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Serdica Math. J. **44** (2018), 1–30

Serdica
Mathematical Journal

Bulgarian Academy of Sciences
Institute of Mathematics and Informatics

PROFESSOR STOYAN YORDANOV NEDEV

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Communicated by V. Valov

Abstract. Professor Stoyan Yordanov Nedev, age 72, died in Sofia, Bulgaria, in the early morning hours of Sunday, May 31, 2015. He was survived by his wife Kalinka, son Yordan and doughter Ivelina. Stoyan Nedev was a reliable friend, a great father, an excellent husband and a gifted mathematician. In this article, we give a a short outline of his life, discuss some of his mathematical contributions and present a list of his major publications.

1. A short outline of the life of Professor Stoyan Yordanov Nedev. Stoyan Nedev was the first child of Ivanka and Yordan Nedev. He was born on November 2, 1942 in the village of General Kolevo, near the town of Varna, Bulgaria. The early childhood of Stoyan Nedev was closely connected to a rural environment in which love for family, for home and homeland, readiness for hard work, respect for others, curiosity and other virtues were built and developed both spontaneously and gradually on a daily basis. These qualities formed a favorable and solid ground for the further development of Stoyan Nedev as an extraordinary personality. His father regularly challenged him with different "brain teasers" which required logical thinking, imagination and creativity. As a high school student in the town of Provadia, Stoyan Nedev stood out among his classmates with his strong quest for knowledge, with his creativity and skills to decipher the essence and the meaning of natural phenomena, mechanical devices, engines, even sentences in foreign to him languages. Notable was also his willingness to share what he had learned. His ambition "to understand" and his ability to explain to others were qualities, which characterized Stoyan Nedev to a very high degree during his entire life as a scientist and as a university professor. He had a natural bend for music and liked singing Bulgarian and foreign language songs. Unlike many others however, he tried to understand and interpret the message in the songs text, even when the text was in a language he did not speak. May be this is one of the earliest appearances of his specific talent to use analogy and generalization in revealing the hidden sense of things.

There were many attractive avenues for professional development Stoyan Nedev could have taken after graduating high school. He chose to become a mathematician. In addition to his natural inclination to this science, there were other reasons for this choice. At the end of the 50s of the last century Sava Kovachev, a young graduate from the mathematics department of Sofia University, was appointed as a school teacher in Provadia. The young and self-confident teacher quickly attracted strong students around him and started to prepare them for solving more difficult competition problems. The efforts soon paid off. The Provadian students performed very well in mathematics Olympiads. At the same time, the "mathematical machines" which today we call computers had appeared on the world stage. They had already proven their potential to accelerate the economic and scientific development. The state leadership in Bulgaria had begun to realize the need for building capacity in the new fields born and developed in the bosom of mathematics. In 1960 an extraordinary contest was organized for the best performers in the National Mathematical Olympiad (about 200) with the aim to select the very best and grant them the right to enroll immediately as students in mathematics at Sofia University. During those times the two-year military service after school was mandatory for all Bulgarian boys and the exemption from this duty of the mathematics students was a sign of the official recognition of the importance of mathematics. Only 29 were admitted as mathematics students at the Faculty of Physics and Mathematics of Sofia University. Among them there were two students of Sava Kovachev – Stoyan Nedev and Radostin Ivanov. The solid preparation in school and the hard work quickly elevated Stoyan Nedev to the top of the elite group of students in the university. His fellow students had a deep respect for his willingness to share his knowledge with them. During breaks or after class, he would explain to the students assembled spontaneously around him the main points of the lecture. As soon as a second year student he was asked by the faculty to conduct the exercise classes for first year students studying Mathematical analysis.

The students of Sofia University in that period had as lecturers distinguished scientists and personalities as Nikola Obreshkov, Boyan Petkanchin, Blagovest Dolapchiev, Ljubomir Iliev, Yaroslav Tagamlitzki, Alipi Mateev and Blagovest Sendov. A rather limited number of mathematical disciplines “beyond the curriculum” were offered as Special Topics Courses. One of them was a systematic course on General Topology delivered for the first time in Bulgaria by Doichin Doichinov. This course and the research seminar on Analysis of Yaroslav Tagamlitzki shaped the initial interest of Stoyan Nedev to General Topology. Together with a group of other excellent students who already exhibited an inclination to research, Stoyan Nedev was sent in 1963 to continue his education in the Mechanics and Mathematics Faculty of the Moscow State University.

At the time when Stoyan Nedev joined Moscow University as the third year undergraduate student, Mekh-Mat Fakulty of Moscow University bristled with activity of lots of special research seminars which attracted students of all levels. The attendance of some seminars was really huge: 80 or even more participants, the average number being around 25. Some of the leaders of these seminars were famous mathematicians of the older generation: A. N. Kolmogorov, I. M. Gel’fand, P. S. Alexandroff, A. N. Tychonoff, A. G. Kurosh. Stoyan joined the seminar for the startup researchers in General Topology conducted by A. V. Arhangel’skii and V. I. Ponomarev, who at the time were Associate Professors at Moscow University and were vigorously working on their second, Doctor of Science, dissertation. A characteristic feature of this seminar was the active presence of young undergraduate students (about twelve of them), each eager to start his own travel in mathematics. Stoyan Nedev immediately felt this atmosphere, apparently, it caught him, all the more so, since his sociable character fitted well. There was no lack of intriguing open problems offered by the leaders of the seminar, and students were actively interacting and competing for the success. Stoyan Nedev was deeply involved in this life, he made many friendly connections in the seminar, and developed a very close relationship with Mitrofan M. Choban, who became an active participant of this seminar approximately at the same time. The friendship and collaboration with Choban became for Stoyan lifelong and resulted, in particular, in many interesting research projects and publications. Since 1964 year he participated in the well-known seminar of the Academician Pavel S. Alexandrov.

The first topic which had attracted Stoyan Nedev deeply, was the concept of distance and its role in Topology. Very soon after the general concept of a topological space had been defined, it had been discovered that even the spaces which satisfy nice and strong separation axioms need not be metrizable. Since the concept of the distance is so deeply built into our intuition, it was natural to modify the concept of the distance by weakening the usual axioms of a metric. Some of the work in this direction had already been done in 1930-ties and later by V. V. Niemytzki and P. S. Alexandroff, M. Fréchet , W. A. Wilson and by some others. In 1966, Arhangel'skii's survey paper in Uspekhi Matematicheskih Nauk had been published, a part of which was devoted to generalized distances and to topologies which they generate. Several new results and problems in this direction were exposed there. The two years before getting the master degree at Moscow University were for Nedev the period of internal ripening, of absorbing the general idea of distance, the rich material at his disposal, working actively not only in the seminar, but also, on his own, in the library and at home. This hard work brought the fruit. Under the scientific guidance of Arhangel'skii and before his graduation from the Moscow State University (in 1966), Stoyan Nedev obtained interesting and promising new topological results and was invited to become a doctoral student. His first result in Topology was as elegant as he himself in those years (end of sixties): he has shown that every Lindelöf symmetrizable space is hereditarily Lindelöf. After that, Nedev wrote a fine Ph. D. dissertation on o-metric spaces with many interesting results. He has also mastered a quite different major direction in applications of General Topology – selection theorems for multi-valued mappings, were he obtained excellent results, some of them jointly with M. M. Choban.

He wrote his dissertation under the supervision of Professor A. V. Arhangel'skii, defended it in 1970 and obtained the scientific degree "Candidate of Physics-Mathematics Sciences" (the equivalent of today's Ph. D. in Physics and Mathematics). After his return to Sofia he was appointed at the Institute of Mathematics of the Bulgarian Academy of Sciences which was later renamed to Institute of Mathematics and Informatics (IMI). He remained affiliated and actively involved with IMI and its Topology department until his retirement. The words "affiliated" and "Involved" are too weak to describe his real contribution for the fulfillment of the mission of IMI. He contributed significantly by his research (that will be presented partially below) by his teaching at different universities in Bulgaria, USA and Algeria, by conducting a research seminar, but also by being a person serving honestly and devotedly the Bulgarian and international mathematical community.

Having a good reputation in the world of mathematics, Professor Stoyan Nedev has been invited to many prestigious international conferences on Topology and its Applications (Russia, Moldova, Poland, Austria, Azerbaijan, Hungary, Czechia, Serbia, Macedonia etc). He passionately and skillfully has organized, in collaboration with colleagues, some national and international conferences on Topology and Topological Algebra. Since 1972, Professor S. Nedev has been scholarly advisor for Ph. Doctor Thesis. Under the guidance of Professor S. Nedev remarkable results were obtained by his post-graduate students. The first post-graduate student of Professor S. Nedev was Professor Vesko Valov who obtained many deep results in the Theory of Set-valued Mappings and Geometrical Topology. Another post-graduate student of him was Professor Valentin Gutev who has many deep results in the Theory of Set-valued Mappings and their Applications in several mathematical areas.

