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TRANSLATIONS OF RELATION SCHEMES

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It this paper we shall be concerned with a class of translations of relation schemes.
Starting from a given relation scheme, translations make it possible to obtain simpler schemes, i. e. those with a less number of attributes and with shorter functional dependencies so that the key-finding problem becomes less cumbersome, etc.
On the other hand, from the set of keys of the relation scheme obtained in this way, the corresponding keys of the original scheme can be found by a single "translation".
In 1 we introduce the notion of Z-translation of a relation scheme, give a classification of the relation scheme and investigate the characteristic properties of some classes of Z-translations.
In 2 we study some properties of the so-called non-translatable relation schemes.
The notation used here is the same as in [1]; $\subset$ means strict inclusion.

1. Definition 1.1. *Let* $S=\langle\Omega, F\rangle$ *be a relation scheme, where* $\Omega=\{A_1, A_2, \ldots, A_n\}$ *is the set of attributes,*

$$F=\{L_i\to R_i \mid \ i=1, 2, \ldots, k;\ L_i,\ R_i\subseteq\Omega\}$$

is the set of functional dependencies, and $Z\subseteq\Omega$, *be an arbitrary subset of* Ω. *We define a new relation scheme* $\langle\Omega_1, F_1\rangle$ *by:*

$$\Omega_1=\Omega\setminus Z$$

$$F_1=\{L_i\setminus Z\to R_i\setminus Z \mid (L_i\to R_i)\in F,\quad i=1,\ldots,k\}.$$

Then $\langle\Omega_1, F_1\rangle$ *is said to be obtained from* $\langle\Omega, F\rangle$ *by a Z-translation, and the notation*

$$\langle\Omega_1, F_1\rangle=\langle\Omega, F\rangle-Z$$

is used.

Remarks 1. Depending on the characteristic properties of the subset Z chosen, the corresponding class of translations has its own characteristic features.

2. With the Z-translation just defined above, a functional dependence of type $\varnothing\to Y$ may occur in $\langle\Omega_1, F_1\rangle$ that has no ordinary semantics, but carries information from the old relation scheme to the new one.

In particular, the possibility that $\varnothing$ turns out to be a key of $\langle\Omega_1, F_1\rangle$ is not excluded.

The next lemma is a fundamental one for the paper.

Lemma 1.1. *Let* $\langle\Omega, F\rangle$ *be a relation scheme and* $\langle\Omega_1, F_1\rangle=\langle\Omega, F\rangle-Z$, $Z\subseteq\Omega$ *then*

a) $X\xrightarrow[F]{}Y$ *implies* $X\setminus Z\xrightarrow[F_1]{}Y\setminus Z$,

b) $X\xrightarrow[F_1]{}Y$ *implies* $X\cup Z\xrightarrow[F]{}Y\cup Z$,

where $X\xrightarrow[F]{}Y$ *means* $(X\longrightarrow Y)\in F^+$ *and, similarly,* $X\xrightarrow[F_1]{}Y$ *for* $(X\longrightarrow Y)\in F_1^+$.

P r o o f. For the part a) of the lemma, we shall prove that

(1) $$X_F^+ \setminus Z \subseteq (X \setminus Z)_{F_1}^+.$$

By the algorithm for finding the closure X^+ of X in [2] with $X_F^{(0)} = X$, $(X \setminus Z)_{F_1}^{(0)} = X \setminus Z$ we have $X_F^{(0)} \setminus Z \subseteq (X \setminus Z)_{F_1}^{(0)}$. Supposing that (1) holds for i, that is

(2) $$X_F^{(i)} \setminus Z \subseteq (X \setminus Z)_{F_1}^{(i)},$$

we prove that (1) holds for $(i+1)$ as well.
Indeed we have

$$X_F^{(i+1)} \setminus Z = (X_F^{(i)} \cup (\bigcup_{\substack{L_J \subseteq X_F^{(i)} \\ (L_J \to R_J) \in F}} R_J)) \setminus Z = (X_F^{(i)} \setminus Z) \cup (\bigcup_{\substack{L_J \subseteq X_F^{(i)} \\ (L_J \to R_J) \in F}} R_J \setminus Z)$$

$$\subseteq (X \setminus Z)_{F_1}^{(i)} \cup (\bigcup_{L_J \subseteq X_F^{(i)}} (R_J \setminus Z))$$

(by virtue of the inductive assumption (2)).
On the other hand, from $L_J \subseteq X_F^{(i)}$ and the inductive assumption (2) we have:

$$L_J \setminus Z \subseteq X_F^{(i)} \setminus Z \subseteq (X \setminus Z)_{F_1}^{(i)}.$$

Consequently: $X_F^{(i+1)} \setminus Z \subseteq (X \setminus Z)_{F_1}^{(i)} \cup (\bigcup_{L_J \subseteq X_F^{(i)}} (R_J \setminus Z)) \subseteq (X \setminus Z)_{F_1}^{(i+1)}$. Thus (1) has been proved.
Now, it is well known that $X \xrightarrow[F]{} Y \Leftrightarrow Y \subseteq X_F^+$. Hence, from $X \xrightarrow[F]{} Y$, we have: $Y \setminus Z \subseteq X_F^+ \setminus Z \subseteq (X \setminus Z)_{F_1}^+$.
That is, $X \setminus Z \xrightarrow[F_1]{} Y \setminus Z$. Similarly, for the part b) of the lemma, we shall prove by induction that

(3) $$X_{F_1}^+ \cup Z \subseteq (X \cup Z)_F^+.$$

By the algorithm for finding the closure X^+ of X we have $X_{F_1}^{(0)} \cup Z \subseteq (X \cup Z)_F^{(0)}$. Supposing that (3) holds for (i), that is

(4) $$X_{F_1}^{(i)} \cup Z \subseteq (X \cup Z)_F^{(i)},$$

we shall prove that (3) also holds for $(i+1)$.
Indeed we have: $X_{F_1}^{(i+1)} \cup Z = X_{F_1}^{(i)} \cup (\bigcup_{L_J \setminus Z \subseteq X_{F_1}^{(i)}} (R_J \setminus Z)) \cup Z = (X_{F_1}^{(i)} \cup Z) \cup (\bigcup_{L_J \setminus Z \subseteq X_{F_1}^{(i)}} (R_J \setminus Z))$
$\subseteq (X \cup Z)_F^{(i)} \cup (\bigcup_{L_J \setminus Z \subseteq X_{F_1}^{(i)}} R_J)$ (by the inductive assumption (4)).

