1 is zero and the covariance matrix Σ of η_1 exists. Then the distribution of the random vector $n^{-1/2}\Sigma_{i=1}^{n}\eta_i$ converges to the normal distribution $\mathcal{N}(0, \Sigma, \cdot)$. (cf. [1]).

If $g_1, \ldots, g_m$ are functions which are integrable and square integrable with respect to ν, the theory of the moment measures enables us to determine the first and the second moments of the random vector

$$\frac{1}{\sqrt{n}}[\int zg_1(x)\Phi^n(d[x, z]), \ldots, \int zg_m(x)\Phi^n(d[x, z])].$$

The moment measures of random point systems are introduced as follows (cf [5]) Let ψ be a random point system on the Polish space $[A, d_A]$ distributed according to Q Let $\mathscr{K}(A)$ be the set of all continuous functions on A with compact support. For any $\mathfrak{L}$-finite measure α on $[A, \mathfrak{A}]$ let $\alpha^{(k)}$ be the k-th power of α, i. e. $\alpha^{(k)}$ is the measure on $[A^k, \mathfrak{A}^k]$ defined by

$$\int \prod_{i=1}^{k} h_i(x_i)\alpha^{(k)}(d[x_1, \ldots, x_k]) = \prod_{i=1}^{k} \int h_i(x)\alpha(dx), \quad h_1, \ldots, h_k \in \mathscr{K}(A).$$

3.2. Definition. *Q is a distribution of k-th order if*

$$\int h(x_1, \ldots, x_k)\rho_Q^k(d[x_1, \ldots, x_k]) := \int (\int h(x_1 \ldots, x_k)\Phi^{(k)}(d[x_1, \ldots, x_k]))Q(d\Phi)$$

***exists and** is finite for each continuous function h on A^k with comvact support. **In this** case ρ_Q^k defines a measure on $[A^k, \mathfrak{A}^k]$ called the k-th **moment** measure of Q.*

If α a $\mathfrak{L}$-finite measure on $[A, \mathfrak{A}]$ for $Q = P_{(\alpha)}$, then the following relations hold (cf. [5], p. 400):

$$\int h(x)\rho^1_{P_{(\alpha)}}(dx) = \int h(x)\alpha(dx), \quad h \in \mathscr{K}(A)$$

and

$$\int h_1(x_1)h_2(x_2)\rho^2_{P_{(\alpha)}}(d[x_1, x_2]) = \int h_1(x)h_2(x)\alpha(dx) \tag{3.3}$$

$$+ \int h_1(x)\alpha(dx) \int h_2(x)\alpha(dx), \quad h_1, h_2 \in \mathscr{K}(A).$$

By usual extension procedures one can prove that (3.3) holds for all functions h, h_1, h_2 which are integrable and square integrable with respect to α. Therefore, if we consider the situation in theorem 2.1., i. e. if we suppose that $\alpha = n\nu \times \frac{1}{2}(\delta_1 + \delta_{-1})$ then

$$\boldsymbol{E}(\frac{1}{\sqrt{n}} \int zg_i(x)\Phi^n(d[x, z])) = 0, \quad 1 \leq i \leq m$$

and

$$\boldsymbol{E}(\frac{1}{n} \int zg_i(x)\Phi^n(d[x, z]) \int zg_j(x)\Phi^n(d[x, z])) = \int g_i(x)g_j(x)\nu(dx), \quad 1 \leq i, j \leq m. \tag{3.4}$$

Let $(\Phi_i)_{i=1}^{r}$ be a finite family of independent random point systems identically distributed according to $P_{(\nu\times\frac{1}{2}(\delta_1+\delta_{-1}))}$. Then the sum $\Sigma_{i=1}^{n}\Phi_i$ possesses the same distribution $P_{(n\nu\times\frac{1}{2}(\delta_1+\delta_{-1}))}$, as the random point system Φ^n (this is just the contents of (1.1.1.)) Consequently, the random vectors

$$[\int zg_1(x)\Phi^n(d[x, z]), \ldots, \int zg_m(x)\Phi^n(d[x, z])]$$

and

$$\sum_{i=1}^{n}[\int zg_1(x)\Phi_i(d[x, z]), \ldots, \int zg_m(x)\Phi_i(d[x, z])]$$

possess the same distribution whereby the vectors

$$([\int zg_1(x)\Phi_i(d[x, z]), \ldots, \int zg_m(x)\Phi_i(d[x, z])])_{i=1}^{n}$$

are independent and identically distributed. The mean vector and the covariance matrix of this distribution exist (cf. (3.4)). Hence the assumptions of 3.1. are fulfilled.

4. Proof of theorem 1.3.6. First of all, there can be stated an assertion concerning the integrability of the term $K_{t-\tau}(y, B)\cdot I_{(0,t)}(\tau)$, $t\in \mathrm{R}^+$, $B\in\mathscr{L}^d$ considered as a function of the variables y and τ. From the identity

$$\int K_{t-\tau}(y, \mathop{\times}_{j=1}^{d}[a_j, b_j])I_{(0,t)}(\tau)d[y, \tau]=t\prod_{j=1}^{d}(b_j-a_j)$$

which is valid for all d-dimensional rectangles $\times_{j=1}^{d}[a_j, b_j]$ we come to the following conclusion denoted by 4.1, 4.2, 4.3 and 4.4.