As a member of different scientifid councils, commissions (including the Higher Attestation Commission of Bulgaria) and juries Professor S. Nedev was known for his objectiveness and for his stimulating attitude toward people aspiring for degrees and positions. In the most difficult years of political transition in the early nineties of the last century Stoyan Nedev served as a deputy director and, for some time, as a director of IMI. The Bulgarian mathematical community will remember him as one of its most respected members.

2. Nedev's main results. In this section we describe the main results published in the works of Stoyan Nedev. We will only mention some relations of these results with the results of other authors. The books [35, 36, 55, 59, 84, 97] contain some overviews of the related results of other authors.

The main topological interests of Stoyan Nedev are in the following areas:

- theory of distances on spaces;
- proximity and uniform structures on spaces;
- set-valued mappings;
- mappings and classes of spaces.

The notion of a distance is one of the principal tools in mathematics. Nonsymmetrical distances or distances without triangle inequality appeared very often in the simulation of various natural or social processes. This fact leads to a general concept of a distance (see [35, 36]).

After Nedev [N2, N11], an *o*-metric on a set X is a real-valued function d on $X \times X$ with the following properties:

(i_m) $d(x, y) \geq 0$ for all $x, y \in X$;

(ii_m) $d(x,y) = 0$ if and only if $x = y$.

A function $d : X \times X \to \mathbb{R}$ with the property (i_m) is a distance on X (see [24, 27]) if we have:

(ii'_m) $d(x,y) + d(y,x) = 0$ if and only if $x = y$.

These notions coincide in the class of T_1-spaces. Let d be a distance on X and $B(x,d,r) = \{y \in X : d(x,y) < r\}$ be the *ball* with radius $r > 0$ centered at a point $x \in X$. The set $U \subset X$ is called *d-open* if for every $x \in U$ there exists $r > 0$ such that $B(x,d,r) \subset U$. The family $\mathcal{T}(d)$ of all d-open subsets is a topology on X generated by d. We say that a topological space X is *o-metrizable*, or a *distance space*, if the topology of X is generated by an o-metric. If X is o-metrizable by a given o-metric d such that for every $r > 0$ and every $x \in X$ the interior of the ball $B(x,d,r)$ contains the point x, then X is called *strongly o-metrizable*. Any distance space is a *sequential* T_0*-space*.

Nedev's work [N11] contains the basis of the theory of distance functions. It provides correlations between the properties of distance functions and the topologies generated by them.

A sequence $\{L_n : n \in \mathbb{N}\}$ of subsets of a space X is a sequential base of the space X at a point x if the following conditions are satisfied:

- $x \in L_{n+1} \subseteq L_n$ for each $n \in \mathbb{N}$;
- if $A = \{x_n : n \in \mathbb{N}\}$ is a sequence of points in X convergent to x, then the set $A \setminus L_n$ is finite for each $n \in \mathbb{N}$;
- any open subset of X containing x contains also some L_n.

A space X is said to be a space with the weak first countable axiom if it is sequential and has a sequential base at every point [10, 24]. Let $\mathcal{B} = \{Q_n x : n \in \mathbb{N}, x \in X\}$ be a family of subsets of a set X with the following properties: $x \in Q_{n+1}x \subseteq Q_n x$ for all $n \in \mathbb{N}$ and $x \in X$, and $\bigcap\{Q_n x : n \in \mathbb{N}\} = \{x\}$ for each $x \in X$. The function $d(x,y) = max\{2^{-n} : y \notin Q_n x\}$ is an o-metric on X and the family $\mathcal{B}$ is a weak base of the topology $\mathcal{T}(d)$.

One of the first Nedev's results describes o-metrizable spaces:

Theorem 1 (see [N2, N11]). *For a T_1-space X the following assertions are equivalent:*

(1) *There exists a distance d on X such that $\mathcal{T}(d)$ is the topology of the space X, i. e. the space X is o-metrizable;*

(2) *X is a space with the weak first countable axiom.*

Let us mention that the presence of a distance on a space X is threefold: it generates both a topology and a proximity on X, and determines metrical properties on X.

Recall that a distance d on X is called:

- a symmetric if $d(x, y) = d(y, x)$ for all $x, y \in X$;
- a quasimetric or a Δ-metric if $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$).

Theorem 2 (see [N2, N11]). *A T_0-space X is quasimetrizable if and only if there exists an open base $\mathcal{B} = \bigcup\{\mathcal{B}_n : n \in \mathbb{N}\}$ such that for each point $x \in X$ and each $n \in \mathbb{N}$ we have $x \in \mathrm{Int}(\bigcap\{U \in \mathcal{B}_n : x \in U\})$, where $\mathrm{Int}H = X \setminus cl_X(X \setminus H)$ is the interior of the set $H \subset X$.*

In [N2, N11] distinct criteria of symmetrizability of spaces were obtained.

There exist countable symmetrizable regular spaces (see [N11]) without a countable base at any point. In this context, the following two results are very important:

Theorem 3 (see [N1, N11]). *Every o-metrizable topological group is metrizable.*

Theorem 4 (see [N1, N11]). *Every symmetrizable orderable space is metrizable.*

Note that Theorem 3 contains the well-known result on metrizability of T_0-groups with the first countable axiom and resolve a problem of Arhangel'skii [10].

A space X with a distance d is first-countable if and only if the following condition is satisfied:

(SD) If A is a non-empty subset of X and $x \in X$, then $x \in cl_X A$ if and only if $d(x, A) = \inf\{d(x, y) : y \in A\} = 0$.

Distances satisfying the condition (SD) are called strong distances. The strong symmetrizable orderable spaces were studied by W. A. Wilson [108, 109] and G. Creede [34]. There are various conditions involving the condition (SD). The first one was proposed by M. Fréchet (see [30, 45, 46, 47, 10]):

(F) *For every $\varepsilon > 0$ there exists $\delta > 0$ such that for every $x, y, z \in X$ with $d(x, y) \leq \delta$ and $d(y, z) \leq \delta$ we have $d(x, z) \leq \varepsilon$.*

V. V. Niemytzki [82] proposed a weaker condition:

(wF) *For every* $\varepsilon > 0$ *and every* $x \in X$ *there exists* $\delta > 0$ *such that for any* $y, z \in X$ *with* $d(x, y) \leq \delta$ *and* $d(y, z) \leq \delta$ *we have* $d(x, z) \leq \varepsilon$.

P. S. Alexandroff and V. V. Niemytzki [6] also proposed a similar condition:

(AN) *For every* $\varepsilon > 0$ *and every* $x \in X$ *there exists* $\delta > 0$ *such that for any* $y, z \in X$ *with* $d(x, y) \leq \delta$ *and* $d(x, z) \leq \delta$ *we have* $d(y, z) \leq \varepsilon$.

Restrictions of that kind permit to establish an important and surprising criteria for paracompactness and metrizability.

A family $\gamma = \{H_\mu : \mu \in M\}$ of subsets of a space X is weakly discrete [N30] if any set of points $\{x_\mu \in H_\mu : \mu \in M\}$ is discrete in X.

Theorem 5 (see [N1, N11]). *Any open cover of a symmetrizable space has a* σ*-weakly discrete refinement.*

Theorem 6 (see [N1, N11]). *A symmetrizable space is Lindelöf if and only if any discrete subset is countable.*

Corollary 7 (see [N1, N11]). *Every symmetrizable Lindelöf space is hereditarily Lindelöf.*

Corollary 8 (see [N1, N11]). *Every symmetrizable hereditarily separable space is hereditarily Lindelöf.*

Corollary 9 (see [N1, N11]). *Every symmetrizable countably compact Hausdorff space is metrizable.*

Corollary 7 is obvious for strongly symmetrizable spaces. The assertion in Corollary 8 was established by J. G. Ceder [29] for strongly symmetrizable spaces, while Corollary 9 for compact spaces was established by A. V. Arhangel'skii and V. Stoyanovskii [10].

Theorem 10 (see [N11]). *Let* d *be a symmetric on a Lindelöf space* X *with the following condition* (wC)*: if a set* L *is not closed in* X*, then for each* $\varepsilon > 0$ *there exist two distinct points* $x, y \in L$ *such that* $d(x, y) < \varepsilon$*. Then for any* $\varepsilon > 0$ *and for any non-empty subspace* Y *of* X *there exists a countable subset* $Z \subseteq Y$ *such that* $Y \subseteq \cup\{B(x, d, \varepsilon) : x \in Z\}$.

