Transform Methods and Special Functions' 2011

6th International Conference, Sofia, October 20-23, 2011

TMSF' 2011
BOOK OF ABSTRACTS

Sofia, 2011

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences

International Steering Committee:

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Website of TMSF' 2011: http://www.math.bas.bg/∼tmsf

On the occasion of 80th anniversary of Professor Peter Rusev

Host:

Institute of Mathematics and Informatics
– Bulgarian Academy of Sciences (IMI-BAS),

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Section "Analysis, Geometry and Topology"

Co-Organizers:

- Faculty of Applied Mathematics and Informatics
  – Technical University of Sofia (FAMI-TUS),
- National Science Fund – Bulgaria: Project D-ID 02/25/2009,
- Union of Scientists in Bulgaria (USB),
- Union of Bulgarian Mathematicians (UBM).

Among the **main topics of "TMSF' 2011"**, are:

- Special Functions, Classical Orthogonal Polynomials
- Classical and Generalized Integral Transforms
- Fractional Calculus and its Applications
- Operational and Convolutional Calculus
- Geometric Function Theory, Functions of One Complex Variable
- Related Topics of Analysis, Several Complex Variables

Proceedings:

A special issue of the international journal "*Mathematica Balkanica*" (Website: http://www.mathbalkanica.info/) will be published with carefully selected and reviewed papers presented at TMSF' 2011.

History of the TMSF International Meetings in Bulgaria

• The First International Workshop "TMSF, Sofia' 94" took place near Sofia (the resort town of Bankya, 20 km far from Sofia) in the period 12-17 August 1994. It was attended by 46 mathematicians and 6 accompanying persons from 15 countries. Proceedings: P. Rusev, I. Dimovski, V. Kiryakova (Eds.), "*Transform Methods & Special Functions, Sofia' 94*", SCT Publishing - Singapore, 1995, 380 pp.

• The Second International Workshop "TMSF, Varna'96" took place in Varna (the Black Sea resort "Golden Sands"), in the period 23-30 August 1996. It was attended by 73 participants and several accompanying persons from 19 countries. Proceedings: P. Rusev, I. Dimovski, V. Kiryakova (Eds.), "*Transform Methods & Special Functions, Varna' 96*", ISSN 954-8986-05-1, IMI - Bulg. Acad. Sci., Sofia, 1998, 614 pp.

• The Third International Workshop "TMSF, AUBG'99"' took place in the town of Blagoevgrad (100km south from Sofia), 13-20 August 1999, with the kind assistance and co-organization of the American University in Bulgaria (AUBG). There were 47 participants from 16 countries. Proceedings: P. Rusev, I. Dimovski, V. Kiryakova (Eds.), Special issues of the journal "*Fract. Calc. Appl. Anal.*", Vol. **2**, No 4 & 5 (1999), IMI - Bulg. Acad. Sci., Sofia, 1999; 382 pp. (`http://www.math.bas.bg/~fcaa/`)

• The Fourth International Workshop "TMSF, Borovets'2003" took place in the frames of the 1st MASSEE (Mathematical Society of South-Eastern Europe) Congress, in Borovets (famous winter resort in the Rila mountain), 15-21 September 2003, with 40 participants from 13 countries. Proceedings: P. Rusev, I. Dimovski, V. Kiryakova (Eds.), Special issue of the journal "*Mathematica Balkanica*", Vol. **18**, No 3 & 4 (2003), Bulg. Acad. Sci. - Nat. Committee for Math., Sofia, 2004, 274 pp. (`http://www.mathbalkanica.info/`)

* The Fifth International meeting "TMSF" was organized as an International Symposium "Geometric Function Theory and Applications' 2010" held in Sofia, at IMI – BAS, in the period 27-31 August 2010, and attracted 50 participants and 7 accompanying persons from 12 countries. Visit `http://www.math.bas.bg/~tmsf/gfta2010/`. Proceedings: V. Kiryakova, S. Owa (Eds.), Special issues of the journal "*Fract. Calc. Appl. Anal.*", Vol. **13**, No 4 & 5 (2010), IMI - Bulg. Acad. Sci., Sofia, 2010; 220 pp. (`http://www.math.bas.bg/~fcaa/`)

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80th ANNIVERSARY OF PROFESSOR PETER RUSEV

Prof. Dr.Sc. Peter Rusev is born 28 August 1931 in the town of Shumen, and this year he is celebrating his 80th Jubilee.

The *main research interests* of Prof. Rusev are in the topics: classical orthogonal polynomials, holomorphic functions and series expansions, special functions and integral transforms. He is *author of more than 95 scientific articles, 2 scientific monographs and many other publications*, and a member of the Editorial Board of the journal *"Fractional Calculus & Applied Analysis"*.

Graduated (1953) at Dept. Math. and Physics - Sofia University, he has obtained Ph.D. degree from Sofia University in 1957, for the thesis "*On the distribution of zeros of a class of entire functions represented in integral form*", under the guidance of Prof. Lyubomir Iliev.

His professional career in Institute of Mathematics and Informatics started in 1958 as a junior researcher, and habilitation (Asso. Prof) in 1968. The next degree - Dr. Sc., he received in 1979 at Bulg. Acad. Sciences for the thesis "*Representation of analytic functions by means of systems of polynomials and functions of Laguerre and Hermite of second kind*". Full Prof. from 1983, at same institute. Fellow of Alexander v. Humboldt Foundation - at Göttingen University (Sept. 1967-June 1968, Sept. 1968-June 1969). Emeritus Prof. from 2002.

Prof. Rusev is *scientific advisor of 7 Ph.D. students* and their theses, as follows:

– Ivan Ramadanoff (Prof., Dr., Univ. de Caen, France, Dirigeur de Recherches); "Some problems of the theory of the Bergman function" (1974);

– Ivanka Kasandrova (Asso. Prof., Dr., Plovdiv Univ.): "Distribution and asymptotic properties of the zeros of a class of entire functions" (1977);

– Johann Davidov (Prof. D.Sc., IMI-BAS): "Holomorphic mappings and representative domains" (1977);

– Valentin Hristov (Asso. Prof., Dr., IMI-BAS):" On the pseudometrics of Caratheodory and Kobayashi" (1978);

– Georgi Boychev (Asso. Prof., Dr., Tracian Univ. at St. Zagora): "Uniform convergence and summability of Jacobi, Laguerre and Hermite series" (1984);

– Lyubomir Boyadjiev (Prof., Dr.Sc., Tech. Univ. Sofia & Kuwait Univ.): "Analytic functions and Laguerre series" (1986);

– Jordanka Paneva-Konovska (Asso. Prof., Dr., Tech. Univ. Sofia): "Basicity and completeness of numerable systems of Bessel functions and polynomials" (1998).