On the other hand, from $L_J \setminus Z \subseteq X_{F_1}^{(i)}$ and (4) we have

$$L_J \subseteq X_{F_1}^{(i)} \cup Z \subseteq (X \cup Z)_F^{(i)}.$$

Consequently: $X_{F_1}^{(i+1)} \cup Z \subseteq (X \cup Z)_F^{(i)} \cup (\bigcup_{L_J \setminus Z \subseteq X_{F_1}^{(i)}} R_J) \subseteq (X \cup Z)_F^{(i+1)}$.

Thus (3) has been proved.

From $X \xrightarrow[F_1]{} Y$ we have $Y \subseteq X^+_{F_1}$ hence $Y \cup Z \subseteq X^+_{F_1} \cup Z \subseteq (X \cup Z)^+_F$ showing that: $X \cup Z \xrightarrow[F]{} Y \cup Z$.
The proof is complete.

Definition 1.2. *Let $S=\langle \Omega, F\rangle$ be a relation scheme. Let $\mathscr{K}(\Omega, F)$ be the set of all keys of S and*

$$H=\bigcup_{X_i \in \mathscr{K}(\Omega, F)} X_i, \quad G=\bigcap_{X_i \in \mathscr{K}(\Omega, F)} X_i.$$

Let us denote $\bar{H}=\Omega \setminus H$. It is easy to prove the following inclusion:

$$\bigcup_{L_i \subseteq G} (R_i \setminus L_i) \subseteq \bar{H}.$$

Indeed, let $x \in \bigcup_{L_i \subseteq G} (R_i \setminus L_i)$. Then there is $(L_j \to R_j) \in F$ such that $x \in R_j$ and $x \bar{\in} L_j$

Let K be an arbitrary key of $S(K \in \mathscr{K}(\Omega, F))$. We shall show that $x \bar{\in} K$.
Since $L_j \subseteq G$, so $L_j \subseteq K$. Suppose that $x \in K$. Then from $x \bar{\in} L_j$ and $L_j \subseteq K$, we have $L_j \subseteq K \setminus \{x\} = K'$.

Obviously: $L_j \longrightarrow R_j \xrightarrow{*} \{x\}, \quad K' \xrightarrow{*} L_j$.

Consequently, $\quad K' \xrightarrow{*} \{x\}$.

Combining with $K' \longrightarrow K'$, we have $K' \xrightarrow{*} K' \cup \{x\} = K$.
This contradicts the fact that K is a key. Hence $\forall K \in \mathscr{K}(\Omega, F): x \bar{\in} K$ i. e. $x \in \bar{H}$. From the inclusion just proved, it is obvious that

$$\bigcup_{L_i=\varnothing} R_i \subseteq \bigcup_{L_i \subseteq G} (R_i \setminus L_i) \subseteq \bar{H}.$$

Taking that into account we can eliminate from a relation scheme all functional dependencies of the form $\varnothing \to R_i$, while preserving its set of all keys.
Now, we give a classification of the relation schemes as follows:

$\mathscr{L}_0=\{\langle \Omega, F\rangle \mid \langle \Omega, F\rangle$ is a relation scheme$\}$

$\mathscr{L}_1=\{\langle \Omega, F\rangle \in \mathscr{L}_0 \mid \Omega=L \cup R\}$.

$\mathscr{L}_2=\{\langle \Omega, F\rangle \in \mathscr{L}_0 \mid L \subseteq R=\Omega\}$

$\mathscr{L}_3=\{\langle \Omega, F\rangle \in \mathscr{L}_0 \mid R \subseteq L=\Omega\}$

$\mathscr{L}_4=\{\langle \Omega, F\rangle \in \mathscr{L}_0 \mid L=R=\Omega\}$, where $L=\bigcup_{i=1}^{k} L_i$ and $R=\bigcup_{i=1}^{k} R_i$

From the above classification, it is easily seen that:

α) $\mathscr{L}_4 \subseteq \mathscr{L}_3 \subseteq \mathscr{L}_1 \subseteq \mathscr{L}_0$

β) $\mathscr{L}_4 \subseteq \mathscr{L}_2 \subseteq \mathscr{L}_1 \subseteq \mathscr{L}_0$

γ) $\mathscr{L}_4 = \mathscr{L}_2 \cap \mathscr{L}_3$.

Figure 1 shows the hierarchy of classes $\mathscr{L}_0, \mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3, \mathscr{L}_4$.
We are now in a position to prove the following theorems.

Theorem 1.1. *Let $\langle \Omega, F\rangle$ be a relation scheme, $Z \subseteq G$ $\langle \Omega_1, F_1\rangle = \langle \Omega, F\rangle - Z$. Then X is a key of $\langle \Omega_1, F_1\rangle$ iff $X \cap Z = \varnothing$ and $X \cup Z$ is a key of $\langle \Omega, F\rangle$.*

P r o o f. First we prove the necessity. Suppose that X is a key of $\langle\Omega_1, F_1\rangle$. Obviously $X\subseteq\Omega_1$, therefore $X\cap Z=\emptyset$. Since X is a key of $\langle\Omega_1, F_1\rangle$, $X\xrightarrow[F_1]{}\Omega_1$. Taking lemma 1.1. into account we get $X\cup Z\xrightarrow[F]{}\Omega_1\cup Z=\Omega$, showing that $X\cup Z$ is a superkey of