Corollary 11 (see [N11]). *Let* d *be a symmetric on a Lindelöf space* X *with the condition* (wC)*. Then* X *is hereditarily separable.*

Corollary 12 (see [N11]). *Let* d *be a strong symmetric on a Lindelöf space* X*. Then* X *is hereditarily separable.*

The following problem of Arhangel'skii is still open:

Problem 1. *Is it true that a Lindelöf symmetrizable space is separable?*

For any distance d Nedev considered the adjoint distance $\hat{d}$ defined by $\hat{d}(x,y) = d(y,x)$. He observed the following curious phenomena:

(s_1) *Let* $\mathcal{T}(d) \subseteq \mathcal{T}(\hat{d})$, *i.e.* $\lim d(x, x_n) = 0$ *implies* $\lim d(x_n, x) = 0$ *for every* $x \in X$. *Then* $\mathcal{T}(d) = \mathcal{T}(d_s)$, *where* $d_s(x,y) = \sup\{d(x,y), d(y,x)\}$. *In particular, if* d *is a quasimetric, then* d_s *is a metric and the topology* $\mathcal{T}(d)$ *is metrizable.*

(s_2) *Let* $\mathcal{T}(\hat{d}) \subseteq \mathcal{T}(d)$. *Then* $\mathcal{T}(d) = \mathcal{T}(d_m)$, *where* $d_m(x,y)$ *is the symmetric* $d_m(x,y) = \min\{d(x,y), d(y,x)\}$.

E. Michael's theorem that a continuous closed image of a paracompact space is paracompact is well known. Theorems of this type, which for metrizable spaces was established earlier by A. V. Arhangel'skii [14], was proved in [N11, Theorem 20]:

Theorem 13 (see [N11])**.** *Let* $\varphi : X \longrightarrow Y$ *be a closed continuous mapping of strongly symmetrizable space* X *onto a paracompact space* Y. *Then* Y *is a hereditarily paracompact space with a* G_δ*-diagonal and admits a continuous condensation onto a metrizable space.*

The conditions of strongly symmetrizability in the above theorem is essential, there exists a closed continuous mapping of a Lindelöf symmetrizable space onto a non-metrizable perfectly normal compact space [N11].

For any two non-empty subsets A and B of a distance space (X,d) we define $d(A,B) = \inf\{d(x,y) : x \in A, y \in B\}$ if $A \neq \varnothing \neq B$, and $d(A,B) = \infty$ if $\varnothing \in \{A,B\}$.

If d is an o-metric on X we consider the following conditions:

(α) For any point $x \in X$ and any $\varepsilon > 0$ there exists $\delta > 0$ such that $d(y,x) < d(y, X \setminus B(x,d,\varepsilon))$ for every $y \in B(x,d,\delta)$.

(α') For any point $x \in X$ and any $\varepsilon > 0$ there exists $\delta > 0$ such that $d(x,y) < d(X \setminus B(x,d,\varepsilon), y)$ for every $y \in B(x,d,\delta)$.

(β) For any point $x \in X$ and any $\varepsilon > 0$ there exists $\delta > 0$ such that $d(y,x) < d(X \setminus B(x,d,\varepsilon), y)$ for every $y \in B(x,d,\delta)$.

(β') For any point $x \in X$ and any $\varepsilon > 0$ there exists $\delta > 0$ such that $d(x,y) < d(y, X \setminus B(x,d,\varepsilon))$ for every $y \in B(x,d,\delta)$.

(Π_1) If $\lim d(x, x_n) = 0$ and $\lim d(x_n, y_n) = 0$, then $\lim d(x, y_n) = 0$.

(Π_2) If $\lim d(x_n, x) = 0$ and $\lim d(x_n, y_n) = 0$, then $\lim d(x, y_n) = 0$.

Theorem 14 (see [N11]). *Let d be an o-metric on a space space X generating its topology and having one of the following requirements:*

(1) *d satisfies condition (α);*

(2) *d is a strong o-metric satisfying condition (α');*

(3) *d satisfies both conditions (β) and (β').*

Then X is a collectionwise normal space.

Corollary 15 (see [N11]). *If d is a quasimetric on a space X satisfying condition (AN), then X is metrizable.*

Corollary 16 (see [N11]). *If a space X is strongly o-metrizable with a distance satisfying condition (Π_1) and o-metrizable with o-metric satisfying conditon (Π_2), then X is metrizable.*

In [N3, N5, N11] Nedev studied mappings between metric spaces and distance spaces and obtained external characteristics of such distance spaces (see also [N7, N8, N10, N12, N13], where some relations between properties of distance functions are obtained). More precisely, the question to what extent the properties of distance functions influence the properties of the corresponding topological spaces was investigated.

In connection with the well known problem of metrizability of normal Moore space, S. Nedev has proved the following result:

Theorem 17 (see [N16, N18]). *If $2^{\aleph_1} > 2^{\aleph_0}$, then for any normal symmetrizable space X the following assertions are equivalent:*

(1) *X is separable;*

(2) *X is a Lindelöf space;*

(3) *X is a hereditary Lindelöf space;*

(4) *X is hereditary separable.*

The key step in the proof of the above theorem is the following statement:

Theorem 18 (see [N3, N11]). *Any symmetrizable space X contains a countable family of closed discrete subspaces whose union is dense in X.*

It is well known that there exists a model of set theory in which $2^{\aleph_1} = 2^{\aleph_0}$ and in which it is possible to construct a normal, separable, non-metrizable (and hence not Lindelöf) Moore space, [105, 106] (note that each Moore space is strongly symmetrizable).

A hereditarily paracompact space with point-countable base which does not have any dense developable subspace was constructed in [90]. Since any first countable space which is union of a countable family of closed discrete subspaces is developable, any strongly symmetrizable space contains a dense developable subspace, which is the union of countably many closed discrete subspaces [88, 89, 90]. Closed discrete subspaces are associated with some important problems in topology:

Question 1. *Under what conditions a space contain a dense σ-discrete metrizable subspace?*

Question 2. *Under what conditions a space contain a dense σ-discrete developable subspace?*

Theorem 19 (see [N30])**.** *Assuming $V = L$, every strongly symmetrizable normal space contains a dense metrizable subspace.*

Papers [N9, N17, N21] are devoted to the general concept of metrizability of spaces. For developing a substantial τ-metric theory, where τ is an infinite cardinal number, it is necessary to introduce the notions of the abstract scale (in the sense of Fréchet and Kurepa). A scale is a partially ordered semigroup S with the following properties:

- there exists an element $0 \in S$ such that $0 \leq x$ for any $x \in S$;
- if $0 < x$, then there exists $z > 0$ such that $z + z \leq x$;
- if $0 < x$ and $0 < y$, then there exists $z > 0$ such that $z \leq x$ and $z \leq y$;
- $(S, +)$ is a commutative semigroup with the neutral element 0;
- if $x, y, z, u \in S$, $x \leq y$ and $u \leq z$, then $x + u \leq y + z$.

Distinct scales were introduced in [7, 8, 21, 32, 44, 45, 46, 53, 54, 60, 79, 80] (see also [35, 36]).

If S is a scale, an S-distance d on a set X is a function $d : X \times X \to S$ such that $x = y$ if and only if $d(x, y) = d(y, x) = 0$. For any $x \in X$ and any $r \in S$ let $B(x, d, r) = \{y \in X : r \not\leq d(x, y)\}$. A subset H of X is d-open if for each $x \in H$ there exist $r \in S$ such that $r > 0$ and $B(x, d, r) \subseteq H$. The family $\mathcal{T}(d)$ of all d-open subsets of X is a topology on X generated by the distance d. A set

$L \subset S$ is said to be co-final in S if for every $x > 0$ there exists $y \in L$ such that $0 < y \leq x$. For any scale S we define the character $\chi(S) = \min\{|L| : L$ *is co-final in* $S\}$. A space X is called τ-0-metrizable if there exists a scale S and an o-metric $d : X \times X \to S$ such that $\chi(S) \leq \tau$ and $\mathcal{T}(d)$ is the topology of space X. The S-o-metrics, S-symmetrics, S-quasimetrics, S-metrics, τ-symmetrizable space, S-quasimetrizable space and τ-metrizable spaces are defined in a similar way. The $\aleph_0$-metrizable spaces is the class of metrizable spaces. X is called a Kurepa-Frechét space if X is S-metrizable over a linear ordered scale S. The concept of S-metrizability permits to obtain a qualitative analysis of metrizability. In particular, the following questions were examined.

Question 3. *Which of the classical criteria of metrizability can be extended for any cardinal* τ*?*

Question 4. *Let* S *be a given scale. Under what conditions a space is* S*-o-metrizable,* S*-metrizable,* S*-quasimetrizable, or* S*-metrizable?*

The metrizability criteria of Alexandroff-Urysohn [4] and Chittenden [30] were extended for each cardinal τ. It was also shown that the metrizability criteria of R. H. Bing [18], Jun-iti Nagata [77] and Ju. M. Smirnov [103] can be extended only under the conditions of normality (see [N9, N21, N17]).

Theorem 20 (see [N9, N17, N21]). *Any Kurepa-Frechét space is paracompact. Moreover, if* X *is a non-metrizable Kurepa-Frechćhet space, then* $\dim X = 0$ *and there exists an infinite cardinal* τ *such that* $\chi(X) \leq \tau$ *and the intersection of any family* ξ *of open sets in* X *is also open provided* $|\xi| < \tau$.

Different variants of the Theorem 20 were obtained by many authors (see [53, 54, 79, 80]).