Prof. Rusev has been advisor of 4 research projects (1989-2000), organizer of a series of international mathematical conferences and co-editor of proceedings of such conferences, namely: 2nd Congress of Bulgarian Mathematicians (Varna 1967, Secretary), Internat. Conf. on Constructive Theory of Functions (Varna 1970, Vice-President), 3rd Congress of Bulgarian Mathematicians (Varna 1972, Vice-President), Internat. Conference on Complex Analysis (Varna 1981, Progr. Comm.), Intern. Colloq. on Complex Analysis and Applications (Golden Sands 1983, Progr. Committee), Internat. Workshops on Transform Methods & Special Functions (Bankya 1994, Varna 1996, Blagoevgrad 1999, Chairman), Internat. Workshop "Transform Methods & Special Functions' 2003" (Progr. Committee), Jubilee Session for the 50th anniversary of IMI (1997, Org. Committee); Intern. Symposium "Geometric Function Theory and applications' 2010" (member of Org. and Progr. Committees), etc.

Prof. Rusev has taken essential role as editor and translator of several volumes, recently published by Bulgarian Academy of Sciences, with selected papers of other famous Bulgarian mathematicians as Nikola Obrechkoff, Lyubomir Tchakaloff, Lyubomir Iliev, etc. He has been also Ed.-in-Chief and author for the parts on special functions, complex analysis, theory of functions, in Physics-Mathematical and Technical Encyclopedia of Bulgarian Academy of Sciences, 1990 and 2000.

In the Institute of Mathematics and Informatics (IMI) - Bulgarian Academy of Sciences, he has been a Scientific Secretary (1964-1969), Vice-Director (1971-1972), member of the Scientific Council of IMI (1995-2007). Chairman of the Specialized Sci. Council on Mathematics and Mechanics at the Higher Attestation Commission, Bulgaria (1998-2004). Expert for the National Science Fund and International foundation "St.St. Cyril and Methodius" on competitions for DAAD scholarships, research projects, etc.

Member of Amer. Math. Soc., Soc. for Didactic of Math. (Germany), Union of Bulgarian Scientists. Reviewer for Math. Reviews and mathematical journals as "Serdica (Bulg. Math. Journal)", "Math. Balkanica", "Revista Technica" (Univ. del Zulia, Venezuela), "Kuwait J. Sci & Eng.", "Soochow J. of Math." (Taipei, Taiwan), "J. Math. Anal. and Appl.", "Bull. Belgian Math. Soc.", "Ann. Polonici Mat.", etc.

Visiting Professor at Trier University and Göttingen University (Germany), International Mathematical Banach Center - Warsaw (Poland), invited talks at many international conferences (Germany, France, Poland, Roumania, Russia, Turkey, Hungary, Serbia), etc.

Teaching activities (at Sofia University, Rousse University, Plovdiv University, Shumen University, IMI - BAS): lectures for undergraduate students (Complex Analysis, Linear Algebra and Analytic Geometry, Differential Equations), lectures for graduate students (Entire Functions, Conformal Mappings, Classical Orthogonal Polynomials, Functions of Several Variables, Zeros of Polynomials), seminars, etc.

For his research, scientific and teaching activities, Prof. Rusev has been awarded by: – Medal "Cyril and Methodius" (1974, 1984); Jubilee medal "1300 Years of Bulgaria" (1981); Jubilee medal "Marin Drinov" - on occasion of 100 years of Academy (1969); Memorial medals "10 years of Shumen University" (1981) and "25 years of Shumen University" (1996); Medal for contributions to IMI on occasion of its 60 years (2007). In 2002, he was promoted as "Doctor Honoris Causa" of the Shumen University.

More details on the CV and lists of publications of Prof. Peter Rusev can be seen at: – http://versita.com/rusev/; – "70th Anniversary of Professor Peter Rusev". *Fract. Calc. Appl. Anal.* **4**, No 3 (2001), pp. 409–416.

Let us pass to Prof. Rusev and his family our best regards and wishes on occasion of his 80th anniversary,

On behalf of the Organizing and Steering Committees of TMSF' 2011,

Virginia Kiryakova

Main directions of P. Rusev's scientific contributions

Prof. Peter Rusev has essential contributions in the following areas, related to the FCAA topics: – special functions and integral representations; – classical orthogonal polynomials; – distributions of zeros of entire functions, defined by means of Fourier transforms; – Bergman function.

List of Publications

Monographs:

Analytic Functions and Classical Orthogonal Polynomials. Bulgarian Mathematical Monographs **3**, Publ. House of Bulg. Acad. of Sci., Sofia, 1984, 135 pp.

Classical Orthogonal Polynomials and Their Associated Functions in Complex Domain. Bulgarian Mathematical Monographs **10**, Prof. Marin Drinov Acad. Publ. House, 2005, 278 pp.

Research Papers and Surveys:

1. On the distribution of the zeros of a class of entire functions represented in integral form, Ph.D. Thesis, Sofia, 1957, 71 pp. (Bulgarian).

2. On the asymptotic behaviour of the zeros of a class of entire functions, Proc. Inst. Math. Bulg. Acad. Sci., 4, no. 2, 1960, 67 - 73 (Bulgarian).

3. Distribution of the zeros of a class of entire functions, Phys. - Math. Journal, 4(37), 1961, 130 - 135 (Bulgarian).

4. Über die Verteilung der Nullstellen einer Klasse ganzer Funktionen, C. R. l'Acad. bulg. Sci., 14, no. 1, 1961, 7 - 9.

5. On the zeros of a class of entire functions, Phys. - Math. Journal, 5(38), 1962, 295 - 298 (Bulgarian).

6. Expansion of analytic functions in Jacobi polynomials, Proc. Inst. Math. Bulg. Acad. Sci., 7, 1963, 61 - 73 (Bulgarian).

7. Similar transformations of the metric spaces and the notion of ε-entropy of a metric space, Phys. - Math. Journal, 6(39), 1963, 37 - 39 (Bulgarian).

