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ANALYTIC METHOD IMPLEMENTATION WITH A TRIGONOMETRY PROBLEM SOLVER

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Abstract. One of the most important aspects when developing a problem solving machine for the purposes of ITS is that the different teaching methods should be fully compatible with the solutions the problem solver provides and should be applicable to them. This paper examines the possibility of implementing a specific teaching method (the Analytic Method) with a specific problem domain (a class of mathematical knowledge - High School Trigonometry).

A simple and convenient control regime is presented along with a set of conventions for applying the analytic method with TRIPS, avoiding most of the pitfalls. As an illustration an ongoing example working within this control regime is provided.

The main objectives of this paper are to consider the potential formalization of the theoretically developed teaching methods and learning tools for the purposes of ITS and to describe an experimental prototype implementation of a specific teaching method, illustrating these ideas.

1. Main Objectives. The presented research serves two goals which are the main objectives of the paper. The obvious objective is to experiment with the well known analytic method, theoretically developed in education for the purposes of learning, and to try to adapt and implement it as a teaching method, suitable for computer formalization. The final goal of this is aiming at the automation of the teaching process. However there is a second and equally important objective, which is to explore the flexibility and the explicitness of a problem solving machine developed for the purposes of ITS. All this is done under the assumption that a domain problem solver will be suitable for the purposes of ITS to the extent it involves flexibility and explicitness. Explicitness demands stating something in details, clearly and fully expressed. Explicitness includes also the requirement for the solution to be self-evident on the basis of being comprehensible for the user. It should provide suitable explanations as well. The flexibility of the Domain Expert implies multiple reasonable solutions and different teaching strategies implementation. By developing a specific teaching method we have tried to assign some context of the above stated general requirements.

2. Architectural Issues for Applying the Analytic Method. The basic architectural presupposition for experimenting the analytic method is the architecture of an overall ITS (Intelligent Tutoring System), details of which are provided in [1] and [2]. Actually this paper would be followed and assessed much more easily if [2] is read first.

The analytic method is viewed in the context of TET (Teaching Expert for Trigonometry problem solving) experimented with TRIPS (TRIgonometry Problem Solver), both of which are agents of a multiexpert ITS.

TET uses the inferences of TRIPS (called solutions) and applies different teaching strategies to them.

Besides making decisions for applying a specific teaching method, TET uses the solutions of TRIPS to generate appropriate tests, supporting the teaching process.

The teaching expert contains different teaching methods (analytic method, brain attack, successive refinement, discovery learning and assessment, cognitive apprenticeship, goal-directed asking etc.) that are chosen by the Cognitive Expert (not described here) and applied by TET. A similar approach for experimenting multiple teaching strategies has been used by Spensley et. al. (1990) in [3]. The current presentation is constrained to the application of only one of the enumerated teaching methods - the analytic one.

At present the teaching methods are not fully implemented on a computer system, but they are rather theoretical devices mainly because of some computer memory problems with the logical environment.

These methods have been conceptualized chiefly to prove that TRIPS is appropriate for use in an ITS.

3. Description of the Analytic Method. The analytic method (AM) is one of the oldest methods, used in teaching and learning. The task of this method is to decompose the problems into subproblems so as to explore and examine the elementary principles these problems are based on. Thus the reasoning acquires a validating character (for more details see [4] and [5]).

The reason of exploring the analytic method with TRIPS is that this teaching method is characterized by strong syllogism and all the reasoning is based on logical inferences, conclusions and deductions.

The analytic method implementation to teaching is based on the confidence that the whole task could be followed, understood and accepted much more comprehensively and could be processed more precisely and directly through its intermediate composite subproblems [6].

The method of analytical reasoning focuses on its ability to analyze and criticize the task argumentation by understanding, estimating and evaluating the relationships among the arguments and the cases of reasoning or parts of them.