Another area of research done by Nedev was proximities in sense of V. A. Efremovich [39, 37, 38, 104]. A proximity δ on a set X is metrizable by a metric (or an o-metric) ρ if $\delta(A, B) = 0$ if and only if $\rho(A, B) = 0$, where A and B are subsets of X. The problem of metrization of proximity spaces was raised by Ju. M. Smirnov [104]. One of the first criteria of metrization of proximity was established by S. Leader [61].

Theorem 21 (see [N15, N19, N26]). *Any o-metrizable proximity is metrizable.*

The following Nedev's problem is still open:

Problem 2. *Is any proximity generated by a countable family of entourages of the diagonal metrizable?*

There are some partial answers of Problem 2.

Theorem 22 (see [N47]). *Any proximity generated by a countable family of entourages of the diagonal is sequential.*

Theorem 23 (see [N47]). *Assume that Ω is a countable family of entourages of the diagonal of a space X and δ is a proximity on X is generated by the family $\Omega \bigcup \{U \circ U^{-1} \circ U \circ U^{-1} : U \in \Omega\}$. Then δ is metrizable.*

H. J. Schmidt [98] introduced a class of spaces, called HS-spaces, and proved that every Hausdorff HS-space is regular. However, in 1985 M. Paoli and E. Ripoli noted that the proof of Schmidt's theorem is incorrect and asked under which conditions it holds. In the article [N44], S. Barov, G. Dimov and S. Nedev, provided a partial answer to this question, presenting a large class of spaces, containing all Hausdorff spaces of countable character, for which Schmidts theorem is true.

Given a space X, denote by $\mathcal{K}(X)$ the set of all compact subsets of X. It has been shown by J. P. R. Christensen [31] that if both X and Y are separable metrizable spaces and $F : \mathcal{K}(X) \longrightarrow \mathcal{K}(Y)$ is a monotone map such that any $L \in \mathcal{K}(Y)$ is covered by $F(K)$ for some $K \in \mathcal{K}(X)$, then Y is complete provided that X is complete. Later on, J. Baars, J. de Groot and J. Pelant [17] showed that Christensen's result fails if the spaces are not separable. The interesting paper [N50] provides some additional conditions, which guarantee the validity of Christensen's result in more general spaces.

A space is suborderable if it is embeddable in a linearly ordered space. The following assertion has important applications:

Theorem 24 (see [N41]). *The Dieudonné completion of a suborderable space is suborderable and paracompact.*

Given a space X let $\exp^*(X)$ be the family of all non-empty subsets of X, $\mathcal{F}(X)$ denotes the family of all non-empty closed subsets of X and $\mathcal{K}^*(X)$ is the family of all non-empty compact subsets of X. A single-valued mapping $\theta : X \longrightarrow \exp^*(Y)$ is called a set-valued mapping. If $\theta, \psi : X \longrightarrow \exp^*(Y)$ are set-valued mappings and $\psi(x) \subseteq \theta(x)$ for all $x \in X$, then ψ is called a selection of θ. If $\theta : X \longrightarrow \exp^*(Y)$ is a set-valued mapping, $\mathcal{L} \subseteq \exp^*(Y)$ and $\theta(x) \in \mathcal{L}$ for all $x \in X$, then we write $\theta : X \longrightarrow \mathcal{L}$. The existence of selections for set-valued maps was one of the main topics of Nedev's research. Of course, it is interesting to find selections having some additional properties, for example, measurable or continuous selections. Conditions for the existence of measurable selections were obtained by H. Lebesgue (1905), N. N. Lusin (1930), P. S. Novikov (????), V. A.

Rohlin (1949), J. von Neumann (1949), K. Kuratowski and C. Ryll-Nardzewski (1965) et al. (see [25, 22, 23]). In 1956 E. Michael proved important theorems about existence of continuous selections (see [64, 65, 66, 67, 68]), and obtained surprising characteristic of paracompactness and collectionwise normality (see [64]).

In 1974 S. Nedev observed that the Michael's proof [64] on the characterization of collectionwise normality is not complete and contains a significant gap. Michael's method was not able to eliminate the gap. A new proof was obtained in [28] applying the Choban's method of covers.

Factorization theorems for single-valued mappings are known for a long time and were applied in different topological problems. An important moment of developing the selection theory is associated with the concept of factorization of set-valued mappings. It was established that the existence of some "good" selections is equivalent with the existence of "good" factorizations [N24].

Let X be normal space and (Y, ρ) be a metric space. The mapping $\Phi : X \to \mathcal{F}(Y)$ is said to have the *selection-factorization property, br. s.f.p.*, if for every closed subset F of X and every locally finite collection γ of open subsets of Y such that $\Phi^{-1}(\gamma) = \{\Phi^{-1}(U) \mid U \in \gamma\}$ covers F there is an open locally finite (in F) covering of F which refines $\Phi^{-1}(\gamma)$.

Next two theorems illustrate the possibility of factorizing set-valued mappings, where $\mathcal{C}(Y)$ denotes the family of all non-empty compact subsets of Y and $\mathcal{C}'(Y) = \mathcal{C}(Y) \cup \{Y\}$.

Theorem 25 (see [N24, N31]). *Let X be a normal space and $\Phi : X \to \mathcal{F}(Y)$ be s.f.p., where Y is a metric space of weight $w(Y) \leq \tau$. Then there are:*

(a) A metric space Z of weight $w(Z) \leq \tau \cdot \aleph_0$;

(b) A continuous mapping $f : X \to Z$;

(c) A l.s.c. mapping $\Psi : Z \to \mathcal{C}(Y)$, such that $\Psi(f(x)) \subset \Phi(x)$ for every $x \in X$; Moreover, if F is a closed G_δ-set in X, then one may assume that $f(F)$ is a closed subset of Z.

Let X be a normal space and $\Phi : X \to \mathcal{F}(Y)$ be a set-valued map, where Y is a metric space. We shall say that the triple (Z, f, Ψ) constitutes a l.s.c. weak-factorization for Φ, where Z is a metric space, $f : X \to Z$ is a continuous (single-valued) map and $\Psi : Z \to \exp^*(Y)$ is l.s.c., if $\Psi(f(x)) \subset \Phi(x)$ for every $x \in X$.

Theorem 26 (see [N31]). *Let X be a T_1-space and $F_0, F_1, F_2, \dots, F_k, \dots$ be a sequence of closed subsets of X. Then the following conditions are equivalent:*

(a) X is τ-collectionwise normal with $\tau \geq \aleph_0$, F_0 is a G_δ-set in X and

$\dim F_k \leq n_k$ *for every* $k = 1, 2, \dots$*;*

(*b*) *every l.s.c. mapping* $\Phi : X \to \mathcal{C}'(Y)$*, where* Y *is a metric space of weight* $w(Y) \leq \tau$*, has an u.s.c. weak-factorization* (Z, f, Ψ) *such that* $f(F_0)$ *is a closed subset of* Z *with* $F_0 = f^{-1}(f(F_0))$ *and* $\mid \Psi(f(x)) \mid \leq n_k + 1$ *for every* $x \in F_k,\ k = 1, 2, \dots$ *.*

The proof of next theorem is also based on factorizations theorems for set-valued mappings (the abbreviations AE and ANE mean absolute extensor and absolute neighborhood extensor, respectively).

Theorem 27 (see [N24])**.** *Let* Y *be a metric space of weight* τ *such that* Y *is an* AE *or* ANE *with respect to the class of all (resp., no more than n-dimensional) metric spaces of weight* $\leq \tau$*. Then* Y *is such an extensor with respect to the class of all (resp., no more than n-dimensional)* τ*-collectionwise normal, perfectly normal spaces. If in addition,* Y *is Čech complete, then* Y *is an* AE *or* ANE *with respect to the class of all (resp., no more than n-dimensional)* τ*-collectionwise normal spaces.*

The factorization method was used implicitly in [22, 23] for characterizing the dimension of normal spaces. In that connection, the problem of characterizing the dimension in terms of factorization arises. This problem was successfully solved by S. Nedev. First, he improved the method of coverings, and then described some covering properties of spaces, like τ-metacompactnees and τ-collectionwise normality [N25, N31, N36]. One of the main results in that direction is the following theorem.