8. On Jacobi polynomials, C. R. l'Acad. bulg. Sci., 16, no. 2, 1963, 117 - 119.

9. Analytic transformations which increase the volume and the Bergman functions, Proc. Inst. Math. Bulg. Acad. Sci., 8, 1964, 195 - 201 (Bulg.).

10. Analytic transformations and the Bergman function, Dokl. Acad. Nauk USSR, 157, no. 2, 1964, 279 - 280 (Russian).

11. Asymptotic properties of the zeros of a class of meromorphic functions, Ann. l'Univ. Sofia, Fac. Math. Inform., Livre 1 - Mathem., 58, 1965, 241 - 271 (Bulgarian).

12. Convergence of series in Jacobi and Bessel poynomials on the boundaries of their regions of convergence, Proc. Inst. Math. Bulg. Acad. Sci., 9, 1966, 73 - 83 (Bulgarian).

13. A class of entire functions with only real zeros, C. R. l'Acad. bulg. Sci., 19, no. 7, 1966, 569 - 570.

14. On a theorem of G. Pólya, C. R. l'Acad. bulg. Sci., 19, no. 8, 1966, 689 - 691.

15. A method for computation of the roots of entire functions of Laguerre's type (with K. Docev and P. Barnev), Proc. Inst. Math. Bulg. Acad. Sci., 10, 1969, 155 - 160 (Bulgarian).

16. A class of analytically uncontinuable series in orthogonal polynomials, Mathem. Annalen, 184, 1969, 61 - 64.

17. On an application of S. N. Bernstein's polynomials, C. R. l'Acad. bulg. Sci., 26, no. 5, 1973, 585 - 586 (Russian).

18. Some results about the distribution of the zeros of the entire functions of the kind $\int_0^1 f(t)\cos zt\,dt$ and $\int_0^1 f(t)\sin zt\,dt$, Proc. Inst. Math. Bulg. Acad. Sci., 15, 1974, 33 - 62 (Bulgarian).

19. Some boundary properties of series in Laguerre polynomials, Serdica, 1, 1975, 64 - 76.

20. Convergence of series in Laguerre polynomils, Ann. l'Univ. Sofia, Fac. math. mech., 67, 1976, 249 - 268 (Bulgarian).

21. Laguerre's functions of the second kind, Ann. l'Univ. Sofia, Fac. math. mech., 67, 1976, 269 - 283 (Bulgarian).

22. Overconvergence of series in Laguerre's polynomials, Ann. l'Univ. Sofia, Fac. math. mech., 67, 1976, 285 - 294 (Bulgarian).

23. Biehler - Hermite's theorem for a class of entire functions, Ann. l'Univ. Sofia, Fac. math. mech., 67, 1976, 295 - 303 (Bulgarian).

24. Recurrence equations, orthogonal polynomials and functions of second kind, Ann. HPI Shummen, 1, 1976, 291 - 308 (Bulgarian).

25. Abel's theorem for Laguerre's series, C. R. l'Acad. bulg. Sci., 29, no. 5, 1976, 615 - 617 (Russian).

26. On the representation of a class of analytic functions by series in Laguerre's polynomials, C. R. l'Acad. bulg. sci., 29, no. 6, 1976, 787 - 789 (Russian).

27. Convergence and (C,1)-summability of Laguerre's series at points of the boundaries of their convergence regions, C. R. l'Acad. bulg. Sci., 29, no. 7, 1976, 947 - 950 (Russian).

28. Hermite functions of second kind, Serdica, 2, 1976, 177 - 190.

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Hermite functions of second kind, Serdica, 2, 1976, 277 - 282.

31. Expansion of analytic functions in Laguerre's series, Ann. l'Univ. Sofia, Fac. math. mech., 68, 1977, 179 - 216 (Bulgarian).

32. On the completeness of the systems of Laguerre's functions of second kind, C. R. l'Acad. bulg. Sci., 30, no. 1, 1977, 9 - 11 (Russian).

33. On the representation of analytic functions by means of Laguerre polynomials, C. R. l'Acad. bulg. Sci., 30, no. 2, 1977, 175 - 178.

34. A theorem of Tauber's type for the summation by means of Laguerre's polynomials, C. R. l'Acad. bulg. Sci., 30, no. 3, 1977, 331 - 334 (Russian).

35. On an inequality for Laguerre's functions of the second kind, C. R. l'Acad. bulg. Sci., 30, no. 5, 1977, 661 - 663 (Russian).

36. Uniqueness of the representation of an analytic function by series in Laguerre's functions of second kind, C. R. l'Acad. bulg. Sci., 30, no. 7, 1977, 969 - 971 (Russian).

37. Representation of analytic functions by the systems of Laguerre's and Hermite's polynomials and functions of second kind, Dr.Sc. Thesis, Sofia, 1978, 202 pp. (Bulgarian).

38. On the representation of analytic functions by Laguerre's series, Dokl. Akad. Nauk USSR, 240, no. 5, 1978, 1025 - 1027 (Russian).

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40. Convergence and (C,1)-summability of Laguerre's series on the boundaries of their convergence regions, Ann. l'Univ. Sofia, Fac. math. mech., 69, 1979, 79 - 106 (Bulgarian).

41. Some results about the representation of analytic functions by Laguerre's series, C. R. l'Acad. bulg. Sci., 32, no. 5, 1979, 569 - 571 (Russian).

42. Analytische Fortsetzung der Laplaceschen Transformation einer Klasse analytischer Funktionen, C. R. l'Acad. bulg. Sci., 32, no. 6, 1979, 719 - 720.

43. On the multiplication of series in Laguerre's functions of second kind, C. R. l'Acad. bulg. Sci., 32, no.7, 1979, 867 - 869 (Russian).

44. On the representation of analytic functions by series in Laguerre's polynomials, Dokl. Acad. Nauk USSR, 249, no.1, 1979, 57 - 59 (Russian).

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46. The representation of analytic function by means of series in Laguerre functions of second kind, Serdica, 5, 1979, 60 -63.

47. Completeness of Laguerre and Hermite functions of second kind, In Proc. "Constructive Functions Theory '77", Sofia, 1980, 469 - 473.

48. Note on asymptotic properties of confluent hypergeometric functions, C. R. l'Acad. bulg. Sci., 33, No 1, 1980, 27 - 30.