Usually when solving a certain problem the usage and the application of the analytic approach is coupled with other methods of teaching and learning. But for the sake of research experiments this approach is isolated and detached by using analytic reasoning tests for example. In the general case sets of questions, provoking analytic reasoning, are used. The purpose of this inquiring in the conventional analytic checks is to test the ability of the subject to understand a given structure of arbitrary relationships among fictitious things or events and to deduce or infer new information from the relationships given. However our case was slightly different, for the ITS is not going to check the analytic skills of the student, but it would rather apply the user's analytic and logical reasoning abilities for teaching purposes. So it would be appropriate to modify a bit the classical framework of the analytic method by introducing a feedback for the sake of tutoring. An example of a teaching feedback, which is not very complex, is the straightforward evaluation of the answers the student gives to the test questions. The most simple feedback is with YES/NO evaluation of the answers, but a smarter solution is to introduce analysis into the feedback again, thus interposing depth at several levels of the teaching process. We shall impose a constraint to the first level of depth by cutting it with YES/NO feedback.

4. Analytical Reasoning Tests. The analytical reasoning tests as a rule involve a set of questions which measure and examine the ability to understand a given structure of relationships ([7], [8]). Each analytical reasoning group consists of:

• a set of about 3 – 7 mutually related statements or conditions (often some other explanatory material is added as well);

• three or more questions which test the degree of understanding the structure, its implications and inklings.

Though each question of a group is based on the same set of conditions, the questions are independent of one another. Answering a question of a group does not influence the answers of the remaining questions.

Each group of questions for analytical reasoning testing is based on a definite set of conditions establishing the interconnections among things and/or the events, [9]. The links are rather ordinary, plain and common, such as:

• Temporal order: (X happened before Y but after Z);

• Spatial order: (X is nearer to Y and Z);

• Cause and effect: (The application of the transformation X causes the possibility of applying the transformation Y).

Some of the relationships may be fixed or constant (at any step only one transformation is to be implemented). Other relationships are variable: (At that step either the transformation X or the trans formation Y should be applied).

A certain number of interrelations and interdependences not provided in the assignment can easily be inferred from the data. When presenting the conditions for the analytical reasoning they must be stated clearly and should be distinct, transparent and obvious [10]. This requirement is imposed because the student should be assisted and facilitated in determining the exact nature of the relationship or relationships involved.

5. Theoretical Application of the Analytic Method. The assignment for applying the analytic method is: First to set up the arguments of a task and to decompose it into subproblems; Second to discuss the results and to observe what inference, based on the solution of the subproblems, can be made.

The phases of theoretical application of the analytical method in problem solving are as follows:

• confronting the problem and encountering the difficulties and the conflicts (set up by TRIPS);

• supplying the required information (data, facts and knowledge), necessary and essential to eliminate the confrontation (TRIPS);

• clarifying and elucidating the essence, the nature and the core of the problem (TRIPS);

• decomposing and disintegrating the problem into subproblems (TRIPS);

• searching for, discovering, and implementing alternative solutions (creative, constructive analytical thinking •to achieve various solutions), (TRIPS & TET);

• evaluating the alternatives and checking up the choices and the options, (TRIPS & TET).

When applying the analytic method for teaching purposes it is assumed, that the student is acquainted with the solutions and has the knowledge and skills to perform them until the opposite is proved.

The general principles used in implementing the analytical method for teaching purposes in problem solving are listed below:

The student's learning and knowledge acquisition is shift from:

• the simple to the more complex;

• from the tangible and the explicit to the intangible and the more abstract;

• from the general and the generic to the specific, the particular and the individual;

• from the known, told and experienced to the unknown, the unfamiliar and the unidentified.

6. Implementation of the Analytic Method. For the purposes of applying the teaching methods of TET with TRIPS a communication protocol has been developed. The solution provided by TRIPS to TET consists of a set of lines and a set of justifications. The set of lines for solving a problem is called a solution printout. The set of the list data structures, dynamically gen erated at each step in the process of problem solving is called justification. All the derivations made over the justifications are called an explanation. The set of a specific solution printout of a problem with the specific justifications and explanations to it is called a variant of that problem. The finite set of all the variants of a given problem (all the alternative solutions) is called a case.