Theorem 28 (see [N47])**.** *Let* τ *be an infinite cardinal,* $\{X_n : n \in \mathbb{N}\}$ *be a sequence of closed subspaces of a* T_1*-space* X *and* $\{m_n : n \in \mathbb{N}\}$ *be a sequence of non-negative integers. The following assertions are equivalent:*

(1) X *is a* τ*-paracompact space and* $\dim X_n \leq m_n$ *for each* $n \in \mathbb{N}$*.*

(2) *For any lower semicontinuous mapping* $\theta : X \longrightarrow \mathcal{F}(Y)$*, where* Y *is a complete metrizable space of weight* $\leq \tau$*, there exists an upper-continuous set-valued mapping* $\varphi : X \longrightarrow \mathcal{C}(Y)$ *such that* $\varphi(x) \subseteq \theta(x)$ *for each* $x \in X$ *and* $|\varphi(x)| \leq m_n + 1$ *for each* $n \in \mathbb{N}$ *and* $x \in X_n$*.*

Corollary 29 (Kuratowski-Morita theorem)**.** *Any n-dimensional metric space is a continuous, perfect,* $(n+1)$*-multiple image of a* 0*-dimensional metric space.*

Corollary 30 (Morita theorem)**.** $\dim X = \operatorname{Ind} X$ *for any metric space* X*.*

Theorem 31 (see [N32]). *If $\theta : X \to \mathcal{C}'(Y)$ is a lower semicontinuous set-valued map, where X is a collectionwise normal space and Y a Banach space, then θ admits a continuous selection provided there is a subset Z of X with* $\dim_X Z = 0$ *such that $\theta(x)$ is convex for all $x \in X \setminus Z$.*

Since the closed convex hull of a compact set in a Banach space is compact and any lower semicontinuous set-valued map θ from a paracompact space X to the non-empty closed subsets of a complete metrizable space Y admits a lower semicontinuous compact-valued selection ([68], or [22, 23] for a regular metacompact space X), Theorem 28 implies the following results of Michael-Pixley and Michael:

Corollary 32 (E. Michael and C. Pixley [70]). *If θ is a lower semicontinuous set-valued map from a paracompact space X to the non-empty closed subsets of a Banach space, then θ admits a continuous selection provided there is a subset Z of X with* $\dim_X Z = 0$ *such that $\theta(x)$ is convex for all $x \in X \setminus Z$.*

Corollary 33 (E. Michael [64]). *If θ is a lower semicontinuous set-valued map from a paracompact space X to the non-empty closed subsets of a Banach space, then θ admits a continuous selection provided the sets $\theta(x)$ are convex for all $x \in X$.*

Let Y be a Banach space and $\mathcal{F}_c(Y)$ be the family of all closed convex subsets of Y. A set-valued mapping $\varphi : X \longrightarrow \mathcal{F}_c(Y)$ is weakly continuous if it is lower semi-continuous and the set $\{x \in X : \varphi(x) \subseteq Y \setminus K\}$ is open in X for every weakly compact subset K of Y. S. Nedev and V. Gutev established a selection theorem for weakly continuous set-valued mappings defined on arbitrary spaces.

Theorem 34 (see[N46]). *Let X be a topological space, Y be a reflexive Banach space, and $\varphi : X \longrightarrow \mathcal{F}_c(Y)$ be weakly continuous. Then φ admits a single-valued continuous selection.*

This result is used in [N46] to solve a problem concerning the set of proper lower semi-continuous convex functions on a reflexive Banach space.

Another selection theorem for set-valued maps with convex values in reflexive Banach space was established by Choban and Nedev:

Theorem 35 (see[N45]). *Every lower semicontinuous closed and convex-valued mapping φ from a generalized order space X into a reflexive Banach space has a single-valued continuous selection.*

For the proof of above theorem, the space X is embedded in a generalized ordered paracompactification Z, such that φ admits a lower semi-continuous extension on Z, and then the Michaels classical continuous selection theorem is applied.

In the theory of selections S. Nedev introduced new important concepts as “selector space” and “inverse selection problem”.

Definition (see [N34]). *Let $\mathcal{K}$ be a class of topological spaces and $\mathcal{L}(X)$, $\mathcal{M}(X)$ be families of non-empty subsets of a space X. The space X is said to be a $\mathcal{K} - \mathcal{L}(X) - \mathcal{M}(X)$-selector if every l.s.c. map mapping $\Phi : Y \to \mathcal{L}(X)$ with $Y \in \mathcal{K}$ has a u.s.c. selection $\Psi : Y \to \mathcal{M}(X)$.*

For simplicity, we say that X is a $\mathcal{K}$-selector if X is $\mathcal{K} - \mathcal{F}(X) - \mathcal{F}(X)$-selector (recall that $\mathcal{F}(X)$ stands for the closed subsets of X).

Below we use the following notations:

1) $\mathcal{N}$ stands for the class of all normal (always Hausdorff) spaces;

2) $\mathcal{P}$ stands for the class of all paracompact members of $\mathcal{N}$;

3) $\mathcal{P}_\tau$ stands for the class of all τ-paracompact members of $\mathcal{N}$;

4) $\mathcal{C}$ stands for the class of all compact members of $\mathcal{N}$;

5) $\{Ç\}$ stands for the one-element class of spaces whose only element is the Cantor set.

The study of the properties of selectors shows that in many known selection theorems the requirement on X to be a complete metric space is (almost) unavoidable. Let's recall, for example, the following results:

Theorem 36 (see [N34]). *Let X be a normal space which is $\mathcal{N} - \mathcal{L}(X) - \mathcal{F}(X)$-selector, where $\mathcal{C}'(X) \subset \mathcal{L}(X) \subset \mathcal{L}(X)$. Then X is a complete separable metric space.*

Theorem 37 (see [N34]). *Every normal space which is $\mathcal{N}$-selector is compact and metrizable.*

Theorem 38 (see [N34]). *Every $X \in \mathcal{P}_{\aleph_0}$ space which is $\mathcal{P}_{\aleph_0}$-selector is completely metrizable.*

Under Cantor set-selector (or Ç-selector) we mean a space X that is $\{Ç\} - \mathcal{F}(X) - \mathcal{F}(X)$-selector.

Theorem 39 (see [N39]). *Every closed subset F of a metric Ç-selector is a Baire space.*

Theorem 40 (see [N39]). *If X is a metric Ç-selector, then either X is scattered (i.e., every closed subset of X has an isolated point) or X contains a copy of Ç.*

The concept of selectors is a source of open problems. We state here two such problems.

Problem 3. *Let $X \in \mathcal{P}$ be a selector for the class $\mathcal{P}$. Is it true that X is p-paracompact or metrizable?*

Recall that p-paracompact spaces are characterized as inverse images of metric spaces under a perfect mappings. For the original definition (see A. V. Arkhangel'skii, [16, 10]).

Problem 4. *Let the metric space X be a $\c{C}$-selector. Is X complete?*

There are also some related questions:

Problem 5. *Is a space $X \in \mathcal{P}$ Čech complete provided X is a $\mathcal{P}$-selector?*

Problem 6. *Let X be a metric space which is a selector for the class of all metric spaces. Is X Čech complete?*

J. van Mill, J. Pelant and R. Pol in [71] answered the last question positively in the case when the u.s.c. selection is assumed to be compact-valued (i.e. if the metric space X is such that every l.s.c. map $\Phi : Y \to \mathcal{F}(X)$, where Y is a metric space, has an u.s.c. compact-valued selection).

3. Main publications of Stoyan Yordanov Nedev.

[N1] S. Y. Nedev. Symmetrizable spaces and ε-mappings *Dokl. Akad. Nauk SSSR* **173** (1967), 523–525 (in Russian); English translation in: *Soviet Math. Dokl.* **8** (1967), 435–437.

[N2] S. Y. Nedev. Generalized-metrizable spaces. *C. R. Acad. Bulgare Sci.* **20** (1967), 513–516 (in Russian).

[N3] S. Y. Nedev. Symmetrizable spaces and final compactness. *Dokl. Akad. Nauk SSSR* **175** (1967), 532–534 (in Russian); English translation in: *Soviet Math. Dokl.* **8** (1967), 890–892.

[N4] S. Y. Nedev. Factor images of metric spaces. *Dokl. Akad. Nauk SSSR* **182** (1968), 1014–1017 (in Russian); English translation in: *Soviet Math. Dokl.* **9** (1968), 1245–1248.

[N5] S. Y. Nedev, M. M. Choban. The metrizability of topological groups. *Vestnik Moskov. Univ. Ser. I Mat. Meh.* **23**, *6* (1968), 18–20 (in Russian. English summary).

[N6] S. Y. NEDEV. Any totally ordered symmetrizable space is metrizable. *Vestnik Moskov. Univ. Ser. I Mat. Meh.* **24**, *6* (1969), 31–33 (in Russian. English summary).

[N7] S. Y. NEDEV. Continuous and semicontinuous *o*-metrics. *Dokl. Akad. Nauk SSSR* **193** (1970), 531–534 (in Russian); English translation in: *Soviet Math. Dokl.* **11** (1970), 975–978.

[N8] S. Y. NEDEV, M. M. CHOBAN. On Δ-metrizable *p*-spaces. *Vestnik Moskov. Univ. Ser. I Mat. Meh.* **25**, *6* (1970), 3–9 (in Russian. English summary).

[N9] S. Y. NEDEV, M. M. CHOBAN. A general concept of metrizability for topological spaces. *Annuaire Univ. Sofia Fac. Math.* **65** (1970/71) (1972), 111–165 (in Russian. English summary).

[N10] S. Y. NEDEV. Continuous and semicontinuous *o*-metrics. *Annuaire Univ. Sofia Fac. Math.* **65** (1970/71), (1972), 235–248 (in Russian. English summary).

[N11] S. Y. NEDEV. *o*-metrizable spaces. *Trudy Moskov. Mat. Obšč* **24** (1971), 201–236 (in Russian); (English translation in : Trans. Moscow Math. Soc. 24 (1974), 213–247).