49. On the representation of analytic functions by means of confluent hypergeometric functions, C. R. l'Acad. bulg. Sci., 33, 1980, 155 - 158.

50. On the asymptotic behaviour of the Laguerre functions of second kind, Ann. l'Univ. Sofia, Fac. Math. Mech., 70, 1981, 123 - 132 (Bulg.).

51. A necessary and sufficient condition for representability of an analytic function by series in Laguerre functions of second kind, Ann. l'Univ. Sofia, Fac. math. mech., 70, 1981, 133 - 140 (Bulgarian).

52. Hankel's transform and series in Laguerre polynomials, Pliska (Studia Mathematica Bulgarica), 4, 1981, 9 - 14.

53. Expansion of analytic functions in series of classical orthogonal polynomials, Banach Center Publ., vol. 11, Warszawa, 1983, 287 - 298.

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55. On the Laplace transform of a class of analytic functions, Bull. de la Societé des sciences et de lettres de Lódz XXXV, 1985, 1 - 5 (Dedicated to Prof. Zygmund Charzyńsky on occasion of his 50 years' scientific work).

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57. Integral representation of analytic functions by means of degenerate hypergeometric functions, Ann. l'Univ. Sofia, Fac. Math. Mech., 72 (1978), 99 - 125 (Bulgarian).

58. Uniform convergence of Laguerre series on boundary arcs of their regions of convergence, Ann. l'Univ. Sofia, 77(1988), Fac.. Math. Mech., Livre 1 - Mathem., 151 - 161 (with G. Boychev) (Bulgarian).

59. On a complete system in $L^2(0,\infty)$, C. R. l'Acad. bulg. Sci., 43, no. 1, 1990, 13 - 14.

60. Müntz type theorems for systems of Tricomi's confluent hypergeometric functions, C. R. l'Acad. bulg. Sci., 43, no. 3, 1990, 25 - 28.

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65. Complete systems of Jacobi associated functions in spaces of holomorphic functions. Analysis 14, 1994, 249 - 255.

66. An improvement of T.M. Cherry's asymptotic formula for the Weber-Hermite function, Transforms Methods and Special functions, Proceedings of International Workshop, Sofia' 94, 272 - 281.

67. Complete systems of Kummer and Weber-Hermite functions in spaces of holomorphic functions, Symp. Gaussiana, Conf.A, 1995, 723-731.

68. Completeness of systems related to Laguerre and Hermite polynomials and associated functions, Analysis 17 (1997), 323 - 334.

69. Holomorphic extension by means of series in Jacobi, Laguerre and Hermite polynomials, Jubilee Proc. "25 years of Shummen's Univ. "Episkop K. Preslavski", Reports - Math. Inform., Part I (1997), 5-12.

70. Complete systems of Gauss hypergeometric functions in spaces of holomorphic functions, Facta Universitatis (Niš), Ser. Math. Inform. 12 (1997), 23-32 (Special Issue dedic. to Prof. D.S. Mitrinović, 1908 - 1995).

71. On the asymptotics of the Weber - Hermite function in the complex plane, Fract. Calc. Appl. Anal., Vol. 1, no. 2 (1998), 151 - 166.

72. Complete systems of Gegenbauer polynomials in spaces of holomorphic functions, Math. Balkanica, 12 (1998), Fasc. 1-2, 171 - 178.

73. Obreshkoff's generalization of Descartes rule, Transform Methods and Special Functions, Varna'96, Proc. of Internat. Workshop, 551 - 556.

74. Zeros of polynomials and entire functions in the works of N. Obreshkoff, Annuaire l'Univ. Sofia, Fac. Mathem. Inform., Livre 1 - Mathematiques, Tome 89 (1995), 37 - 46.

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77. Complete systems of Laguerre polynomials with large parameter, C. R. l'Acad. Bulg. Sci., 52 (1999), No. 5-6, 21 - 22.

78. Komplexe Zahlen und algebraische Gleichungen, Beitrage zum

Mathematikunterricht, 1999, 429 - 432.

79. Perron's type asymptotic formulas for the Laguerre polynomials, Fract. Calc. Appl. Anal., 2 (1999), No. 5, 689 - 698.

80. Complete systems of Tricomi functions in spaces of holomorphic functions, Ann. l'Univ. Sofia, , Fac. Mathem. Inform. Livre 1 - Mathematiques, Tome 88 (1994), Livre 3, 401 - 407.

81. On the asymptotics of a function related to Tricomi's confluent hypergeometric function, Math. Balkanica, 13 (1999), No. 3-4, 409 - 418.

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83. Complete system of Hermite associated functions, Serdica Math. J., 26 (2000), 221 - 228.

84. An inequality for Hermite's polynomials in the complex plane, C.R. l'Acad. Bulg. Sci. 53 (2000), No 10, 13 - 16.

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90. A note on a classical generating function for the Jacobi polynomials, Fract. Calc. Appl. Anal. 8, No 1 (2005), 63-66.

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93. Laguerre polynomials and the nonreal zeros of Riemann's ζ-function. C. R. Acad. Bulg. Sci. 63, No 1 (2010), 1547 - 1550.

94. Zero distribution of a class of finite Fourier transforms, Math. Balkanica 25, No 1-2 (2011), 79 - 93.

95. Zeros of entire Fourier transforms (joint with D. Dimitrov), East J. of Approximations 17, No 1 (2011), 1 - 110.

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SOLUTIONS OF FRACTIONAL DIFFUSION-WAVE EQUATIONS IN TERMS OF H-FUNCTIONS

Bader AL-SAQABI

Department of Mathematics, Faculty of Science
Kuwait University, Safat 13060, KUWAIT
e-mail: k-b-s@hotmail.com

The method of integral transforms based on joint application of a fractional generalization of the Fourier transform and the classical Laplace transform is utilized for solving Cauchy-type problems for the space-time fractional diffusion-wave equations expressed in terms of the Caputo time-fractional derivative and the Weyl space-fractional operator.

The solutions obtained are in integral form whose kernels are Green functions expressed in terms of the Fox H-functions. The results derived are of general nature and include already known results as particular cases.

