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A CONSTRUCTION OF ALMOST ANTI-SELF-DUAL CONNECTIONS ON KUMMER SURFACES

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Using a partition of unity we glue Eguchi—Hanson metric near a singular point of Kummer surface and the Euclidean metric. The resulting metric is Hermitian one. It determines canonically a connection, which is called almost anti-self-dual. Two L_p estimates on its curvature are proved. This work is the first step of Taubes's iteration procedure.

1. Introduction. If the first Chern class of a compact complex two-dimensional Kähler manifold vanishes, then there exist two types of such manifolds: Abelian manifolds and K3 surfaces. By definition K3 surface is a two-dimensional compact complex manifold whose first Betti number $b_1=0$ and whose first Chern class $c_1=0$. We are interested in finding a metric g on a two-manifold which satisfies Einstein vacuum equations

$$\mathrm{Ricci}(g)=0.$$

For instance, in the case of torus T in $\mathbf{C}^n$ the unique solution is the restriction to T of the Euclidean flat metric in $\mathbf{C}^n$, but it is not interesting from differential geometrical point of view.

For the present K3 surfaces are the unique simply connected compact manifolds on which there exists nontrivial Ricci — flat metric ([10]). The construction of this metric in explicit form or in appropriate approximation is of great interest both for the mathematicians ([4]) and for the physicists ([6]). Actually, N. Hitchin has set in [4] the problem of finding of the K3 metric explicitly and proposed a method of attacking based on twistor theory.

One important particular case of K3 surfaces are the so-called Kummer surfaces, which are viewed in the following way.

Let Λ be a lattice in $\mathbf{C}^2$, generated by four vectors, linearly independent over $\mathbf{R}$, $\Lambda\cong\mathbf{Z}^4$. Consider the complex torus $T=\mathbf{C}^2/\Lambda$, which is a compact complex manifold. Let

$$\sigma\colon\ T\to T$$

be the involution, defined by

$$\sigma(x)=-x,$$

where $x\in T$. We introduce the following relation of equivalence on T:

$$x\sim y \ \text{ iff } \ \sigma(x)=y.$$

Denote $X=T/\sim\ =T/\sigma$. X is said to be Kummer surface.

One verifies that $c_1(X)=0$. It is proved that $b_1(X)=0$ ([8]), i. e. X is certainly of type K3.

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It is easy to see that X has 16 singular points and near a singular point it can be embedded locally in $\mathbf{C}^3$. In fact, near a singular point it can be identified locally with the cone $y_0^2=y_1y_2$ in $\mathbf{C}^3$.

The purpose of this paper is to construct an almost anti-self-dual connection on the Kummer surface X. Denote it by ∇_0 and let F^{∇_0} be its curvature.

Our main result in this paper is the following

Proposition 1. *These exists a constant $c_1>0$ such that for all $p\geqq 1$ and sufficiently small λ*

$$\| F^{\nabla_0} \|_{L_p} \leqq c_1 \lambda^{4/p} \tag{1}$$

$$\| P_+ F^{\nabla_0} \|_{L_p} \leqq c_1 \lambda^{4/p}. \tag{2}$$

Here $P_+=(1+*)/2$, where $*$ is the Hodge-star operator and c_1 is independent of λ.

Definition. *A connection which satisfies (2) is called almost anti-self-dual connection.*

How do we construct the connection ∇_0?

First we blow up the 16 singular points and let $\widehat{X}$ be the resulting non-singular surface. Under this transformation every singular point is replaced by a copy of $\mathbf{CP}^1$, the complex one-dimensional projective space. In a neighbourhood (ball) of every distinct projective line in $\widehat{X}$, which has sufficiently small radius λ, there exists the metric of Eguchi-Hanson (see [1, 2] and [3]). Denote this metric by g_{EH}. Outside the neighbourhood of radius 2λ we consider the Euclidean metric g_E. Let (α, β) be an appropriate partition of unity subordinate to these balls. Set

$$h=\alpha g_{EH}+\beta g_E, \tag{3}$$

h is a Hermitian metric, but it is not Kähler one, while the metrics g_{EH} and g_E are Kähler. The metric h determines canonically a unique connection ∇_0. This is the connection we have looked for. We hope that (3) yields a good approximation of the Kähler-Einstein-Calabi-Yau metric ([10]).