4.1. If the density of the measure ν on $[\mathrm{R}^{d+1}, \mathfrak{R}^{d+1}]$ is bounded, then for every positive real number t and every set B from $\mathfrak{L}^d$ it holds

$$\int K_{t-\tau}(y, B)I_{(0,t)}(\tau)\nu(d[y, \tau])<\infty.$$

The assertions 4.1. and 2.2. are justified.

4.2. For every finite sequence $(B_i)_{i=1}^{m}$ from $\mathfrak{L}^d$ and every finite sequence $(t_i)_{i=1}^{m}$ from R^+ the distribution of the random vector

$$[D_{t_1}^{(n)}(B_1), \ldots, D_{t_m}^{(n)}(B_m)]$$

converges weakly to the Dirac measure corresponding to the vector

$$[\int K_{t_1-\tau}(y, B_1)I_{(0,t_1)}(\tau)f(y, \tau)d[y, \tau], \ldots, \int K_{t_m-\tau}(y, B_m)I_{(0,t_m)}(\tau)(f(y, \tau)d[y, \tau]].$$

Proof. From (1.3 3.) we know that for all $B\in\mathfrak{L}^d$ and $t\in\mathrm{R}^+$ it holds

$$D_t^{(n)}(B)=\frac{1}{n}\int(K_{t-\tau}(y, B)I_{(0,t)}(\tau)+\delta_y(B)I_{\{t\}}(\tau))(I_{T^+}(y, \tau)-I_{T^-}(y, \tau))\widetilde{\Phi}^n(d[y, \tau]).$$

Let $(B_i)_{i=1}^{m}$ be a finite sequence from $\mathfrak{L}^d$ and let $(t_i)_{i=1}^{m}$ be a finite sequence from R^+. The function $|f|$ is the density of the intensity measure μ_1 which was assumed to be bounded. Applying 4.1., we obtain (for $i=1, \ldots, m$)

$$E(D_{t_i}^{(n)}(B_i))=\int K_{t_i-\tau}(y, B_i)I_{(0,t_i)}(\tau)f(y, \tau)d[y, \tau]$$

$$=\int K_{t_i-\tau}(y, B_i)I_{(0,t_i)}(\tau)(I_{T^+}(y, \tau)-I_{T^-}(y, \tau))\mu_1(d[y, \tau]).$$

Thus the assumption of 2.2. is satisfied. Further, Proposition 4.1 and the relations

$$0 \leq K_\lambda^2(y, B) \leq K_\lambda(y, B) \leq 1, \quad y \in \mathrm{R}^d, \ B \in \mathfrak{L}^d, \ \lambda \in \mathrm{R}^+$$

lead to Proposition 4.3.

4.3. If the density of the measure ν on $[\mathrm{R}^{d+1}, \mathfrak{A}^{d+1}]$ is bounded, then for all t from R^+ and all B from $\mathfrak{L}^d$ we have

$$\int K_{t-\tau}^2(y, B)\, I_{(0,t)}(\tau)\nu(d[y, \tau]) < \infty.$$

Propositions 4.1. and 4.3 are used in order to prove

4.4. For each finite sequence $(B_i)_{i=1}^m$ from $\mathfrak{L}^d$ and each finite sequence $(t_i)_{i=1}^m$ from R^+ the distribution of the random vector

$$[C_{t_1}^{(n)}(B_1), \ldots, C_{t_m}^{(n)}(B_m)]$$

converges weakly to the normal distribution $N(0, \Sigma, \cdot)$ whereby

$$\Sigma_{ij} = \int K_{t_i-\tau}(y, B_i) K_{t_j-\tau}(y, B_j) I_{(0, \min\{t_i, t_j\})}(\tau)\sigma^2(y, \tau) d[y, \tau], \quad 1 \leq i, j \leq m.$$

P r o o f. From (1.3.4.) we know that for each $B \in \mathfrak{L}^d$ and $t \in \mathrm{R}^+$

$$C_t^{(n)}(B) = \frac{1}{\sqrt{n}} \int z(K_{t-\tau}(y, B)\, I_{(0,t)}(\tau) + \delta_y(B) I_{\{t\}}(\tau))\Phi^n(d[y, \tau, z])$$

Let $(B_i)_{i=1}^m$ be a finite sequence from $\mathfrak{L}^d$ and $(t_i)_{i=1}^m$ be a finite sequence from R^- The density σ^2 of the measure μ_2 is bounded. Thus the functions g_i defined by

$$g_i(y, \tau) := K_{t_i-\tau}(y, B_i) I_{(0,t_i)}(\tau) + \delta_y(B_i) I_{\{t_i\}}(\tau), \quad i = 1, \ldots, m$$

are integrable and square integrable with respect to μ_2, i. e. the assumptions of Theorem 2.1. are fulfilled by

$$[C_{t_1}^{(n)}(B_1), \ldots, C_{t_m}^{(n)}(B_m)]$$

The first and the second moment of the distribution of this vector can be computed by means of (3.4.). So the proof of 4.4. follows from Theorem 2.1.

The assertions 4.2. and 4.4. imply the validity of Theorem 1.3.6. because for each finite sequence $(t_i)_{i=1}^m$ from R^+ and all finite sequences $(B_i)_{i=1}^m$ from $\mathfrak{L}^d$ we have

$$[u_{t_1}^{(n)}(B_1), \ldots, u_{t_m}^{(n)}(B_m)] =$$
$$[A_{t_1}(B_1) + C_{t_1}^{(n)}(B_1) + D_{t_1}^{(n)}(B_1), \ldots, A_{t_m}(B_m) + C_{t_m}^{(n)}(B_m) + D_{t_m}^{(n)}(B_m)].$$

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