MSC 2010: 35R11, 42A38, 26A33, 33E12
Key Words and Phrases: Caputo fractional derivative, fractional diffusion-wave equations, Laplace transform, fractional Fourier transform

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HYPERBOLIC FOURTH-R QUADRATIC EQUATION AND HOLOMORPHIC FOURTH-R POLYNOMIALS

Lilia APOSTOLOVA

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bontchev str., Block 8, Sofia 1113, BULGARIA
e-mail: liliana@math.bas.bg

Fourth-dimensional analogue of the complex numbers and holomorphic functions is proposed by S. Dimiev in the paper "Double complex analytic functions", *Applied Complex and Quaternionic Approximation*, Edizione Nova Cultura, Roma, 2009, 58–75. These numbers, named double-complex numbers, or fourth-**R** numbers, form a commutative associative algebra with zero divisors. Other fourth-dimensional algebra is this one of the hyperbolic double-complex numbers, or hyperbolic fourth-**R** numbers. It generalizes the known two-dimensional hyperbolic complex numbers $x+jy$, where $j^2 = +1$ and x, y are real numbers. Hyperbolic double-complex numbers form also an algebra with zero divisors.

In the talk we obtain the square roots of the hyperbolic fourth-**R** numbers and give the solutions of the hyperbolic fourth-**R** quadratic equation. Then we list the holomorphic fourth-**R** polynomials.

MSC 2010: 30C10; 32A30; 30G35

Key Words and Phrases: fourth-**R** numbers, hyperbolic fourth-**R** numbers, holomorphic fourth-**R** polynomials, hyperbolic fourth-**R** quadratic equation

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AN ALGEBRAIC APPROACH TO TEMPERED DISTRIBUTIONS

Dragu ATANASIU [1], **Piotr MIKUSIŃSKI** [2], **Dennis NEMZER** [3,§]

[1] School of Engineering – Borås University
Borås, SE 501 90, SWEDEN, e-mail: dragu.atanasiu@hb.se

[2] Department of Mathematics – University of Central Florida
Orlando, FL 32816-1364, USA, e-mail: piotrm@mail.ucf.edu

[3] Department of Mathematics – California State University, Stanislaus
One University Circle, Turlock, CA 95382, USA
e-mail: jclarke@csustan.edu

Using a method similar to the one of Yosida (*Operational Calculus, A Theory of Hyperfunctions*) allowing to give a simplified version of Mikusiński's operational calculus, we construct a linear space $\mathcal{B}$. A typical element of $\mathcal{B}$ has the form $\frac{Qf}{E^n}$, where Q is a polynomial, $f \in L^2(\mathbb{R})$, $E(x) = e^{-|x|}$, and E^n denotes the convolution of E with itself n times. By using the identification $f \leftrightarrow \frac{f*E}{E}$, $L^2(\mathbb{R})$ can be thought of as a subspace of $\mathcal{B}$.

Let $F = \frac{Qf}{E^n}$. Define a map $\mathscr{F} : \mathcal{B} \to \mathcal{S}'(\mathbb{R})$ ($\mathcal{S}'(\mathbb{R})$ is the space of tempered distributions) by

$$\mathscr{F}F = \frac{(1+x^2)^n}{2^n}\, Q(iD)\widehat{f},$$

where D is the differentiation operator on $\mathcal{S}'(\mathbb{R})$ and $\widehat{f}$ is the Fourier transform of the L^2 function f.

It is not difficult to show that $\mathscr{F}(\frac{f*E}{E}) = \widehat{f}$, and hence, $\mathscr{F}$ may be considered an extension of the Fourier transform on $L^2(\mathbb{R})$.

After defining a convergence structure on $\mathcal{B}$, we show that $\mathscr{F}$ is a linear bijective bicontinuous mapping of $\mathcal{B}$ onto $\mathcal{S}'(\mathbb{R})$.

MSC 2010: 42B10, 44A35, 46F12

Key Words and Phrases: convolution quotients, Fourier transform, Schwartz distributions, tempered distributions

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STRICT L^p SOLUTIONS FOR NONAUTONOMOUS FRACTIONAL EVOLUTION EQUATIONS

Emilia BAZHLEKOVA

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
"Acad. G. Bontchev" Str., Block 8, Sofia - 1113, BULGARIA
e-mail: e.bajlekova@gmail.com

Nonautonomous problems are important especially as a transient case between the linear and the nonlinear theory. We study the nonautonomous linear problem for the fractional evolution equation

$$D_t^\alpha u(t) + A(t)u(t) = f(t), \quad \text{a.a. } t \in (0, T),$$

where D_t^α is the Riemann-Liouville fractional derivative of order $\alpha \in (0, 2)$, $\{A(t)\}_{t\in[0,T]}$ is a family of linear closed operators densely defined on a Banach space X and the forcing function $f(t) \in L^p(0, T; X)$. Strict L^p solvability of this problem is proved under some additional assumptions on the family of operators $\{A(t)\}$. The proofs are based on regularity theorems for the corresponding autonomous problem.

MSC 2010: 26A33, 34A08, 34K37
Key Words and Phrases: Riemann-Liouville fractional derivative, fractional evolution equation, maximal L^p regularity, real interpolation space

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FROM THE CLASSICAL ORTHOGONAL POLYNOMIALS TO THE FRACTIONAL DIFFUSION EQUATIONS

Lyubomir BOYADJIEV [1,2]

[1] Department of Mathematics, Kuwait University, KUWAIT

[2] On leave from: Faculty of Applied Mathematics and Informatics
Technical University of Sofia, BULGARIA, e-mail: boyadjievl@yahoo.com

Fractional differential equations emerge as a useful tool for modeling anomalous phenomena in nature. The talk is aimed primarily on the space-time fractional diffusion equations utilized in modeling of diffusion processes in a porous medium.

Special attention is put on the method of integral transforms based on using a fractional generalization of the Fourier transform and the classical Laplace transform for solving Cauchy-type problem for the time-space fractional diffusion equation expressed in terms of the Caputo time fractional derivative and generalized Riemann-Liouville space-fractional derivative.

The application of a generalized Laplace-type integral transform for solving fractional diffusion equations with non-constant coefficients is also discussed.

MSC 2010: 35R11, 44A10, 44A20, 26A33, 33C45

Key Words and Phrases: fractional order partial differential equations, diffusion equation, integral transforms method, Riemann-Liouville and Caputo fractional derivatives

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HERMITE SERIES WITH POLAR SINGULARITIES

Georgi BOYCHEV

Department of Informatics and Mathematics, Trakia University
Stara Zagora – 6000, BULGARIA, e-mail: GBoychev@hotmail.com

Series in Hermite polynomials with poles on the boundaries of their strips of convergence are considered.