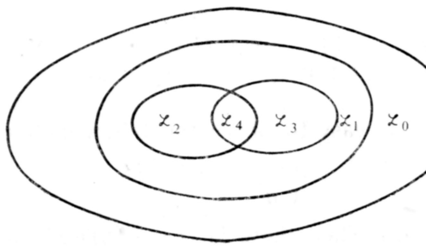

Fig. 1

$\langle\Omega, F\rangle$. Were $X\cup Z$ not a key of $\langle\Omega, F\rangle$ then there would exist a key $\bar{X}$ of $\langle\Omega, F\rangle$ such that

$$Z\subseteq\bar{X}\subset X\cup Z.$$

Consequently, there would exist an $X_1\subset X$ such that $\bar{X}=X_1\cup Z$, $X_1\cap Z=\emptyset$. Since $\bar{X}$ is supposed to be a key of $\langle\Omega, F\rangle$, $X_1\cup Z\xrightarrow[F]{}\Omega$. Applying lemma 1.1, clearly

$$(X_1\cup Z)\setminus Z\xrightarrow[F_1]{}\Omega\setminus Z,$$

that is $X_1\xrightarrow[F_1]{}\Omega_1$. This contradicts the hypothesis that X is a key of $\langle\Omega_1, F_1\rangle$. Thus $X\cup Z$ is a key of $\langle\Omega, F\rangle$.

We now turn to the proof of sufficiency. Suppose that $X\cap Z=\emptyset$ and $X\cup Z$ is a key of $\langle\Omega, F\rangle$. We have to show that X is a key of $\langle\Omega_1, F_1\rangle$.

Since $X\cup Z$ is a key of $\langle\Omega, F\rangle$ we have $X\cup Z\xrightarrow[F]{}\Omega$. By virtue of lemma 1.1, we get $(X\cup Z)\setminus Z\xrightarrow[F_1]{}\Omega\setminus Z$. Consequently (from $X\cap Z=\emptyset$): $X\xrightarrow[F_1]{}\Omega_1$, showing that X is a superkey of $\langle\Omega_1, F_1\rangle$. Assume that X is not a key of $\langle\Omega_1, F_1\rangle$. Then, there would exist a key $\bar{X}$ of $\langle\Omega_1, F_1\rangle$ such that $\bar{X}\subset X$ and $\bar{X}\xrightarrow[F_1]{}\Omega_1$. Applying lemma 1.1, it follows:

$$\bar{X}\cup Z\xrightarrow[F]{}\Omega_1\cup Z=\Omega,$$

where

$$\bar{X}\cup Z\subset X\cup Z.$$

This contradicts the fact that $X\cup Z$ is a key of $\langle\Omega, F\rangle$. Hence X is a key of $\langle\Omega_1, F_1\rangle$.

The proof is complete.

T h e o r e m 1.2. *Let $\langle\Omega, F\rangle$ be a relation scheme, $Z\subseteq\Omega$, $Z\cap H=\emptyset$ and $\langle\Omega_1, F_1\rangle=\langle\Omega, F\rangle-Z$.*
Then X is a key of $\langle\Omega_1, F_1\rangle$ iff X is a key of $\langle\Omega, F\rangle$.

P r o o f. (i) (The necessity).
Suppose that X is a key of $\langle\Omega_1, F_1\rangle$. Obviously $X\xrightarrow[F_1]{}\Omega_1$. By virtue of lemma 1.1, we have

$$X \cup Z \xrightarrow[F]{} \Omega_1 \cup Z = \Omega,$$

showing that $X \cup Z$ is a superkey of $\langle \Omega, F \rangle$. Hence, there exists a key $\bar{X}$ of $\langle \Omega, F \rangle$ such that $\bar{X} \subseteq X \cup Z$. Since $Z \cap H = \emptyset$, then $\bar{X} \cap Z = \emptyset$. From this, it is easy to see that $\bar{X} \subseteq X$. There are two possible cases:

a) $\bar{X} = X$. Then obviously X is a key of $\langle \Omega, F \rangle$.

b) $\bar{X} \subset X$. Since $\bar{X}$ is a key of $\langle \Omega, F \rangle$, $\bar{X} \xrightarrow[F]{} \Omega$.

Applying lemma 1.1, we have $\bar{X} \setminus Z \xrightarrow[F_1]{} \Omega \setminus Z$, that is $\bar{X} \xrightarrow[F_1]{} \Omega_1$.

This contradicts the fact that X is a key of $\langle \Omega_1, F_1 \rangle$.

(ii) (The sufficiency).

Suppose that X is a key of $\langle \Omega, F \rangle$. We have to prove that X is also a key of $\langle \Omega_1, F_1 \rangle$. By the definition of keys, we have $X \xrightarrow[F]{} \Omega$. Applying lemma 1.1: $X \setminus Z \xrightarrow[F_1]{} \Omega \setminus Z = \Omega_1$. Since $Z \cap H = \emptyset$, it follows $X \cap Z = \emptyset$. Consequently, $X \xrightarrow[F_1]{} \Omega_1$ showing that X is a superkey of $\langle \Omega_1, F_1 \rangle$.

Now, assume the reverse that X is not a key of $\langle \Omega_1, F_1 \rangle$. Then there would exist a key $\bar{X}$ of $\langle \Omega_1, F_1 \rangle$ such that $\bar{X} \subset X$. Obviously $\bar{X} \xrightarrow[F_1]{} \Omega_1$. We invoke lemma 1.1. to deduce

$$\bar{X} \cup Z \xrightarrow[F]{} \Omega_1 \cup Z = \Omega,$$

showing that $\bar{X} \cup Z$ is a superkey of $\langle \Omega, F \rangle$. Consequently, there exists a key $\bar{\bar{X}}$ of $\langle \Omega, F \rangle$ such that $\bar{\bar{X}} \subseteq \bar{X} \cup Z$, $\bar{\bar{X}} \cap Z = \emptyset$.

From this $\bar{\bar{X}} \subseteq \bar{X} \subset X$. This contradicts the hypothesis that X is a key of $\langle \Omega, F \rangle$.