The metric h, as an approximation of the K3 metric, was deduced (using the words of N. Hitchin, [4], p. 115) "heuristically" by D. Page [7], but in this paper we use it in a concrete way. The Proposition 1 is only the first step of the iteration scheme of C. Taubes [9], which will enable us to obtain stronger results concerning existence of anti-self-dual connections and metrics on Kummer surfaces. They will appear in a separate paper.

In the next section we introduce some notations and give explicit form of the metric h. In section 3 we prove Proposition 1 using a technical lemma which is proved in section 4.

In conclusion of this introduction we would like to thank Professor Andrei Todorov for proposing the problem, his useful advices and the help in preparation of the work.

2. Definition of the almost anti-self-dual connection. Let $\{p_j\}$, $j=1,\ldots,16$, be the set of the singular points of the Kummer surface X. In fact

$$p_j\in\{(0,0,0,0,);\ (0,0,0,1/2);\ (0,0,1/2,0);\ \ldots;\ (1/2,1/2,1/2,1/2)\}.$$

Real coordinates are used above. Let B_λ^j be the ball of radius λ and centre at p_j. We choose $\lambda<1/2$ to be so small that

$$B_{2\lambda}^j\cap B_{2\lambda}^s=\varnothing \quad \text{if} \quad j\neq s.$$

Define

$$\alpha_j(x)=\begin{cases}1, & x\in B_\lambda^j\\ 0, & x\bar{\in} B_{2\lambda}^j\end{cases},\quad 0\leq\alpha_j\leq 1.$$

Set $\alpha(x)=\Sigma_{j=1}^{16}\alpha_j(x)$ and $\beta(x)=1-\alpha(x)$, or $\alpha+\beta=1$. Then

$$\alpha(x)=\begin{cases}1, & \text{if } x\in B_\lambda^j \text{ for some } j=1,\dots,16;\\ 0, & \text{if } x\bar{\in} B_{2\lambda}^s \text{ for every } s.\end{cases}$$

After making 16 σ-processes, instead of $\widehat{B}_\lambda^j$ we write B_λ^j again with no confusion.

Since for every point $p'\in\widehat{X}$ we have $\alpha(p')=\alpha_j(p')$ for some j, it is sufficient to work in a neighbourhood of a singular point $p_j=p$. In the ball $B_\lambda=B_\lambda^j$ we know the metric of Eguchi-Hansan

$$g_{EH}=(\sqrt{1+ct}/(1+|z|^2)^2+c|z|^2|\xi|^2/\sqrt{1+ct})\,dz\otimes d\bar{z}+((c/2)(1+|z|^2)\bar{z}\xi/\sqrt{1+ct})dz\otimes d\bar{\xi}$$
$$+((c/2)(1+|z|^2)z\bar{\xi}/\sqrt{1+ct})d\xi\otimes d\bar{z}+((c/4)(1+|z|^2)^2/\sqrt{1+ct})d\xi\otimes d\bar{\xi},$$

where

$$t=(1+|z|^2)^2|\xi|^2,\quad z,\xi\in\mathbf{C},$$

$c>0$ is an arbitrary constant ([1]). In order to prove Lemma 2, section 4, we choose $c=4$.

g_{EH} is a Kähler metric on the bundle $L\to\mathbf{CP}^1$, where L is biholomorphically equivalent to the cone $\{(y_0,y_1,y_2)\in\mathbf{C}^3\colon y_0^2=y_1y_2\}$. See [1]. But our Kummer surface defines the same bundle after making 16 σ-processes, that is, g_{EH} is the metric near the singular point we need.