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COMMUTANTS OF STURM-LIOUVILLE OPERATORS ON THE HALFLINE $\mathbb{R}_+$

Nikolai BOZHINOV [1], **Ivan DIMOVSKI** [2,§]

[1] Department of Mathematics, University of National and World Economy
Studentski Grad, 1700 – Sofia, BULGARIA, e-mail: niki_bojinov@abv.bg

[2] Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bontchev Str., Block 8, 1113 – Sofia, BULGARIA
e-mail: dimovski@math.bas.bg

The linear operators $L : C[0,+\infty) \to C[0,+\infty)$ with an invariant subspace $C_h^1 = \{f \in C^1[0,+\infty) : f'(0) - hf(0) = 0\}$ commuting with a Sturm-Liouville operator $D = \frac{d^2}{dx^2} - q(x)$ in $C^2[0,+\infty)$ are characterized as the operators of the form

$$Lf(x) = \Phi_\xi\{\,T^\xi f(x)\,\},$$

where Φ is a linear functional on $C^1[0,+\infty)$ and T^ξ is the generalized translation operator of B.M. Levitan [1] with C_h^1 as an invariant subspace.

It is proven that the subspace of these operators with an invariant subspace

$$C_{h,\Phi}^1 = \{f \in C_h^1 : \Phi(f) = 0\},$$

which commute with D are the operators of the form

$$Lf(x) = \lambda f(x) + m * f\,,$$

where λ is a constant, $m \in C[0,+\infty)$, and $*$ is the non-classical convolution found by the authors [2] in 1978.

References: [1] B.M. Levitan, *Theory of Generalized Translation Operators.* Nauka, Moscow, 1973 (In Russian); [2] N. Bozhinov, I. Dimovski, Boundary value operational calculi for linear differential operator of second order. *Compt. Rend. Acad. Bulg. Sci.*, **31**, No 7 (1978), 815-818.

MSC 2010: 44A35, 44A40, 34B24

Key Words and Phrases: operational calculus, Sturm-Liouville operator, translation operators, commutant

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EXACT SOLUTIONS OF A NONLOCAL PLURIPARABOLIC PROBLEM

Georgi CHOBANOV [1,§], **Ivan DIMOVSKI** [2]

[1,2] Institute for Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonthcev Str., Block 8
Sofia - 1113, BULGARIA
e-mails: [1] chobanov@math.bas.bg , [2] dimovski@math.bas.bg

A generalization of the classical Duhamel principle for the pluriparabolic equation

$$\frac{\partial u}{\partial t_1} + \ldots + \frac{\partial u}{\partial t_n} = \frac{\partial^2 u}{\partial x^2} + F(x, t_1, \ldots, t_n) \quad \text{in} \quad 0 \le x \le a,\ 0 \le t_k < \infty,$$

with nonlocal BVC of the form $\chi_{k,\tau}\{u(x, t_1, \ldots, t_{k-1}, \tau, t_{k+1}, \ldots, t_n)\} = 0$ and $u(0, t_1, \ldots, t_n) = 0$, $\Phi_\xi\{u(x, t_1, \ldots, t_n)\} = \phi(t_1, \ldots, t_n)$ is proposed.

To this end, two nonclassical convolutions $\phi \overset{t_1 \ldots t_n}{*} \psi$ and $F \overset{x t_1 \ldots t_n}{*} G$ are introduced for functions of $t_1, \ldots, t_n$ only and of $x, t_1, \ldots, t_n$, respectively. The corresponding Duhamel representation takes the form

$$u(x, t_1, \ldots, t_n) = \frac{\partial^n}{\partial t_1 \ldots \partial t_n}(\Omega \overset{t_1 \ldots t_n}{*} \phi) + \frac{\partial^{n+2}}{\partial x^2 \partial t_1 \ldots \partial t_n}(\Omega \overset{x t_1 \ldots t_n}{*} F),$$

where Ω is the solution of the same problem for the homogeneous equation with the special choice $\phi \equiv 1$.

MSC 2010: 44A35, 44A40
Key Words and Phrases: operational calculus, nonlocal boundary conditions, pluriparabolic equations

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FRACTIONAL NONLINEAR AUTONOMOUS SYSTEMS: STABILITY ANALYSIS AND LIMIT CYCLES

Bohdan DATSKO [1,§], Yury LUCHKO [2]

[1] Department of Mathematical Modeling of Selforganisation Phenomena
Institute of Applied Problems for Mechanics and Mathematics, NASU,
Naukova Street 3b, Lviv - 79060, UKRAINE
e-mail: b_datsko@yahoo.com

[2] Beuth Technical University of Applied Sciences
Luxemburger Str. 10
Berlin - 13353, GERMANY
e-mail: luchko@beuth-hochschule.de

We investigate the stability and limit cycles of the two component autonomous system of the fractional differential equations with the nonlinear feedbacks in the form

$$\tau \cdot D_\alpha u = F(u, A),$$

where

$$D_\alpha w = (d^{\alpha_1} u_1/dt^{\alpha_1}, d^{\alpha_2} u_2/dt^{\alpha_2})^T$$

is the fractional differential operator,

$$\alpha_1, \alpha_2 \in (0,2),\ u=(u_1(t), u_2(t))^T,\ F(u,A)=(f_1(u_1,u_2,A), f_2(u_1,u_2,A))^T,$$

is a nonlinear vector-function depending on an external parameter A, the vector $\tau = (\tau_1, \tau_2)^T$ represents the characteristic times of the system. The fractional derivatives of order $\alpha \in \mathbb{R}_+$ are understood in the Riemann-Liouville or Caputo sense.

It was shown that the characteristic times and fractional derivative orders play an important role for the instability conditions and nonlinear system dynamics. Stationary solutions in such type system can be unstable

for wider range of parameters than in the system with integer derivatives. This fact takes place even for fractional order indices less than one. By computer simulation it was shown that different kinds of complex attractors including strange attractors can be observed in the simple fractional nonlinear systems. The attracting trajectories may intersect and different way of system evolution can lead to the same attractor. From the other side, the change in initial conditions can lead to different attractors. An overall picture of non-linear dynamics in fractional systems with positive and negative feedbacks is presented.