Every case is conveyed to TET. TET selects a variant and uses analytical tests that matches it. In the general case matching is accomplished by unifying the constraints of the test with the justifications of the variant. On success this test is activated and presented to the student.The answers of the student are evaluated by TET with Yes/No or Right/Wrong. The general algorithm for applying the analytic method is given on Fig.1, where A1 and A2 are standard measures for the test samples and R1 and R2 are the values of the results, obtained by testing the student. The test samples from the algorithm are presented in the Appendix. When the tests over a given variant or variants are exhausted, cognitive expertise is used so as to make a decision on whether to continue the application of the selected teaching method, whether to choose another teaching method, or to end up the teaching session. The application of the analytic method is terminated also when the justifications of the variant do not match the constraints of any of the tests in a test sample. A test sample is a set of analytic tests applied successively at a given step of the general algorithm.

One of the implementation problems is that every test for analytical reasoning is specific and it is formalized in a particular way. For example the constraints in the assumption of the Test Sample No.1 are general for all trigonometric problems, so it can be applied always, no matter what the discussed problem is, but the tests from the Test Sample 2 are quite different and it is difficult or even impossible to develop general specifications for applying them, so we shall restrict to a specific application.

Let us consider Test Sample No.1. Following the protocol TRIPS generates a variant and passes down this variant to TET. It is interesting to investigate how the wrong answers of that test (solution printouts) should be generated by TET. The wrong answers are derived from the right answers by violating one or more than one of the constraints. The most "natural thing to do" is to skip some lines of the solution.

The first constraint may be violated by dropping out any one of the solution lines except for the first and the last one. The second constraint may be violated by repeating some of the lines in the solution. The 3rd constraint is set up on the basis of the assigned list from the justifications of a variant. The 4th constraint may be violated by skipping a line from the solution printout at random. The 5th constraint is checked out or violated on the basis of the contradictory lists of the justifications of a variant. The 6th constraint may be violated by applying two opposite transformations, taken from the contradictory list, successively. And the 8th constraint may be violated by cutting off the solution printout before the first simplifying transformation (TS or AS), is applied (it can be read from the list of the corresponding justification).

The techniques used for implementing the analytic tests are quite simple, but the more simple the techniques are the more efficient its computer program realization is.