Since the real dimension of a K3 surface is four, then there exists a local coordinate chart (φ, U) such that

$$p\in U,\quad \varphi\colon U\to\mathbf{R}^4 \text{ and } \varphi(p)=0.$$

Assume λ is such that $B_{2\lambda}\subset U$. We identify $\varphi(B_\lambda)$, $\varphi(B_{2\lambda})$ and B_λ, $B_{2\lambda}$ correspondingly and instead of $\varphi(B)$ we shall write simply B. Introduce real normal coordinates x_1, x_2, x_3, x_4. Then we make the change

$$z=x_1+\sqrt{-1}x_2,\quad \xi=x_3+\sqrt{-1}x_4. \tag{4}$$

The "normality" of coordinates means (see [9], p. 161) that

a) $\varphi(p)=0\in\mathbf{R}^4$;

b) the components of the Riemannian metric η:

$$\eta^{ij}(p')=(\varphi^*(dx^i),\ \varphi^*(dx^j)),\quad \varphi(p')=x,$$

satisfy

$$\eta^{ij}(p)=\delta^{ij},\quad d\eta^{ij}|_p=0,\quad |\eta^{ij}(p')-\delta^{ij}|\leq|\varphi(p')|^2\rho(p)=|x|^2\rho(p),$$

for all $p'\in U$. $|.|$ is Euclidean norm in $\mathbf{R}^4$ and $\rho(p)<\infty$ is a constant which depends on Riemannian structure of $\widehat{X}$.

Let λ_1 be sufficiently small and $\lambda_1^2\rho(p)\ll 1$. Then for $\lambda<\lambda_1/2$ we have the inclusion $B_{2\lambda}=\{p'\in U\colon |x|<2\lambda\}\subset\{p'\colon |x|<\lambda_1\}$. Hence, for every $p'\in B_{2\lambda}$:

$$|\eta^{ij}(p')-\delta^{ij}|\leq|x|^2\rho(p)<\lambda_1^2\rho(p)\ll 1.$$

After the change of variables (4), we get that the metric $h=\alpha g_{EH}+\beta g_E$ can be expressed in the form

$$(5)\qquad h=(h_{ij})=\begin{pmatrix} A & 0 & C & -D \\ 0 & A & D & C \\ C & D & B & 0 \\ -D & C & 0 & B \end{pmatrix},$$

where

$$A=\alpha[\sqrt{1+4t}/(1+x_1^2+x_2^2)^2+4(x_1^2+x_2^2)(x_3^2+x_4^2)/\sqrt{1+4t}]+\beta,$$

$$B=\alpha(1+x_1^2+x_2^2)^2/\sqrt{1+4t}+\beta,$$

$$C=2\alpha(1+x_1^2+x_2^2)(x_1x_3+x_2x_4)/\sqrt{1+4t},\quad D=2\alpha(1+x_1^2+x_2^2)(x_2x_3-x_4x_1)/\sqrt{1+4t}$$

and $t=(1+x_1^2+x_2^2)^2(x_3^2+x_4^2)$. ∇_0 will denote the unique connection corresponding to h, which is determined by the Christoffel symbols

$$\Gamma^i_{jk}=h^{im}\left(\frac{\partial h_{km}}{\partial x^j}+\frac{\partial h_{jm}}{\partial x^k}-\frac{\partial h_{jk}}{\partial x^m}\right)/2$$

where

$$(6)\qquad h^{-1}=(h^{ij})=\frac{1}{AB-C^2-D^2}\begin{pmatrix} B & 0 & -C & D \\ 0 & B & -D & -C \\ -C & -D & A & 0 \\ D & -C & 0 & A \end{pmatrix}$$

is the inverse matrix of the matrix h and we have used Einstein's summation convention.

3. Proof of Proposition 1. First we shall introduce L_p norms.

The metric η (see the last section) determines a Hodge operator $*_\eta$ and Euclidean metric determines Hodge operator $*$. By $|.|$ we will denote the norm (of a matrix with p-forms as elements) which corresponds to $*$, and by $|.|_\eta$ — the norm corresponding to $*_\eta$. Let $\omega=\omega_{i_1\ldots i_p}dx^{i_1}\wedge\cdots\wedge dx^{i_p}$ be a p-form and n be the real dimension of considered manifold. Then (see [2]):

$$*\omega=(1/p!)\varepsilon^{i_1\ldots i_p}{}_{j_1\ldots j_{n-p}}\,\omega_{i_1\ldots i_p}dx^{j_1}\wedge\ldots\wedge dx^{j_{n-p}}\quad(0\le p\le n)$$

$$*_\eta\omega=(\sqrt{\eta}/p!)\,\eta^{i_1j_1}\ldots\eta^{i_pj_p}\omega_{i_1\ldots i_p}\varepsilon_{j_1\ldots j_pj_{p+1}\ldots j_n}dx^{j_{p+1}}\wedge\ldots\wedge dx^{j_n},$$

where $\eta=\det(\eta_{ij})$ and ε is the fully antisymmetrical tensor.