MSC 2010: 34N05; 65R20; 26A33

Key Words and Phrases: fractional differential equations, nonlinear systems, stability analysis, limit cycles

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EXISTENCE OF HOLOMORPHIC FUNCTIONS ON TWISTOR SPACES

Johann DAVIDOV [1], **Oleg MUSHKAROV** [2], §

[1] Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonchev St., Block 8
1113 – Sofia, BULGARIA

e-mails: [1] jtd@math.bas.bg , [2] muskarov@math.bas.bg

A natural question in the study of complex and almost-complex manifolds is whether there exist non-constant holomorphic functions. In our talk we shall discuss this problem for the twistor spaces of Riemannian or pseudo-Riemannian four-manifolds.

MSC 2010: 32C28, 32L25, 53C15, 32A99
Key Words and Phrases: (almost) complex manifolds, holomorphic functions, twistor spaces

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MATHEMATICS AND MUSIC – SO FAR AND SO CLOSE

Georgi DIMKOV

Department of Analysis, Geometry and Topology
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bontchev Str., Block 8, 1113 – Sofia, BULGARIA
e-mail: gdimkov@math.bas.bg

Which are the reasons to speak that Mathematics is too far from Music and vice versa? Let us try to explore this situation.

The purpose of all kind musical compositions is to give pleasure to people of different aesthetical tastes, as well as to provoke joyful or sad emotions. Music affects immediately and directly.

A mathematical result is used by special circumstances and conditions. As shows the History of Mathematics some results found their immediate application and other (including whole theories) were applied years and centuries later.

One more difference is the following. Mathematics uses 100 % logical and consecutive reasoning. Contrariwise in music as well as in the other arts the logic is very often broken to effect some special emotion.

From another point of view Mathematics and Music play similar role for other sciences and arts. Musical compositions, inspired by poetry, dramatic works, paintings etc., make deeper the feeling of the author's idea. The main purpose of Mathematics is to create tools to be used by natural, humanitarian, social and other sciences.

Now - back to the end of the title.

To compose and perform music the musicians use sound. This sound must fulfil some specific conditions and is called musical tone or simply tone. The European music uses about 90 tones with different pitches. Look on the classical musical instruments. They are divided into three main groups: string instrument, wind instruments (wooden and brass) and percussion. A wooden instrument, generally speaking, is a wooden pipe with perforations. The performer uses these perforations previously drilled in the factory. To play a string instrument is completely different. The violin has only four strings. The violinist changes the pitch of the strings by touching them on different strictly determined places. Starting the study in the childhood, the artist learns to find immediately these places. So we arrived at the question: Which law determines the touching places on the string as well as the distances between the perforations of the flute?

During the millennia people found the answer of these questions on base of their experience. But the developing of the sciences insisted to give an answer which does not depend on aesthetical reasons but does not cause contradictions.

This was a challenge for the scientists for about 5000 years. The first consecutive investigations in this domain are realized by Pythagoras.

The main topic of the talk is the construction of Pythagoras and the developing of the aesthetical ideas as well as the difficulties connected with these ideas.

MSC 2010: 00A65, 97M80

Key Words and Phrases: Mathematics and Music; musical composition; musical tone; construction of Pythagoras

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COMMUTANTS OF THE SQUARE OF DIFFERENTIATION ON THE HALF-LINE

Ivan H. DIMOVSKI [1], **Valentin Z. HRISTOV** [2]

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
"Acad. G. Bonchev" Str., Block 8, 1113 Sofia, BULGARIA
e-mails: [1] dimovski@math.bas.bg , [2] valhrist@bas.bg

Let C_h^1 denote the space of the smooth functions $f(x)$ on the real half-line $\mathbb{R}_{\geq 0} = [0, \infty)$ satisfying the initial value condition $f'(0) - hf(0) = 0$ with fixed real h. We characterize the continuous linear operators $M : C_h^1 \to C_h^1$ which commute with the square $D^2 = \dfrac{d^2}{dx^2}$ of the differentiation operator $D = \dfrac{d}{dx}$ on the subspace C_h^2 of the twice continuously differentiable functions of C_h^1. The explicit representation of such operators is

$$Mf(x) = \Phi_y\{T^y f(x)\}$$

with a linear functional Φ on C_h^1, where

$$T^y f(x) = \frac{1}{2}\{f(x+y) + f(|x-y|)\} + \frac{h}{2}\int_{|x-y|}^{x+y} f(t)dt$$

is a generalized translation operator in the sense of B. M. Levitan.

The kernel space of this operator, denoted by MP_Φ, is called the space of the mean-periodic functions for D^2 determined by Φ. It is proved that the space MP_Φ is invariant under the resolvent operator of D^2 with the boundary value conditions $y'(0) - hy(0) = 0$ and $\Phi\{y\} = 0$. A convolution structure $* : C_h^1 \times C_h^1 \to C_h^1$ is introduced in C_h^1, such that MP_Φ is an ideal in the convolution algebra $(C_h^1, *)$. This result is used for effective solution of ordinary differential equations of the form $P(D^2)y = f$ with a polynomial P in mean-periodic functions from MP_Φ.

MSC 2010: Primary: 447B37; Secondary: 47B38, 47A15

Key Words and Phrases: commutant, generalized translation operator, mean-periodic function, convolution algebra, ideal

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EXACT SOLUTIONS OF NONLOCAL BVPs FOR THE MULTIDIMENSIONAL HEAT EQUATION

Ivan DIMOVSKI [1,§], **Yulian TSANKOV** [2]

[1] Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
"Acad. G. Bonchev" Str., Block 8 Sofia – 1113, BULGARIA
email: dimovski@math.bas.bg

[2] Faculty of Mathematics and Informatics
Sofia University "St. Kliment Ohridsky"
"J. Boucher" Str., N 5, Sofia-1164, BULGARIA
email: ucankov@fmi.uni-sofia.bg

It is proposed a general approach for obtaining of exact solutions of the multidimensional heat equation

$$u_t = u_{x_1x_1} + ... + u_{x_nx_n} + F(x_1, ..., x_n, t), \quad 0 < t,\, 0 < x_j < a_j,$$

determined by: a time-nonlocal initial conditions of the form

$$\chi_\tau\{u(x_1, ..., x_n, \tau)\} = f(x_1, ..., x_n),$$

and space-local

$$u(x_1, ..., x_{j-1}, 0, x_{j+1}, ..., x_n, t) = g_j(x_1, ..., x_{j-1}, 0, x_{j+1}, ..., x_n, t)$$

and space-nonlocal boundary value conditions of the form

$$\Phi_{j,\xi}\{u(x_1, ..., x_{j-1}, \xi, x_{j+1}, ..., x_n, t)\} = h_j(x_1, ..., x_{j-1}, x_{j+1}, ..., x_n, t),$$

$j = 1, ..., n$, where χ and Φ_j are given functionals.