[N12] S. Y. NEDEV, M. M. CHOBAN. On the theory of *o*-metrizable spaces. I. *Vestnik Moskov. Univ. Ser. I Mat. Meh.* **27**, *1* (1972), 8–15 (in Russian. French summary).

[N13] S. Y. NEDEV, M. M. CHOBAN. On the theory of *o*-metrizable spaces. II. *Vestnik Moskov. Univ. Ser. I Mat. Meh.* **27**, *2* (1972), 10–17 (in Russian. English summary).

[N14] S. Y. NEDEV, M. M. CHOBAN. On the theory of o-metrizable spaces. III. *Vestnik Moskov. Univ. Ser. I Mat. Meh.* **27**, *3* (1972), 10–15 (in Russian. English summary).

[N15] N. G. HADŽIIVANOV, S. Y. NEDEV. *o*-metrizable proximities are metrizable. *C. R. Acad. Bulgare Sci.* **26** (1972), 1289–1291 (in Russian).

[N16] S. Y. NEDEV. Estimation of the cardinality of a discrete subset of a normal space. *C. R. Acad. Bulgare Sci.* **25** (1972), 1319–1321 (in Russian).

[N17] S. Y. Nedev, M. M. Choban. Non-archimedean metrics, and Kurepa-Fréchet spaces. *Annuaire Univ. Sofia Fac. Math. Méc.* **67** (1972/73) (1976), 221–234 (in Russian. English summary).

[N18] S. Y. Nedev. An estimate for the cardinality of a discrete subset of a normal space, and some of its applications. *Mathematics and mathematical education* (Proc. Second Spring Conf. Bulgarian Math. Soc., Vidin, 1973). Sofia, Izdat. Bulgar. Akad. Nauk., 1974, 163–166 (Bulgarian. Russian, English summaries).

[N19] S. Y. Nedev, M. M. Choban. Certain classes of proximity spaces and their metrization. *C. R. Acad. Bulgare Sci.* **27** (1974), 591–594 (in Russian).

[N20] S. Y. Nedev. Multivalued sections of multivalued mappings. *C. R. Acad. Bulgare Sci.* **27** (1974), 725–728 (in Russian).

[N21] S. Y. Nedev, M. M. Choban. Linear ordering and o-metrization of topological spaces. *C. R. Acad. Bulgare Sci.* **27** (1974), 729–732 (in Russian).

[N22] S. Y. Nedev. Metrizable spaces as images of zero-dimensional metric spaces. *C. R. Acad. Bulgare Sci.* **27** (1974), 877–879 (in Russian).

[N23] S. Y. Nedev, M. M. Choban. Linear ordering and o-metrization of topological spaces. Papers presented at the Fifth Balkan Mathematical Congress (Belgrade, 1974). *Math. Balkanica* **4** (1974), 461–464 (in Russian).

[N24] S. Y. Nedev, M. M. Choban. Factorization theorems for multivalued mappings, multivalued cross sections and topological dimension. Papers presented at the Fifth Balkan Mathematical Congress (Belgrade, 1974). *Math. Balkanica* **4** (1974), 457–460 (in Russian).

[N25] S. Y. Nedev. Four theorems of E. Michael on continuous selections. *Bulgar. Akad. Nauk Izv. Mat. Inst.* **15** (1974), 389–393 (in Russian. Bulgarian, English summaries).

[N26] S. Y. Nedev, M. M. Choban. o-metrics and proximity spaces. Metrization of proximity spaces. *Serdica* **1**, *1* (1975), 12–28 (in Russian).

[N27] S. Y. Nedev. Selected theorems on multivalued sections and extensions. *Serdica* **1**, *3* (1975), 285–294 (in Russian).

[N28] S. Y. Nedev. Multivalent selections and topological dimension. *Mathematics and mathematical education* (Proc. Third Spring Conf. Bulgar. Math. Soc., Burgas, 1974). Sofia, Bulgar. Akad. Nauk, 1976, 195–198 (in Bulgarian).

[N29] S. Y. Nedev. A theorem of Wilkinson. *C. R. Acad. Bulgare Sci.* **30**, *3* (1977), 319–321 (in Russian).

[N30] A. V. Arhangel'skii, S. Y. Nedev. Some remarks on semi-metrizable spaces and their subspaces. *C. R. Acad. Bulgare Sci.* **31**, *5* (1978), 499–500.

[N31] S. Y. Nedev. Selection and factorization theorems for set-valued mappings. *Serdica* **6**, *4* (1980) (1981), 291–317.

[N32] S. Y. Nedev. A selection theorem. *C. R. Acad. Bulgare Sci.* **35**, *7* (1982), 873–876.

[N33] S. Y. Nedev, V. M. Valov. On metrizability of selectors. *C. R. Acad. Bulgare Sci.* **36**, *11* (1983), 1363–1366.

[N34] S. Y. Nedev, V. M. Valov. Normal selectors for the normal spaces. *C. R. Acad. Bulgare Sci.* **37**, *7* (1984), 843–846.

[N35] S. Y. Nedev, V. M. Valov. Some properties of selectors. *C. R. Acad. Bulgare Sci.* **38**, *12* (1985), 1593–1596.

[N36] S. Y. Nedev. Single-valued and set-valued selections. Topics on topology, Graz-Sofia 1985, Ber. No. 261, 4 pp., Berichte, 259–261, Forschungszentrum Graz, Graz, 1986.

[N37] S. Y. Nedev. A selection example. *C. R. Acad. Bulgare Sci.* **40**, *11* (1987), 13–14.

[N38] S. Y. Nedev, J. Vilimovský. A note on regularity and metrizability of proximities. *C. R. Acad. Bulgare Sci.* **41**, *5* (1988), 13–14.

[N39] V. Gutev, S. Y. Nedev, J. Pelant, V. Valov. Cantor set selectors. Proceedings of the Symposium on General Topology and Applications (Oxford, 1989). *Topology Appl.* **44**, *1–3* (1992), 163–166.

[N40] S. Barov, G. Dimov, S. Y. Nedev. On a theorem of H.-J. Schmidt. *C. R. Acad. Bulgare Sci.* **46**, *3* (1993), 9–11.

[N41] M. Choban, S. Y. Nedev. Paracompact extensions and orderability of spaces. *Izv. Akad. Nauk Respub. Moldova Mat. 3* (1996), 43–51.

[N42] M. Choban, S. Y. Nedev. Strong P-embeddings. *C. R. Acad. Bulgare Sci.* **49**, *1* (1996), 9–12.

[N43] M. Choban, S. Y. Nedev. Continuous extensions and collectionwise normal embeddability via selections. *C. R. Acad. Bulgare Sci.* **49**, *2* (1996), 5–8.

[N44] S. Barov, G. Dimov, S. Y. Nedev. On a question of M. Paoli and E. Ripoli. *Boll. Un. Mat. Ital. A (7)* **10**, *1* (1996), 127–141.

[N45] M. Choban, S. Y. Nedev. Continuous selections for mappings with generalized ordered domain. *Math. Balkanica (N.S.)* **11**, *1–2* (1997), 87–95.

[N46] V. Gutev, S. Y. Nedev. Continuous selections and reflexive Banach spaces. *Proc. Amer. Math. Soc.* **129**, *6* (2001), 1853–1860

[N47] S. Ivanov, S. Y. Nedev, J. Pelant. On proximities generated by countable families of entourages. *Comment. Math. Univ. Carolin.* **45**, *3* (2004), 535–541.

[N48] E. Mihaylova, S. Y. Nedev. Metrization of proximities. *Math. Balkanica* **19**, *1–2* (2005), 167–174.

[N49] M. M. Choban, E. P. Mihaylova, S. Y. Nedev. On selections and classes of spaces. *Topology Appl.* **155**, *8* (2008), 797–804.

[N50] S. Nedev, J. Pelant, V. Valov. A non-separable Christensen's theorem and set tri-quotient maps. *Topology Appl.* **156**, *7* (2009), 1234–1240.

[N51] M. M. Choban, E. P. Mihaylova, S. I. Nedev. Selections, paracompactness and compactness. Serdica Math. J. **36**, *4* (2010), 299–328

[N52] E. Mihaylova, S. Nedev. Selections and selectors. *Topology Appl.* **158**, *2* (2011), 134–140.

[N53] M. Choban, E. Mihaylova, S. Nedev. Around one Michael's theorem on selections. *Math. Balkanica (N.S.)* **26**, *3–4* (2012), 313–324.

[N54] M. M. Choban, E. Mihaylova, S. Nedev, D. Stavrova. Paracompactness, selections and factorization principles. *C. R. Acad. Bulgare Sci.* **66**, *2* (2013), 163–170.

REFERENCES

[1] J. M. Aarts, D. J. Lutzer. Completeness properties designed for recognizing Baire spaces. *Dissertationes Math. (Rozprawy Mat.)* **116** (1974), 48 pp.

[2] P. Alexandrov. On the metrization of topological spaces. *Bull. Acad. Polon. Sci. Sér. Math. Astr. Phys.* **8** (1960), 135–140.