If F is a matrix with elements p-forms ω^i_j, then $|F|_\eta=(1/p!)\,\eta^{i_1j_1}\ldots\eta^{i_pj_p}\omega^i_{ji_1\ldots i_p}\times\omega^j_{ij_1\ldots j_p}$, in particular $|F|=(1/p!)\,\omega^i_{ji_1\ldots i_p}\omega^j_{ii_1\ldots i_p}$, and

$$\|F\|_{L_s}=C_{n,s}\{\int_X|F|_\eta^{s/2}\sqrt{\eta}\,d^nx\}^{1/s}.$$

If $n=4$, $p=2$, then $0<c_{n,p}\le5$.

Recall that F^{∇_0} is the curvature of the connection ∇_0.

Lemma 1. *There are two constants k_1 and k_2 independent of λ, such that*

$$|F^{\nabla_0}|_\eta \leqq k_1 |F^{\nabla_0}|$$

and

$$|P_+ F^{\nabla_0}|_\eta \leqq k_2 |F^{\nabla_0}|$$

in the ball $B_{2\lambda}$.

Proof. See (8.20), p. 162 and (8.21), p. 163 in [9].

The following lemma will be proved in Section 4.

Lemma 2. *There exists a constant $C_0>0$ independent of λ, such that in the ball $B_{2\lambda}$ the estimate*

$$|R^i_{jkl}| \leqq C_0 \tag{7}$$

holds for all $i, j, k, l=1, 2, 3, 4$. R^i_{jkl} are the components of the Riemannian tensor corresponding to h (or ∇_0).

Now we are ready for the

Proof of the Proposition 1. Outside the ball $B_{2\lambda}$: $h \equiv g_E$ and therefore ∇_0 is flat: $F^{\nabla_0} \equiv 0$. Thus, it is sufficient to prove the assertion in $B_{2\lambda}$.

$$|F^{\nabla_0}| = R^i_{jkl} R^j_{ikl}/8$$

(summation in all indices).

From Lemma 2 we get that $|F^{\nabla_0}| \leqq |R^i_{jkl}| . |R^j_{ikl}|/8 \leqq 32C_0^2$.

Then Lemma 1 gives $|F^{\nabla_0}|_\eta \leqq k_1 \, 32\, C_0^2 \leqq k^2$, $|P_+ F^{\nabla_0}|_\eta \leqq k_2 \, 32\, C_0^2 \leqq k^2$. Therefore

$$\|F^{\nabla_0}\|_{L_p} \leqq 5 \{ \int_{|x|<2\lambda} |F^{\nabla_0}|_\eta^{p/2} \sqrt{\eta}\, d^4x\}^{1/p} \leqq 5k \{ \int_{|x|<2\lambda} dV\}^{1/p}$$

$$\leqq 5\tilde{k}\{\int_0^{2\lambda} \rho^3 d\rho\}^{1/p} \{ \int_{|w|=1} dw\}^{1/p} \leqq 5\tilde{k} . 2^{-2/p} . 2^{4/p} . w_4^{1/p} . \lambda^{4/p} = 5\tilde{k}(4w_4)^{1/p}\lambda^{4/p} \leqq 5\tilde{k} . 4w_4 . \lambda^{4/p}.$$

The last inequality holds because $p \geqq 1$ and $4w_4 = 4 \int_{|w|=1} dw \geqq 1$. Set $C_1 = 20\tilde{k}w_4$. Then $\|F^{\nabla_0}\|_{L_p} \leqq C_1 \lambda^{4/p}$.

The proof of the estimate (2) (see Introduction) is similar.