The proof is complete.

Basing on theorems 1.1 and 1.2, next we investigate only the class of Z-translations with $Z \neq \emptyset$, $Z = Z_1 \cup Z_2$, $Z_1 \cap Z_2 = \emptyset$, $Z_1 \subseteq G$, $Z_2 \cap H = \emptyset$. Bearing this in mind, if

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - Z,$$

then applying theorem 1.1 and 1.1 one after another to the Z_2-translation and the Z_1-translation, we have: X is a key of $\langle \Omega_1, F_1 \rangle$ if and only if $X \cap Z = \emptyset$ and $X \cup Z_1$ is a key of $\langle \Omega, F \rangle$. For the sake of convenience, we use in the sequel the notation

$$\langle \Omega, F \rangle \underset{\rho = (Z_2, Z_1)}{\Longrightarrow} \langle \Omega_1, F_1 \rangle,$$

where the meaning of ρ is obvious.

To continue, let us recall a result in [1]. Let $S = \langle \Omega, F \rangle$ be a relation scheme, where $\Omega = \{A_1, \ldots, A_n\}$ — the set of attributes, $F = \{L_i \longrightarrow R_i \mid L_i, R_i \subseteq \Omega, i = 1, \ldots, k\}$ — the set of functional dependencies.

Let us denote

$$L = \bigcup_{i=1}^{k} L_i, \quad R = \bigcup_{i=1}^{k} R_i.$$

Then, the necessary condition for which X is a key of S is that $\Omega \setminus R \subseteq X \subseteq (\Omega \setminus R) \cup (L \cap R)$. For $V \subseteq \Omega$ we denote $\bar{V} = \Omega \setminus V$. It is easily seen that $\overline{L \cup R} \subseteq \Omega \setminus R \subseteq G$,

$L \setminus R \subseteq \Omega \setminus R \subseteq G$, $R \setminus L \subseteq \bar{H}$, consequently $(R \setminus L) \cap H = \emptyset$, and we have the following lemma:

Lemma 1.2. *Let $S = \langle \Omega, F \rangle$ be a relation scheme, $Z \subseteq G$, where G is the intersection of all the keys of S.*

Then $(Z^+ \setminus Z) \cap H = \emptyset$, where H is the union of all the keys of S.

Proof. Assume the reverse that $(Z^+ \setminus Z) \cap H \neq \emptyset$. Then, there would exist an attribute $A \in Z^+$, $A \notin Z$ and $A \in H$. Consequently, there exists a key X of $S = \langle \Omega, F \rangle$ such that $A \in X$.
Since $A \in Z^+$ and $A \notin Z$ we infer that $Z \subseteq X \setminus A$. Hence

$$X \setminus A \overset{*}{\longrightarrow} Z \overset{*}{\longrightarrow} Z^+ \overset{*}{\longrightarrow} A$$

with $A \in X$.

This constradicts the fact that X is a key of S.

The proof is complete.

From the results mentioned above the following theorems are obvious.

Theorem 1.3. *Let $S = \langle \Omega, F \rangle$ be a relation scheme belonging to $\mathcal{L}_0$,*

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - \overline{L \cup R}.$$

Then $$\langle \Omega, F \rangle \underset{\rho = (\overline{L \cup R},\ \overline{L \cup R})}{\Longrightarrow} \langle \Omega_1, F_1 \rangle$$

with $$\langle \Omega_1, F_1 \rangle \in \mathcal{L}_1.$$

Proof. As remarked above $\overline{L \cup R} \subseteq G$. Applying Theorem 1.1. to the Z-translation with $Z = \overline{L \cup R}$, we have

$$\langle \Omega, F \rangle \underset{\rho = (\overline{L \cup R},\ \overline{L \cup R})}{\Longrightarrow} \langle \Omega_1, F_1 \rangle.$$

The theorem 1.3 is illustrated by Fig. 2.

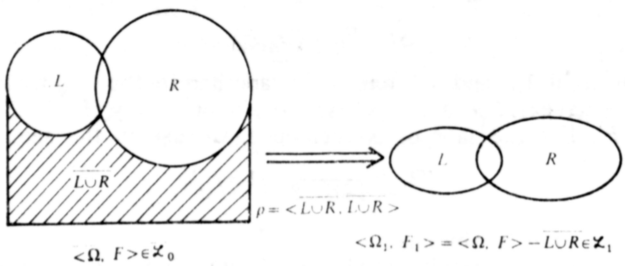

Fig. 2

Example 1. Let there be given $S = \langle \Omega, F \rangle$ with $\Omega = \{a, b, c, d, e\}$, $F = \{c \to d, d \to e\}$ We have $\overline{L \cup R} = ab$. Consider $\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - ab$. Obviously, $\Omega_1 = \{c, d, e\}$, $F_1 = \{c \to d, d \to e\}$. It is easily seen that c is the unique key of $\langle \Omega_1, F_1 \rangle$, hence abc is the unique. key of $\langle \Omega, F \rangle$.

Theorem 1.4. *Let $\langle \Omega, F \rangle$ be a relation scheme of $\mathcal{L}_0$,*

$$\langle \Omega_1, F_1\rangle = \langle \Omega, F\rangle - (\overline{L \cup R} \cup (L \setminus R)).$$

Then
$$\langle \Omega, F\rangle \xrightarrow[\rho=(\overline{L\cup R}\cup(L\setminus R),\overline{L\cup R}\cup(L\setminus R))]{} \langle \Omega_1, F_1\rangle$$
with
$$\langle \Omega_1, F_1\rangle \in \mathscr{L}_2.$$