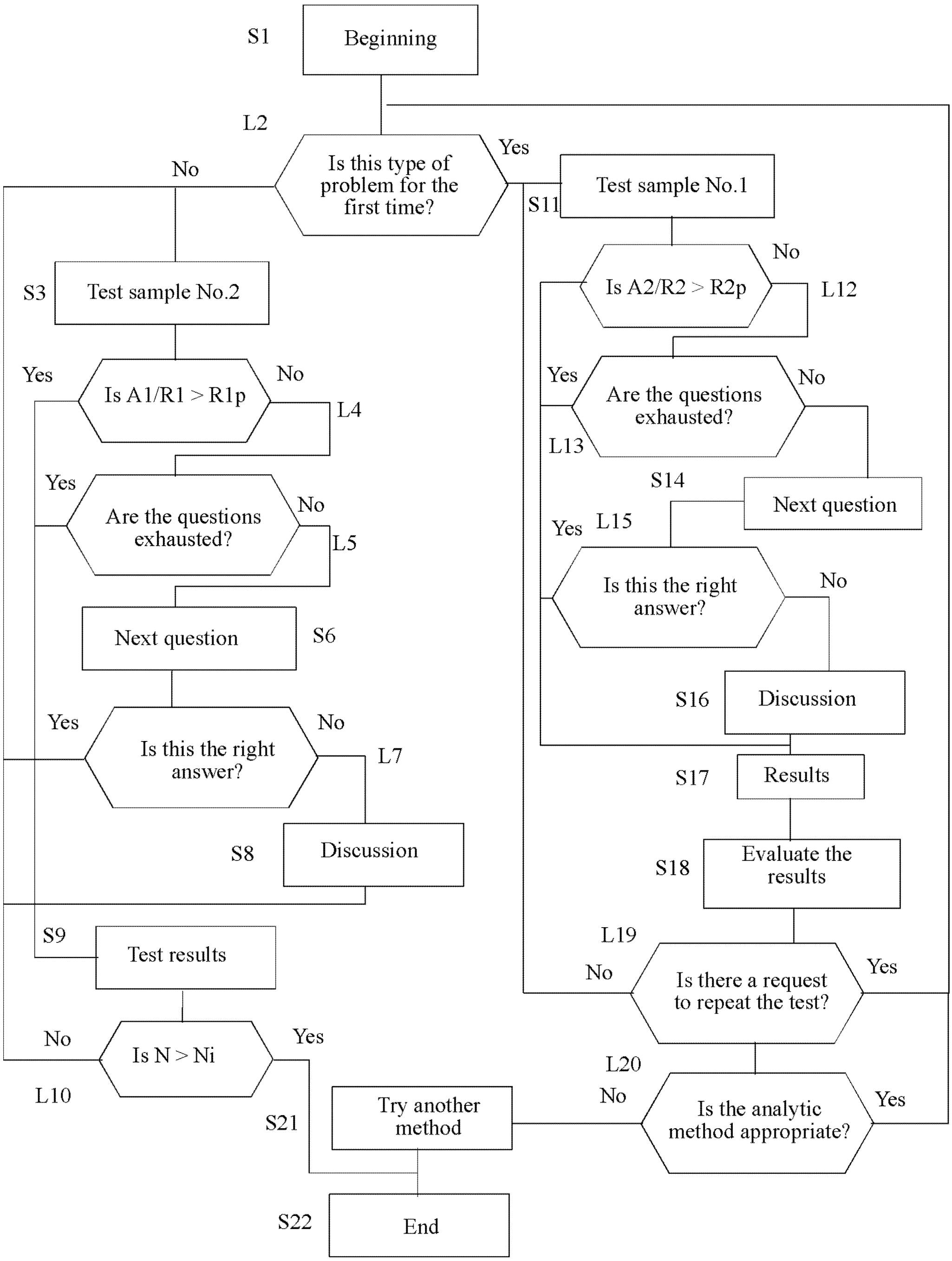

Fig.1 Analytic Method Implementation.

7. Contributions. As a part of an ITS, TET is an attempt to design and implement an operational Domain Expert-Teaching Expert protocol in a real world environment. The contributions of TET are:

• integrating different types of knowledge into one model by building up compatibility techniques;

• conceptualizing of detached teaching expert methods;

• developing an approach for computer implementation of purely theoretical teaching methods;

Since the analytic method of TET considered here is well-known and very old one (probably one of the oldest methods for learning) the specific contributions, related to that method are:

• formalizing the method;

• making the analytical method and its implementation compatible with specific domain knowledge;

• developing knowledge bases for representing the conceptualized knowledge in a logic programming environment (PROLOG);

The implementation of formalized teaching methods to a specific problem domain is a thankless research work, taking out a lot of time and efforts with minimal results. This research with TRIPS is forthcoming and its objective is to bridge the gap between AI, Cognitive science and Education in ITS.

Conclusion. The the proposed analytic algorithm is a pathological case of analysis and does not pretend to cover the theoretical method. But the algorithm is rather flexible and provides opportunities to experiment with various types of analytic tests.

The computer formalization is still somewhat primitive, although it captures the style and the essence of the teaching method it is intended to represent.

Refinement and sophistication of this method, along with the extension of the test procedures is among the major goals of future research work.

APPENDIX

Test Sample Type 1

A problem for trigonometric simplification is subjected to the following constraints:

•1• The solution of the problem must be written down LINE by LINE;

•2• The transition from one line to another is made by transformations;

•3• The following transformations are allowed: [AS,AD,TS,TD,SB,CD];

•4• For the transition from line to line EXACTLY one transformation should be used;

•5• Some of the transformations are opposite of others e.g. [AS,AD], [TS,TD];

•6• Two opposite transformations cannot be applied successively;
•7• The simplification of the expression is evaluated in terms of:
- Number of monomials
- Complexity of the trigonometric functions
- Number of different subterms

•8• At least one simplification transformation should be applied in the solution.

?1? Which of the following examples satisfies the above stated constraints?