4. Main technical lemma. We are going to prove Lemma 2 in the previous section.

It is well-known that

$$R^i_{jkl} = (1/2)\, h^{is} [\frac{\partial^2 h_{sk}}{\partial x^l \partial x_j} - \frac{\partial^2 h_{jk}}{\partial x^l \partial x^s} - \frac{\partial^2 h_{sl}}{\partial x^j \partial x^k} + \frac{\partial^2 h_{jl}}{\partial x^k \partial x^s}]$$

$$+ (1/4)\, h^{is} h^{tp} [(\frac{\partial h_{tk}}{\partial x^s} + \frac{\partial h_{ts}}{\partial x^k} - \frac{\partial h_{sk}}{\partial x^t}) (\frac{\partial h_{pj}}{\partial x^l} + \frac{\partial h_{pl}}{\partial x^j} - \frac{\partial h_{jl}}{\partial x^p})$$

$$- (\frac{\partial h_{tl}}{\partial x^s} + \frac{\partial h_{ts}}{\partial x^l} - \frac{\partial h_{ls}}{\partial x^t}) (\frac{\partial h_{pj}}{\partial x^k} + \frac{\partial h_{pk}}{\partial x^j} - \frac{\partial h_{kj}}{\partial x^p})].$$

Then it is sufficient to obtain upper estimates on the quantities

$$|h^{ij}|, \quad |\frac{\partial h_{ij}}{\partial x^k}| \text{ and } |\frac{\partial^2 h_{ij}}{\partial x^k \partial x^l}|.$$

Everywhere below we assume that $|x|^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 \leqq 4\lambda^2 \leqq 1$ $(x \in B_{2\lambda})$.

1. First we shall estimate $|h^{ij}|$. We have

(i) $$|D|\leqq 2|\alpha|(1+x_1^2+x_2^2)(x_2x_3+x_4x_1)/\sqrt{1+4t}\leqq 4$$

(ii) $$|C|=C\leqq 2\alpha(1+x_1^2+x_2^2)(x_1x_3+x_2x_4)/\sqrt{1+4t}\leqq 4$$

(iii) $$|B|=B\leqq\alpha(1+x_1^2+x_2^2)^2/\sqrt{1+4t}+\beta\leqq 5$$

(iv) $$|A|=A=\alpha(1+4(x_3^2+x_4^2)(1+x_1^2+x_2^2)^3)/(\sqrt{1+4t}(1+x_1^2+x_2^2)^2)+\beta\leqq 34.$$

Further $AB-C^2-D^2=\alpha^2+\beta^2+\alpha\beta[\sqrt{1+4t}(1+x_1^2+x_2^2)^2+(4(x_1^2+x_2^2)(x_3^2+x_4^2)+(1+x_1^2+x_2^2)^2)/\sqrt{1+4t}]\geqq\alpha^2+\beta^2=2\alpha^2-2\alpha+1\geqq 1/4$.
The last inequality holds since the discriminant of the equation $2\alpha^2-2\alpha+3/4=0$ is negative.

From (6), 1.(i), (ii), (iii), (iv) and from the fact that $AB-C^2-D^2\geqq 1/4$ it follows that

(8) $$|h^{ij}|\leqq C_0'$$

for some constant $C_0'>0$.

2. We shall obtain estimates on the first derivatives of the metric h.

(i) $$\frac{\partial D}{\partial x^k}=2\frac{\partial\alpha}{\partial x^k}(1+x_1^2+x_2^2)(x_2x_3-x_4x_1)/\sqrt{1+4t}$$
$$+2\alpha[u\sqrt{1+4t}+2(1+x_1^2+x_2^2)(x_1x_4-x_2x_3)v/\sqrt{1+4t}]/(1+4t),$$

where $u=(\partial/\partial x^k)[(1+x_1^2+x_2^2)(x_2x_3-x_4x_1)]$ and $v=(\partial/\partial x^k)[(1+x_1^2+x_2^2)^2(x_3^2+x_4^2)]$. Then

$$\left|\frac{\partial D}{\partial x^k}\right|\leqq 16\lambda^2\left|\frac{\partial\alpha}{\partial x^k}\right|+2\alpha(|u|(1+8\lambda)+4|v|).$$