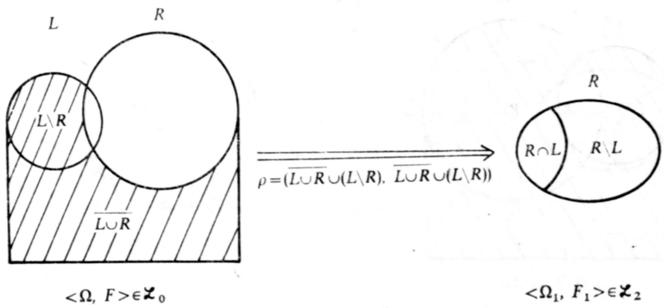

Fig. 3

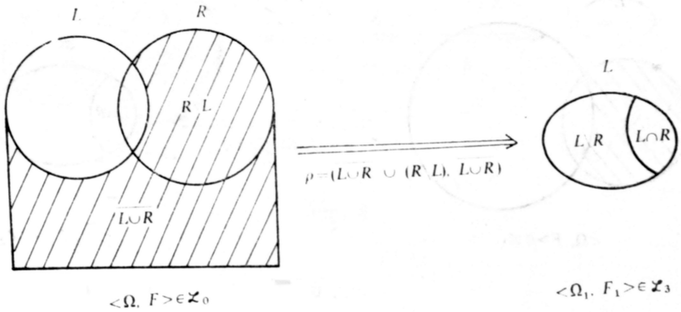

Fig. 4

P r o o f. It is clear that $Z=\overline{L \cup R} \cup (L \setminus R)=\Omega \setminus R \subseteq G$. Theorem 1.4 now follows from applying theorem 1.1 to the Z-translation. Theorem 1.4 is illustrated by Fig. 3.

Theorem 1.5. *Let $S=\langle \Omega, F\rangle$ be a relation scheme of $\mathscr{L}_0$,*

$$\langle \Omega_1, F_1\rangle = \langle \Omega, F\rangle - ((\overline{L \cup R}) \cup (R \setminus L)).$$

Then
$$\langle \Omega, F\rangle \xrightarrow[\rho=(\overline{L\cup R}\cup(R\setminus L),\ \overline{L\cup R})]{} \langle \Omega_1, F_1\rangle$$

with $\langle \Omega_1, F_1\rangle \in \mathscr{L}_3$.

P r o o f. As remarked above, $R \setminus L \subseteq \bar{H}$. Let $Z=\overline{L \cup R} \cup (R \setminus L)=Z_1 \cup Z_2$, where $Z_1=\overline{L \cup R} \subseteq G$, $Z_2=R \setminus L$, $Z_2 \cap H=\emptyset$. Theorem 1.5 now follows from consecutive applications of theorems 1.2 and 1.1 one after another to the Z_2-translation and the Z_1-translation. Theorem 1.5 is illustrated by Fig. 4.

T h e o r e m 1.6. *Let $S=\langle \Omega, F\rangle$ be a relation scheme of $\mathscr{L}_0$,*

$$\langle\Omega_1,\ F_1\rangle=\langle\Omega,\ F\rangle-(\overline{L\cup R}\cup(L\setminus R)\cup(R\setminus L)).$$

Then $$\langle\Omega,\ F\rangle\ \xrightarrow[\rho=(\overline{L\cup R}\cup(L\setminus R),\ \cup(R\setminus L),\ \overline{L\cup R}\cup(L\setminus R))]{}\ \langle\Omega_1,\ F_1\rangle,$$

with $\langle\Omega_1,\ F_1\rangle\in\mathscr{L}_4$.

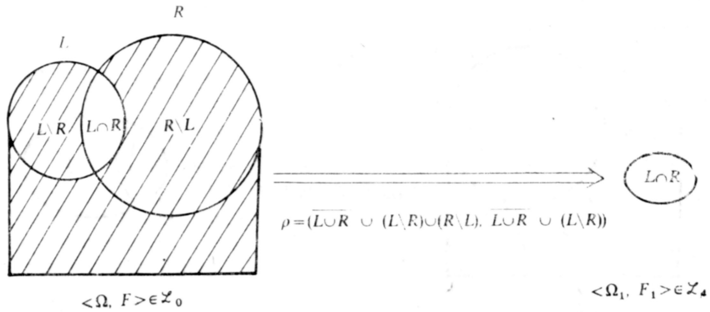

Fig. 5

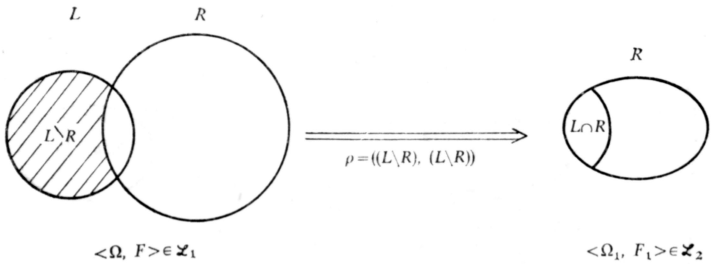

Fig. 6

P r o o f. Let $Z=\overline{L\cup R}\cup(L\setminus R)\cup(R\setminus L)=Z_1\cup Z_2$, where $Z_1=\overline{L\cup R}\cup(L\setminus R)=\Omega\setminus R\subseteq G$, $Z_2=R\setminus\overline{L}\subseteq H$, or equivalently $Z_2\cap H=\varnothing$. It is obvious that $\langle\Omega_1,\ F_1\rangle$ is obtained from $\langle\Omega,\ F\rangle$ by the Z-translation. The proof of theorem 1.6 is straight-forward. Theorem 1.6 is illustrated by Fig. 5.

Similarly, we can prove the following theorems:

T h e o r e m 1.7. *Let* $S=\langle\Omega,\ F\rangle$ *be a relation scheme of* $\mathscr{L}_1$,

$$\langle\Omega_1,\ F_1\rangle=\langle\Omega,\ F\rangle-(L\setminus R).$$

Then $$\langle\Omega,\ F\rangle\ \xrightarrow[\rho=(L\setminus R,\ L\setminus R)]{}\ \langle\Omega_1,\ F_1\rangle,$$

where $\langle\Omega_1,\ F_1\rangle\in\mathscr{L}_2$.

Theorem 1.7 is illustrated by Fig. 6.