Example 1:
1. $2 - \sin^2(\alpha) + \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
2. $1 + 1 - \sin^2(\alpha) + \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
3. $1 + \cos^2(\alpha) + \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
4. $1 + 2 * \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
5. $1 + 2 * \cos^2(\alpha) + \cos(\alpha) * \mathrm{tg}(\alpha) + \cos(\alpha)^2$
6. $1 + 3 * \cos^2(\alpha) + \cos(\alpha) * \mathrm{tg}(\alpha) + \cos(\alpha)^2$
7. $1 + 3 * \cos^2(\alpha) + \cos(\alpha) * \mathrm{tg}(\alpha) + \cos(\alpha)^2$

Example 2:
1. $2 - \sin^2(\alpha) + \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
2. $1 + \cos^2(\alpha) + \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
3. $1 + 2 * \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
4. $1 + 3 * \cos^2(\alpha) + \cos(\alpha) * \mathrm{tg}(\alpha) + \cos(\alpha)^2$
5. $1 + 3 * \cos^2(\alpha) + \cos(\alpha) * \mathrm{tg}(\alpha) + \cos(\alpha)^2$

...

TRIPS can be used to generate solutions violating 0, 1 or more than one of the constraints. In the considered Example 1 none of the constraints is violated, though the solution itself is somewhat difficult to handle.

?2? Which of the following sequences of transformations CANNOT be implemented in solving problems for trigonometric simplification?

A) AS,AD,TS,SB,TD
B) AS,TS,AD,TS,AS
C) AS,AS,AS,AS
D) AD,TD,AD
E) TS,AS,TS,AS
F) SB,AS,SB

TRIPS could be used to generate alternative real world examples over the presented set of operators. These examples are chosen from the already solved problems.

?3? If we know that AS and TS reduce V^2 and SB,TD, and AD increase it, which of the following sequences could have completed the simplification of the expression?

A) AS,TS,AS,TS
B) AD,SB,AS,TD
...

The sequence of the operators chosen by the student is to be verified by TRIPS in terms of plausibility.

?4? Let us have the following solution:

1. $2 - \sin^2(\alpha) + \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
2. $1 + 1 - \sin^2(\alpha) + \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
3. $1 + \cos^2(\alpha) + \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
4. $1 + 2 * \cos^2(\alpha) + \cos(\alpha) * (\mathrm{tg}(\alpha) + \cos(\alpha))$
5. $1 + 2 * \cos^2(\alpha) + \cos(\alpha) * \mathrm{tg}(\alpha) + \cos(\alpha)^2$
6. $1 + 3 * \cos^2(\alpha) + \cos(\alpha) * \mathrm{tg}(\alpha) + \cos(\alpha)^2$
7. $1 + 3 * \cos^2(\alpha) + \cos(\alpha) * \mathrm{tg}(\alpha) + \cos(\alpha)^2$

Without using evaluations, choose a couple of successive steps which result in simplifying the expression?

A) 1-2 & 2-3
B) 2-3 & 5-6
...

TRIPS can be used to explore wrong answers by applying analysis recursively as a feed-back.

Test Sample Type 2

A certain trigonometric problem is solved in five steps: S1, S2, S3, S4 and S5. For solving the problem all the five transformations – Op1, Op2, Op3, Op4, Op5 – must be used. The solution is subjected to the following restrictions:

1) In the solution Op1 must be applied earlier than Op4 and Op5
2) Op2 must be applied earlier than Op3 and earlier than Op4
3) It is not allowed to apply more than one transformation at a given step

?1? Which of the following sequences is an acceptable solution path, where the transformations are listed as applied in the sequence from step S1 to S5?

A) Op1, Op2, Op3, Op4, Op5
A) Op2, Op3, Op4, Op3, Op2
...

TRIPS could be used either to generate solutions over these sets of operators or to generate an appropriate question and choices over a given problem. This is done by matching a readily available test with the currently solved domain problem.

?2? Which of the following couples of transformations can be executed at step S1 and step S2 respectively in some of the solutions?

A) Op1 and Op4

B) Op2 and Op5

...

?3? If Op5 is applied earlier than Op1 in the solution, which of the following choices must be true for that solution as well?

A) Op1 is applied earlier than Op2

B) Op3 is applied earlier than Op2

...

With TRIPS the student, would be able to check the answers.

?4? The step in the solution, subjected to the constraints of the assignment, which allows the application of each of the five transformations and at the same time provides the opportunity to apply the rest of the four transformations at the next steps is:

A) S1

B) S2

...

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