It is easy to verify that $|u|\leqq 3$ and $|v|\leqq 16$. Hence

$$\left|\frac{\partial D}{\partial x^k}\right|\leqq 16\lambda^2\left|\frac{\partial\alpha}{\partial x^k}\right|+138.$$

In order to estimate the first and second derivatives of α we need the following lemma:

Lemma. ([5]). *The C^∞ function α can be chosen such that there exists independent of λ positive constant $\tilde{C}$ for which the estimates*

(9) $$\left|\frac{\partial\alpha}{\partial x^k}\right|\leqq\tilde{C}^2\lambda^{-1}$$

(10) $$\left|\frac{\partial^2\alpha}{\partial x^k\partial x^l}\right|\leqq 4\tilde{C}^3\lambda^{-2}$$

hold.

Using this lemma we get $\left|\frac{\partial D}{\partial x^k}\right|\leqq 8\tilde{C}^2+138.$

(ii) Similarly we get that $\left|\frac{\partial C}{\partial x^k}\right|\leqq 8\tilde{C}^2+138.$

(iii) Differentiating B we obtain

$$\frac{\partial B}{\partial x^k}=\frac{\partial\alpha}{\partial x^k}\,[(1+x_1^2+x_2^2)^2-\sqrt{1+4t}\,]/\sqrt{1+4t}+\alpha[(\partial/\partial x^k)\,(1+x_1^2+x_2^2)^2.\sqrt{1+4t}$$
$$-2v(1+x_1^2+x_2^2)^2/\sqrt{1+4t}]/(1+4t).$$
$$\left|\frac{\partial B}{\partial x^k}\right|\leqq\left|\frac{\partial\alpha}{\partial x^k}\right|.\left|(1+x_1^2+x_2^2)^2-\sqrt{1+4t}\right|+150.$$

But $(1+x_1^2+x_2^2)^2-\sqrt{1+4t}=(x_1^2+x_2^2)^2+2(x_1^2+x_2^2)-\sum_{k=1}^{\infty}\binom{1/2}{k}4^kt^k$ and therefore

$$|(1+x_1^2+x_2^2)^2-\sqrt{1+4t}|\leqq 16\lambda^4+8\lambda^2+\sum_{k=1}^{\infty}\left|\binom{1/2}{k}\right|4^kt^k\leqq 16\lambda^2+8\lambda^2$$
$$+\sum_{k=1}^{\infty}4^kt^k\leqq 24\lambda^2+\sum_{k=1}^{\infty}4^k(16\lambda^2)^k$$

since $t\leqq 16\lambda^2$. We want that $64\lambda^2<1/2$, i. e. $\lambda<1/(8\sqrt{2})$. Then the series $\Sigma_{k=1}^{\infty}(64\lambda^2)^k$ is convergent and

$$\sum_{k=1}^{\infty}(64\lambda^2)^{k-1}<\sum_{k=1}^{\infty}(1/2)^{k-1}=2.$$

Thus $\left|\frac{\partial B}{\partial x^k}\right|\leqq(24\lambda^2+64\lambda^2.2)\left|\frac{\partial\alpha}{\partial x^k}\right|+150$ and from (9) we obtain

$$\left|\frac{\partial B}{\partial x^k}\right|\leqq 76\tilde{C}^2+150.$$

(iv) On the analogy of (iii) we prove that

$$\left|\frac{\partial A}{\partial x^k}\right|\leqq \text{const.}$$

From 2. (i), (ii), (iii) and (iv) we obtain

$$\left|\frac{\partial h_{ij}}{\partial x^k}\right|\leqq C_0'' \tag{11}$$

for some $C_0''>0$ which is independent of λ.

3. After twice differentiating and applying Hörmander's lemma (see (9), (10)) we conclude that there exists a constant $C_0'''>0$ which is independent of λ such that

$$\left|\frac{\partial^2 h_{ij}}{\partial x^k\,\partial x^l}\right|\leqq C_0'''. \tag{12}$$

The proof of this fact is lengthy but there are no new points in it and we omit it

From (8), (11), (12) and from the formula in the beginning of the proof we obtain the desired estimate on the components of the Riemannian tensor.

This completes the proof of the Proposition 1.

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