Theorem 1.8. *Let* $S=\langle\Omega,\ F\rangle$ *be a relation scheme of* $\mathscr{L}_1$, $\langle\Omega_1, F_1\rangle=\langle\Omega, F\rangle-(R\setminus L)$. *Then* $\langle\Omega,\ F\rangle\xrightarrow[\rho=(R\setminus L,\ \varnothing)]{}\langle\Omega_1,\ F_1\rangle$, *where* $\langle\Omega_1,\ F_1\rangle\in\mathscr{L}_3$.

Theorem 1.8 is illustrated by Fig. 7.

T h e o r e m 1.9. *Let* $S=\langle\Omega,\ F\rangle$ *be a relation scheme of* $\mathscr{L}_1$, $\langle\Omega_1,\ F_1\rangle=\langle\Omega,\ F\rangle-((L\setminus R)\cup(R\setminus L))$.

Then $\langle\Omega, F\rangle \xRightarrow[\rho=((L\setminus R)\cup(R\setminus L),\, L\setminus R)]{} \langle\Omega_1, F_1\rangle$, *where* $\langle\Omega_1, F_1\rangle \in \mathscr{L}_4$.
Theorem 1.9 is illustrated by Fig. 8.

Theorem 1.10. *Let* $\langle\Omega, F\rangle$ *be a relation scheme of* $\mathscr{L}_2$, $\langle\Omega_1, F_1\rangle=\langle\Omega, F\rangle-(R\setminus L)$. *Then* $\langle\Omega, F\rangle \xRightarrow[\rho=(R\setminus L,\, \varnothing)]{} \langle\Omega_1, F_1\rangle$, *where* $\langle\Omega_1, F_1\rangle \in \mathscr{L}_4$.

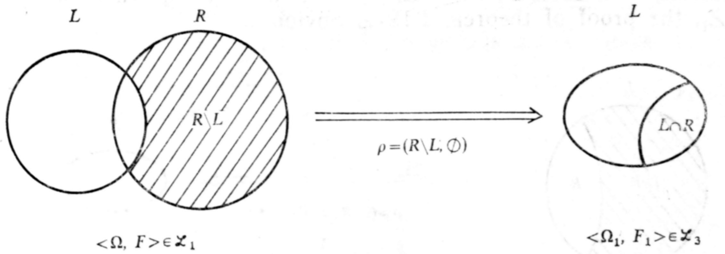

Fig. 7

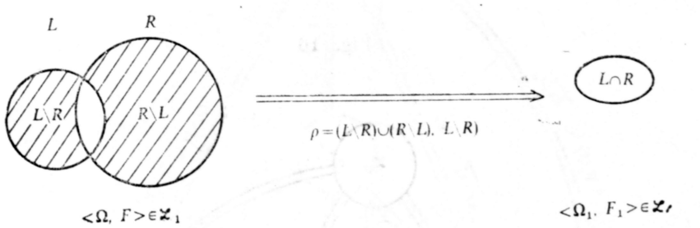

Fig. 8

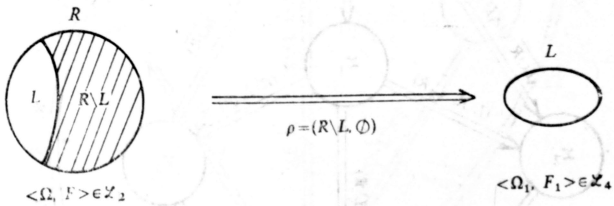

Fig. 9

Theorem 1.10 is illustrated by Fig. 9.

Theorem 1.11. *Let* $\langle\Omega, F\rangle$ *be a relation scheme of* $\mathscr{L}_3$, $\langle\Omega_1, F_1\rangle=\langle\Omega, F\rangle-(L\setminus R)$. *Then* $\langle\Omega, F\rangle \xRightarrow[\rho=(L\setminus R,\, L\setminus R)]{} \langle\Omega_1, F_1\rangle$, *where* $\langle\Omega_1, F_1\rangle \in \mathscr{L}_4$.
Theorem 1.11 is illustrated by Fig. 10.
Combining theorems 1.3—1.11, we have the diagram of translations as illustrated on Fig. 11.

Now, the following theorem follows from theorems 1.1, 1.2 and lemma 1.3.

Theorem 1.12. *Let* $\langle\Omega, F\rangle$ *be a relation scheme of* $\mathscr{L}_0$, $\langle\Omega_1, F_1\rangle=\langle\Omega, F\rangle-\{\overline{L\cup R}\cup(L\setminus R)^+\cup(R\setminus L)\}$.

Then $\langle\Omega, F\rangle \xrightarrow[\rho=(\overline{L\cup R}\cup(L\setminus R)^{+}\cup(R\setminus L),\ \overline{L\cup R}\cup(L\setminus R))]{} \langle\Omega_1, F_1\rangle$, *where* $\langle\Omega_1, F_1\rangle \in \mathscr{L}_4$.

Proof. Put $Z=\overline{L\cup R}\cup(L\setminus R)\cup[(L\setminus R)^{+}\setminus(L\setminus R)]\cup(R\setminus L)=Z_1\cup Z_2$, where $Z_1=\overline{L\cup R}\cup(L\setminus R)=\Omega\setminus R\subseteq G$, $Z_2=[(L\setminus R)^{+}\setminus(L\setminus R)]\cup(R\setminus L)$. Clearly $Z_2\cap H=\emptyset$.
Applying theorem 1.2 to $\langle\Omega', F'\rangle=\langle\Omega, F\rangle-Z_2$, and then, theorem 1.1 to $\langle\Omega_1, F_1\rangle=\langle\Omega', F'\rangle-Z_1$, the proof of theorem 1.12 is obvious.