[3] P. S. Aleksandrov. Some results in the theory of topological spaces obtained within the last twenty-five years. *Uspehi Mat. Nauk* **15** (1960), *2(92)*, 25–95 (in Russian); English translation in: *Russian Math. Surveys* **15**, *2* (1960), 2383.

[4] P. Alexandroff, P. Urysohn. Une condition nécésare et suffisante pour qu'une classe (L) soit une classe (D), *C. R. Acad. Sci., Paris* **177** (1923), 1274–1276.

[5] P. Alexandroff, P. Urysohn. Mémoire sur les espaces topologiques compacts dédié à Monsieur D. Egoroff. *Verhandelingen Amsterdam* **14**, *1* (1929), 93 pp.

[6] P. Alexandroff, V. Niemytzki. Conditions of metrizability of topological spaces ad the axiom of countability. *Mat. Sbornik* **3**, *3* (1938), 663–672 (in Russian).

[7] M. Ja. Antonovskii, V. G. Boltyanskii, T. A. Sarymsakov. An outline of the theory of topological semi-fields. *Uspehi Mat. Nauk* **21**, *4(130)* (1966), 185–218; (in Russian); English translation in: *Russian Math. Surveys* **21**, *4* (1966), 163–192.

[8] A. Appert, Ky-Fan. Espaces topologiques intermediates. Problème de la distanciation. Actualites Sci. Ind., no. **1121**. Paris, Hermann & Cie, 1951, 160 pp (in French).

[9] A. V. Arhangel'skii. A class of spaces which contains all metric and all locally compact spaces. *Mat. Sb. (N.S.)* **67(109)**, *1* (1965), 55–88 (in Russian); English translation in: *Amer. Math. Soc. Transl.* **92** (1970), 1–39.

[10] A. V. Arhangel'skii. Mappings and spaces. *Uspehy Mat. Nauk* **21**, *4(130)* (1966) 133–184 (in Russian); English translation in: *Russian Math. Surveys* **21**, *4* (1966), 115–162.

[11] A. V. Arhangel'skii. Behavior of metrizability in factor mapping. *Dokl. Akad. Nauk SSSR* **164**, *2* (1965), 247–250 (in Russian).

[12] A. V. Arhangel'skii. Some metrization theorems. *Uspehi Mat. Nauk* **18**, *5(113)* (1963), 139–145 (in Russian).

[13] A. V. Arhangel'skii. On the metrization of topological spaces. *Bull. Acad. Polon. Sci. Sér. Math.* **8** (1960), 589–595 (in Russian).

[14] A. V. Arhangel'skii. Perfect mappings and condensations. *Dokl. Akad. Nauk SSSR* **176**, *5* (1967), 983–986 (in Russian); English translation in: *Soviet Math. Dokl.* **8** (1967), 1217–1220.

[15] A. V. Arkhangel'skii, V. V. Filippov. Spaces with bases of finite rank. *Mat. Sbornik* **87(129)**, *2* (1972), 147–158 (in Russian); English translation in: *Math. USSR-Sb.* **16**, *2* (1972), 147–158.

[16] C. E. Aull. Topological spaces with a σ-point finite base. *Proc. Amer. Math. Soc.* **29** (1971), 411–416.

[17] J. Baars, J. de Groot, J. Pelant. Function spaces of completely metrizable spaces. *Trans. Amer. Math. Soc.* **340**, *2* (1993), 871–883.

[18] R. H. Bing. Metrization of topological spaces. *Canad. J. Math.* **3** (1951), 175–186.

[19] H. R. Bennett. Quasi-developable spaces. Topology Conference (Arizona State University, Tempe, Ariz. 1967), Arizona State University, Tempe, Ariz., 1968, 314–317.

[20] D. K. Burke. On subparacompact spaces. *Proc. Amer. Math. Soc.* **23** (1969), 655–663

[21] E. Čech. Topologické Prostory. Prague, Publishing House of the Czechoslovak Academy of Sciences, 1959 (in Czech); English translation: Topological spaces. London-New York-Sydney, Interscience Publishers John Wiley & Sons, 1966.

[22] M. M. Choban. Multi-valued mappings and Borel sets. I. Trudy Moskov. Mat. Obšč **22** (1970), 229–250 (in Russian); English translation in : *Trans. Moscow Math. Soc.* **22** (1970), 258–280.

[23] M. M. Choban. Multi-valued mappings and Borel sets. II. Trudy Moskov. Mat. Obšč **22** (1970), 277–301 (in Russian); English translation in : *Trans. Moscow Math. Soc.* **22** (1970), 286–310.

[24] M. M. Choban. Fixed points of mappings defined on spaces with distance. *Carpathian J. Math.* **32**, *2* (2016), 173–188.

[25] M. M. Choban. Descriptive set theory and topology. Current problems in mathematics. Fundamental directions, Vol. **51**, 173–237, 247, Itogi Nauki i Tekhniki, Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1989 (in Russian); English translation in: Encyclopaedia of Math. Sciences, **51**. Berlin, Springer, 1995, 157–219.

[26] M. M. Choban. σ-paracompact spaces. *Vestnik Moskov. Univ. Ser. I Mat. Meh.* **24**, *1* (1969), 20–27 (in russian).

[27] M. M. Choban, I. A. Budanaev. Distances on monoids of strings and their applications. Proceedings MFOI-2016 – Conference on Mathematical Foundations of Informatics, July 25–29, 2016, Chişinău, Republic of Moldova, 144–159.

[28] M. M. Choban, V. M. Valov. A certain theorem of Michael on sections. *C. R. Acad. Bulgare Sci.* **28**, *7* (1975), 871–873 (in Russian).

[29] J. G. Ceder. Some generalization of metric spaces. *Pacific J. Math.* **11** (1961), 105–125.

[30] E. W. Chittenden. On the equivalence of Écart and voisinage. *Trans. Amer. Math. Soc.* **18**, *2* (1917), 161–166.

[31] J. P. R. Christensen. Necessary and sufficient conditions for the measurability of certain sets of closed subsets. *Math. Ann.* **200** (1973), 189–193.

[32] J. Colmez. Espaces à écart généralisé régulier. *C. R. Acad. Sci. Paris* **244** (1947), 372–373.

[33] H. H. Corson, E. Michael. Metrizability of certain countable unions. *Illinois J. Math.* **8** (1964), 351–360.

[34] G. Creede. Concerning semi-stratifiable spaces. *Pacific J. Math.* **32**, *1* (1970), n 47–54.

[35] E. Deza, M. M. Deza. Dictionary of Distances. Amsterdam, Elsevier, 2006.

[36] M. M. Deza, E. Deza. Encyclopedia of Distances. Berlin, Springer, 2014.

[37] V. A. Efremovich. On proximity geometry of Riemannian manifolds. *Uspehy Matem. Nauk* **8**, *5* (1953), 189–191 (in Russian); English translation in : *Amer. Math. Soc. Transl. (2)* **39** (1964), 167–170.

[38] V. A. Efremovich, E. S. Tikhomirova. Equimorphisms of hyperbolic spaces. *Izv. Akad. Nauk SSSR, Ser. Mat.* **28** (1964), 1139–1144.

[39] R. Engelking. General Topology. Warszawa, PWN, 1977.

[40] M. E. Estill. Concerning abstract spaces. *Duke Math. J.* **17** (1950), 317–327.

[41] M. E. Estill. Separation in non-separable spaces. *Duke Math. J.* **18** (1951), 623–629.

[42] B. Fitzpatrick Jr. On dense subspaces of Moore spaces. *Proc. Amer. Math. Soc.* **16** (1965), 1324–1328.

[43] B. Fitzpatrick Jr. On dense subsets of Moore spaces. II. *Fund. Math.* **61** (1967), 91–92.

[44] M. Frechet. Sur quelques points du Calcul fonctionnel. *Rend. Circ. Mat. Palermo* **12** (1906), 1–74 (in French).

[45] M. Frechet. Les espances abstraits et leur théorie considérée comme introduction à lanalyse générale. Paris, Gauthier-Villars, 1928 (in French).

[46] M. Frechet. De l'écart numérique á l'écart abstrait. *Portugalie Mathematica* **5**, (1946), 121–131 (in French).

[47] A. H. Frink. Distance functions and the metrization problem. *Bull. Amer. Math. Soc.* **43**, *2* (1937), 133–142.

[48] R. W. Heath. Arc-wise connectedness in semi-metric spaces. *Pacific. J. Math.* **12**, *4* (1962), 1301–1319.

[49] R. W. Heath. On open mappings and certain spaces, satisfying the first countability axion. *Fund. Math.* **57**, *1* (1965), 91–96.

[50] R. W. Heath. On spaces with point countable bases. *Bull. Acad. Polon Sci. Sér Sci. Math. Astronom. Phys.* **13** (1965), 393–395.

[51] R. W. Heath. On certain first countable spaces. *Ann. Math. Stud.* **60** (1966), 103–113.

[52] R. W. Heath. Screenability, pointwise paracompactness, and metrization of Moore spaces. *Canad. J. Math.* **16** (1964), 763–770.