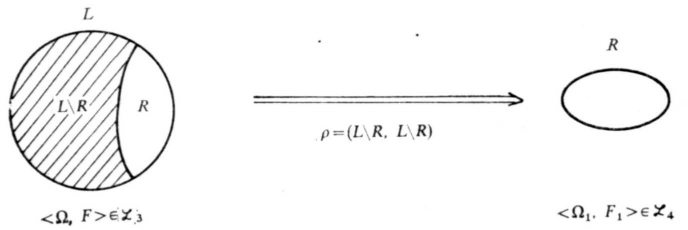

Fig. 10

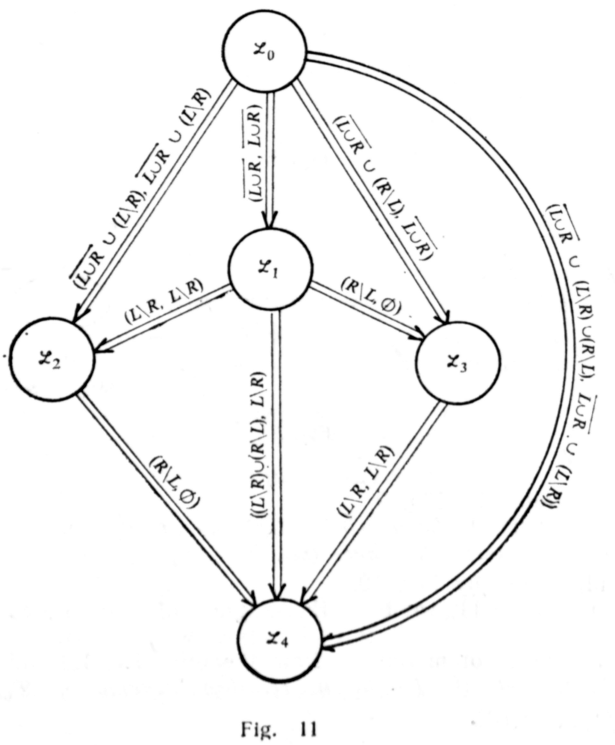

Fig. 11

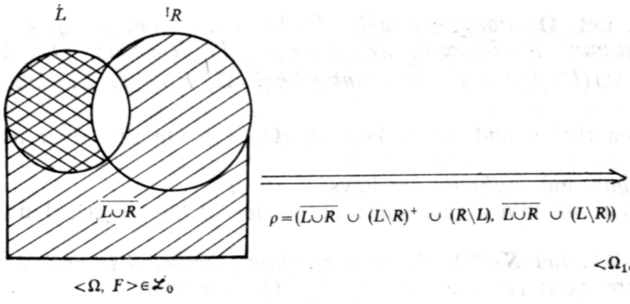

Fig. 12

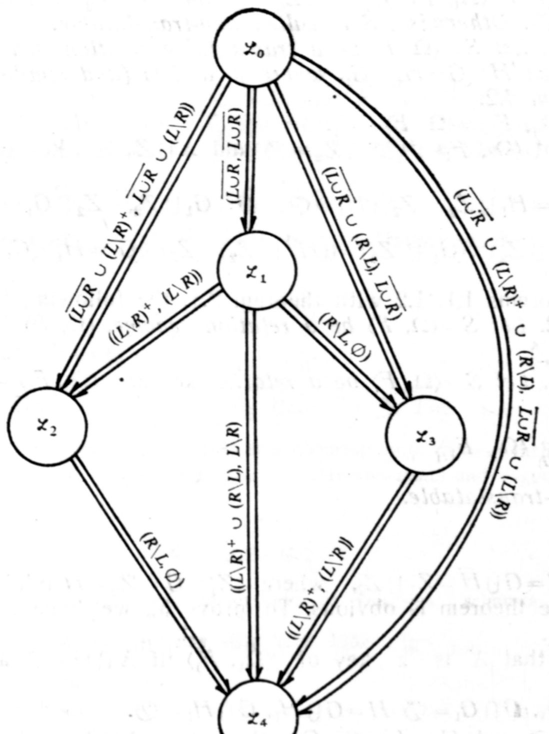

Fig. 13

Theorem 1.12 is illustrated by Fig. 12.

The "double hashing" part is $(L\setminus R)^{+}$.

From the just mentioned results, we have the following diagram of translations of relation schemes (Fig. 13).

Example 2. Let $\Omega = abhgqmnvwkl$, $F = \{a \to b,\ b \to h,\ g \to q,\ kv \to w,\ w \to vl\}$. We have $L = abgkvw$; $R = bhqwvl$, $R \setminus L = hql$; $L \setminus R = kga$; $(L \setminus R)^+ = kgabhq$; $\overline{L \cup R} = mn$; $(R \setminus L) \cup (L \setminus R)^+ \cup (\overline{L \cup R}) = mnkgabhql$ $\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - mnkgabhql = \langle wv, \{v \to w,\ w \to v\} \rangle$.

It is easily seen that v and w are keys of $\langle \Omega_1, F_1 \rangle$. On the other hand, $(\overline{L \cup R}) \cup (L \setminus R) = mnkga$.
Consequently $mnkgav$ and $mnkgaw$ are keys of $\langle \Omega, F \rangle$.

2. In this section we investigate some properties of the so-called non-translatable relation schemes.

Definition 2.1. *Let $S = \langle \Omega, F \rangle$ be a relation scheme. S is called translatable if and only if there exist certain sets $Z_1, Z_2 \subseteq \Omega$ such that:*

(i) $Z_1 \neq \emptyset$;

(ii) *X is a key of $\langle \Omega_1, F_1 \rangle$ iff $X \cap Z_2 = \emptyset$ and $X \cup Z_2$ is a key of $\langle \Omega, F \rangle$, where $\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - Z_1$. Otherwise, S is called non-translatable.*

Theorem 2.1. *Let $S = \langle \Omega, F \rangle$ be a translatable relation scheme with Z_1, Z_2 as defined above. Then $H \setminus G = H_1 \setminus G_1$, where H and G (and similarly H_1 and G_1) are defined in definition 1.2.*

Proof. Let $\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - Z_1$.
Since X is a key of $\langle \Omega_1, F_1 \rangle$ iff $X \cap Z_2 = \emptyset$ and $X \cup Z_2$ is a key of $\langle \Omega, F \rangle$, it follows

$$H = H_1 \cup Z_2, \quad Z_2 \cap H_1 = \emptyset, \quad G = G_1 \cup Z_2, \quad Z_2 \cap G_1 = \emptyset,$$

hence $H \setminus G = (H_1 \cup Z_2) \setminus (G_1 \cup Z_2) = ((H_1 \cup Z_2) \setminus Z_2) \setminus G_1 = H_1 \setminus G_1$ (because $Z_2 \cap H_1 = \emptyset$).

Combining theorems 1.1, 1.2 with theorem 2.1, the following theorem is obvious

Theorem 2.2. *Let $S = \langle \Omega, F \rangle$ be a relation scheme, $\langle \Omega, F \rangle$ is non-translatable iff $H = \Omega$ and $G = \emptyset$.*

Theorem 2.3. *Let $S = \langle \Omega, F \rangle$ be a relation scheme, $\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - (G \cup \overline{H})$. Then:*

a) $\langle \Omega, F \rangle \xrightarrow[\rho = (G \cup \overline{H},\ G)]{} \langle \Omega_1, F_1 \rangle$.

b) $\langle \Omega_1, F_1 \rangle$ *is non-translatable.*

c) $\langle \Omega_1, F_1 \rangle \in \mathscr{L}_4$.

Proof. Let $Z = G \cup \overline{H} = Z_1 \cup Z_2$, where $Z_1 = G$, $Z_2 = \overline{H}$ (clearly $Z_2 \cap H = \emptyset$). Hence part a) of the theorem is obvious. To prove b), we have only to show that $G_1 = \emptyset$ and $H_1 = \Omega_1$.
From a) it is clear that X is a key of $\langle \Omega_1, F_1 \rangle$ iff $X \cap G = \emptyset$ and $X \cup G$ is a key of $\langle \Omega, F \rangle$.
Therefore, $G = G \cup G_1$, $G \cap G_1 = \emptyset$ $H = G \cup H_1$, $G \cap H_1 = \emptyset$.
Hence $G_1 = G \setminus G = \emptyset$ and $H_1 = H \setminus G$. On the other hand, we have $\Omega_1 = \Omega \setminus (G \setminus \overline{H}) = (\Omega \setminus \overline{H}) \setminus G = H \setminus G = H_1$.
To prove c) we have to show that $L^1 = R^1 = \Omega_1$ where L^1 and R^1 are the union of all the left sides and right ones of all functional dependencies of F_1, respectively.

It is known [1] that $\Omega_1 \setminus R^1 \subseteq G_1 = \emptyset$. On the other hand, $R^1 \subseteq \Omega_1$. Hence $R^1 = \Omega_1$. There remained to prove $L^1 = \Omega_1$. Where that is false, there would exist an $A \in \Omega_1 \setminus L^1$. Since $R^1 = \Omega_1$, we have $A \in R^1$ and $A \bar{\in} L^1$. From $\Omega_1 = H_1$, there exists a key X of $\langle \Omega_1, F_1 \rangle$ such that $A \in X$ and $X \xrightarrow{*} \Omega_1$. Since $A \bar{\in} L^1$ it follows from [1] that $X \setminus A \xrightarrow{*} \Omega_1 \setminus A$.

Evidently, $L^1 \subseteq \Omega_1 \setminus A$ and from this, $X \setminus A \xrightarrow{*} \Omega_1 \setminus A \xrightarrow{*} L^1 \xrightarrow{*} R^1 \xrightarrow{*} A$. This contradicts the fact that X is a key of $\langle \Omega_1, F_1 \rangle$, hence $L^1 = \Omega_1$.

The proof is complete.

From the proof of c) we conclude that all non-translatable relation schemes are of type $\mathscr{L}_4$.

Theorem 2.4. *Let $S = \langle \Omega, F \rangle$ be a relation scheme of $\mathscr{L}_4$ satisfying the following conditions*:

(i) $L_i \cap R_i = \emptyset, \quad \forall i = 1, 2, \ldots, k,$

(ii) *for each L_i, $i = 1, \ldots, k$ there exists a key X_i such that $L_i \subseteq X_i$.*

Then $\langle \Omega, F \rangle$ is a non-translatable relation scheme.

Proof. We have to prove that $H = \Omega$ and $G = \emptyset$. In fact, from $\langle \Omega, F \rangle \in \mathscr{L}_4$ we have $L = R = \Omega$. By virtue of the hypothesis of the theorem we have

$$\Omega = L = \bigcup_{i=1}^{k} L_i \subseteq \bigcup_{i=1}^{k} X_i \subseteq H \subseteq \Omega.$$

Consequently, $H = \Omega$.

To prove $G = \emptyset$ we first show that if $L_i \rightarrow R_i$ and X_i is a key such that $L_i \subseteq X_i$, then $X_i \cap R_i = \emptyset$. Assume the reverse that $X_i \cap R_i \neq \emptyset$. Then, there would exist an $A \in X_i \cap R_i$. Since $L_i \cap R_i = \emptyset$, clearly $A \notin L_i$. Therefore $L_i \subseteq X_i \setminus A$. On the other hand,

$$X_i \setminus A \xrightarrow{*} L_i \xrightarrow{*} R_i \xrightarrow{*} A,$$

showing that X_i is not a key of $\langle \Omega, F \rangle$. This is a contradiction. From $X_i \cap R_i = \emptyset$, it follows:

$$X_i \subseteq \Omega \setminus R_i.$$

Thus $G \subseteq \bigcap_{i=1}^{k} X_i \subseteq \bigcap_{i=1}^{k} (\Omega \setminus R_i) = \Omega \setminus \bigcup_{i=1}^{k} R_i$.

Since $R = \Omega$ clearly $G \subseteq \Omega \setminus \Omega = \emptyset$, showing that $G = \emptyset$. The proof is complete.

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