[53] R. E. Hodel. Extensions of metrization theorems to higher cardinality. *Fund. Math.* **87**, *3* (1975), 219–229.

[54] M. Hušek, H.-C. Reichel. Topological characterizations of linearly uniformizable spaces. *Topology Appl.* **15**, *2* (1983), 173–188.

[55] M. Husek, J. van Mill (Eds). Recent Progress in General Topology II. Amsterdam, Elsevier, 2002.

[56] F. B. Jones. Metrization. *Amer. Math. Monthly* **73** (1966), 571–576.

[57] H. J. K. Junnila. Neighbornets. *Pacific J. Math.* **76**, *1* (1978), 83–108.

[58] Ya. A. Kofner. On a new class of spaces and some problems in the theory of symmetrizability. *Dokl. Akad. Nauk SSSR* **187**, *2* (1969), 270–273 (in Russian).

[59] K. Kunen, J. E. Vaughan (Eds). Handbook of Set-Theoretic Topology. Amsterdam, North-Holland, 1984.

[60] G. Kurepa. Tableaux ramifiés d'ensembles. Espaces pseudo-distanciés. *C. R. Acad. Sci., Paris* **198** (1934), 1563–1565 (in French).

[61] S. Leader. Metrization of proximity spaces. *Proc. Amer. Math. Soc.* **18** (1967), 1084–1088.

[62] L. F. McAuley. A note on complete collectionwise normality and paracompactness. *Proc. Amer. Math. Soc.* **9**, *5* (1958), 796–799.

[63] E. A. Michael. Point-finite and locally finite coverings. *Canad. J. Math.* **7** (1955), 275–279.

[64] E. A. Michael. Continuous selections. I. *Ann. of Math. (2)* **63** (1956), 361–382.

[65] E. A. Michael. Continuous selections. II. *Ann. of Math. (2)* **64** (1956), 562–580.

[66] E. A. Michael. Continuous selections. III. *Ann. of Math. (2)* **65** (1957), 375–390.

[67] E. A. Michael. Selected selection theorems. *Amer. Math. Montly 63*, *4* (1956), 233–238.

[68] E. A. Michael. A theorem on semi-continuous set-valued functions. *Duke Math. J.* **26** (1959), 647–651.

[69] E. A. Michael. Some problems. In: Open Problems in Topology (Eds J. van Mill, G. M. Reed) Amsterdam, North-Holland, 1990, pp. 271–277.

[70] E. A. Michael, C. Pixley. A unified theorem on continuous selections. Pacific J. Math. **87**, *1* (1980), 187–188.

[71] J. van Mill, J. Pelant, R. Pol. Selections that characterize topological completeness. *Fund. Math.* **149**, *2* (1996), 127–141.

[72] A. Miscenko. Spaces with point-countable base. *Dokl. Akad. Nauk SSSR* **144** (1962), 985–988 (in Russian); English translation in: *Soviet Math. Dokl.* **3** (1962), 855–858.

[73] R. L. Moore. Foundations of point set theory. Revised edition. AMS Colloquium Publications, Vol. **XIII**. Providence, R.I., AMS, 1962.

[74] R. L. Moore. A set of axioms for plane analysis situs. *Fund. Math.* **25** (1935), 13–28.

[75] K. Morita. Some properties of M-spaces. *Proc. Japan Acad.* **43** (1967), 869–872.

[76] K. Nagami. Paracompactness and strong screenability. *Nagoya Math. J.* **8** (1955), 83–88.

[77] J. Nagata. On a necessary and sufficient condition of metrizability. *J. Inst. Polytech. Osaka City Univ. Ser. A. Math.* **1** (1950), 93–100.

[78] L. J. Norman. A sufficient condition for quasi-metrizability of a topological space. *Portugal. Math.* **26** (1967), 207–211.

[79] P. Nyikos, H.-C. Reichel. On the structure of zerodimensional spaces. *Indag. Math.* **37** (1975) 120–136.

[80] P. Nyikos, H.-C. Reichel. On uniform spaces with linearly ordered bases. II. *Fund. Math.* **93**, *1* (1976), 1–10.

[81] V. Niemytzki. On the "third axiom of metric spaces". *Trans Amer. Math. Soc.* **29**, *3* (1927), 507–513.

[82] Über die Axiome des metriohen Raumes. *Math. Ann.* **104**, *1* (1931), 666–671 (in German).

[83] M. Paoli, E. Ripoli. An immersion in topologies on hyperspaces. *Boll. Un. Mat. Ital. A (6)* **4**, *1* (1985), 111–118.

[84] E. Pearl (ed.). Open Problems in Topology II. Amsterdam, Elsevier, 2007.

[85] C. Pixley, P. Roy. Uncompletable Moore spaces. Proceedings of the Auburn Topology Conference (Auburn Univ., Auburn, Ala., 1969; dedicated to F. Burton Jones on the occasion of his 60th birthday). Auburn, Ala., Auburn Univ., 1969, 75–85.

[86] V. Ponomarev. Axioms of countability and continuous mappings. *Bull. Acad. Polon. Sci. Ser. Math. Astr. Phys.* **8** (1960), 127–133.

[87] C. W. Proctor. Metrizable subsets of Moore spaces. *Fund. Math.* **66** (1969/1970), 85–93.

[88] G. M. Reed. Concerning first countable spaces. *Fund. Math.* **74**, *3* (1972), 161–169.

[89] G. M. Reed. Concerning first countable spaces. II. *Duke Math. J.* **40** (1973), 677–682.

[90] G. M. Reed. Concerning first countable spaces. III. *Trans. Amer. Math. Soc.* **210** (1975), 169–177.

[91] G. M. Reed. Concerning normality, metrizability and the Souslin property in subspaces of Moore spaces. *General Topology and Appl.* **1**, *3* (1971), 223–246.

[92] G. M. Reed. Concerning completable Moore spaces. *Proc. Amer. Math. Soc.* **36** (1972), 591–596.

[93] G. M. Reed. On screenability and metrizability of Moore spaces. *Canad. J. Math.* **23** (1971), 1087–1092.

[94] G. M. Reed. On completeness conditions and the Baire property in Moore spaces. TOPO 72–general topology and its applications (Proc. Second Pittsburgh Internat. Conf., Pittsburgh, Pa., 1972; dedicated to the memory of Johannes H. de Groot). Lecture Notes in Math. vol. **378**, Berlin, Springer, 1974, 368–384.

[95] G. M. Reed. On chain conditions in Moore spaces. *General Topology and Appl.* **4** (1974), 255–267.

[96] G. M. Reed. On continuous images of Moore spaces. *Canad. J. Math.* 26 (1974), 1475–1479.

[97] D. Repovš, P. V. Semenov. Continuous selections of multivalued mappings. Mathematics and its Applications vol. **455**. Dordrecht, Kluwer Academic Publishers, 1998.

[98] H.-J. Schmidt. Hyperspaces of quotient and subspaces I. Hausdorff topological spaces. *Math. Nachr.* **104** (1981), 271–280.

[99] I. Shishkov. Extensions of l.s.c. mappings into reflexive Banach spaces. *Set-Valued Anal.* **10**, *1* (2002), 79–87.

[100] I. Shishkov. Selections of set-valued mappings with hereditarily collectionwise normal domain. *Topology Appl.* **142**, *1–3* (2004), 95–100.

[101] I. Shishkov. On a Conjecture of Choban, Gutev and Nedev. *Math. Balkanica (N.S.)* **18**, *1–2* (2004), 193–196.

[102] M. Sion, G. Zelmer. On quasi-metrizability. *Canad. J. Math.* **19** (1967), 1243–1249.

[103] Yu. M. Smirnov. On metrization of topological spaces. *Uspekhy Mat. Nauk* **6**, *6(46)* (1951), 100–111 (in Russian); English translation in: *Amer. Math. Soc. Translation* **1953**, *91* (1953), 17 pp.

[104] Yu. M. Smirnov. On the completeness of proximity spaces. *Trudy Moskov. Mat. Obšč.* **3** (1954), 271–306.

[105] F. D. Tall. A note on normality and collectionwise normality. *Topology Proc.* **7**, *2* (1982), 267–277.

[106] F. D. Tall. The normal Moore space problem. Topological structures. II. Proc. Sympos. Topology and Geom., Amsterdam, 1978. Math. Centre Tracts vol. **116**. Amsterdam, Math. Centrum, 1979, 243–261.

[107] F. D. Tall. Set-theoretic consistency results and topological theorems concerning the normal Moore space conjecture and related problems. Ph.D. Thesis, University of Wisconsin, Madison, Wis., 1969, 105 pp.

[108] W. A. Wilson. On semi-metric spaces. *Amer. J. Math.* **53**, *2* (1931), 361–373.

[109] W. A. Wilson. On quasi-metric spaces. *Amer J. Math.* **53**, *3* (1931), 675–684.

[110] J. N. Younglove. Concerning dense metric subspaces of certain non-metric spaces. *Fund. Math.* **48** (1959), 15–25.

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Received July 5, 2017