A generalization of Duhamel principle is proposed. The multidimensional case when the initial value condition is a local one, i.e. of the form $u(x_1, ..., x_n, 0) = f(x_1, ..., x_n)$ reduces to n one-dimensional cases.

In the Duhamel representation of the solution it is used a multidimensional non-classical convolution. This explicit representation can be used for numerical calculation of the solution.

MSC 2010: 44A35, 44A45, 44A40, 35K20, 35K05

Key Words: heat equations, nonlocal BVP, non-classical convolutions

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TWO PATHWAYS TO SUBORDINATION IN TIME-FRACTIONAL DIFFUSION

Rudolf GORENFLO [1,§], **Francesco MAINARDI** [2]

[1] Department of Mathematics an Informatics, Free University of Berlin
Arnimallee 3, Berlin, D – 14195, GERMANY
e-mail: gorenflo@mi.fu-berlin.de

[2] Department of Physics, University of Bologna
Via Irnerio 46, Bologna, I-40126, ITALY
e-mail: francesco.mainardi@unibo.it

First pathway: The uncoupled spatially one-dimensional Continuous Time Random Walk (CTRW) under power law regime is split into three distinct random walks: a walk (RW1) along the line of natural time, happening in operational time, a walk (RW2) along the line of space, happening in operational time, a walk (RW3) (the walk (RW1) inverted) along the line of operational time, happening in natural time. Via the general integral equation of CTRW and appropriate rescaling, the transition to the diffusion limit is carried out separately for each of these three random walks. Combining the limits of (RW1) and (RW2) we get the method of parametric subordination for generating particle paths, whereas combination of (RW2) and (RW3) yields the subordination integral formula for the sojourn probability density in space-time fractional diffusion.

Second pathway: Via Fourier-Laplace manipulations of the relevant fractional differential equation we obtain the subordination integral formula that teaches us how a particle path can be constructed by first generating the operational time from the physical time and then generating in operational time the spatial path. By inverting the generation of the operational time from the physical time we arrive at the method of parametric subordination.

Comment: The fascinating fact is that by parametric subordination a non-Markovian process can be produced by combination of two Lévy processes.

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MSC 2010: 26A33, 33E12, 33C40, 44A10, 45K05, 60G52

Key Words and Phrases: integral transforms, special functions, fractional calculus, Mittag-Leffler functions, Wright functions, stable probability densities, random walks, diffusion, subordination

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PASSAGE OF A LONG WAVE OVER A VERTICAL BARRIER: CALCULATION OF THE LOCAL DISTURBANCES BY A COMPLEX VARIABLE METHOD

Allaoua GUERZIZ [1,§], **Abdelhamid LAOUAR** [2]

Department of Physics, University of Annaba
P.O. Box 12, 23000 – Annaba, ALGERIA

e-mails: [1] a_guerzizw@yahoo.fr, [2] laouar.abdelhamid@univ-annaba.org

We apply a method of calculating the local disturbances caused by a long surface wave in the vicinity of an immersed vertical barrier. For that, we introduce the generalized theory of shallow water with the first order approximation and a method of the complex variable.

MSC 2010: 74J15, 76B15, 41A99, 30E10
Key Words and Phrases: gravity waves, local disturbances, obstacle, shallow water

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SELF-GRAVITATING STABILITY OF A ROTATING FLUID LAYER SANDWICHED IN A DIFFERENT FLUID

Alfaisal A. HASAN

Basic and Applied Sciences Department
– College of Engineering and Technology
Arab Academy for Science & Technology and Maritime Transport
(AASTMT)
P.O. Box 2033, Elhorria, Cairo, EGYPT
e-mail: alfaisal772001@yahoo.com

The self-gravitating instability of the present model has been discussed by using the simple linear theory. The problem is formulated, and the dispersion relation, valid for all kinds of perturbation, is derived and discussed. The self-gravitating force is destabilizing for small range of the wavenumber while it is stabilizing in the other ranges depending on the densities ratio of the fluids. For high values of Ω, the rotating force is stabilizing and it may suppress: the self-gravitating instability. In absence of the self-gravitating force effect it is found that the growth rate is zero and the model is marginally stable and so the rotating force has no direct influence on the stability of the model, since there is no term for it.

MSC 2010: 76E25, 76U05
Key Words and Phrases: self-gravitating stability, rotating fluid layer, superposed fluids

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FINANCIAL DERIVATIVES AND INTEGRAL TRANSFORMS

Shyam KALLA

Institute of Mathematics, Vyas Institutes of Higher Education
15 B, Pal-Link Road Jodhpur, 342008, INDIA

e-mail: shyamkalla@yahoo.com; shyamkalla@gmail.com

An option on an underlying asset is an asymmetric contract that entitles the holder to buy or sell the underlying asset at a specified price on or before a certain date. *American options* can be exercised at any time up to the expiration date, whereas *European options* can be exercised only on expiry date. The early exercise feature of such options leads to a free boundary value problem under the *Black-Scholes* framework.

Here first we present a brief introduction about *financial mathematics* and *mathematical Brownian motion*. Basic terminology including stochastic calculus and Ito's Lemma is introduced.

The Mellin transform is defined and some of its properties including convolution are discussed. We apply the Mellin transform technique to the Black-Scholes model and derive an analytic expression for the price of an European option and new integral representations for the price and free boundary of American options in one dimension.

MSC 2010: 33C20, 44A05, 26A33, 60J65
Key Words and Phrases: financial mathematics, Black-Scholes equation, integral transforms, financial derivatives, options

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EPQ MODEL WITH IMPERFECT QUALITY RAW MATERIAL

Abdul-Nasser EL-KASSAR [1,§],
Moueen SALAMEH [2], **Mokarram BITAR** [3]

[1] School of Business - Lebanese American University
P.O. Box 13-5053, Chouran Beirut - 1102 2801, LEBANON
e-mail: abdulnasser.kassar@lau.edu.lb

[2] Engineering Management Program - American University of Beirut
P.O. Box 11-0263, Riyad El Soleh Beirut - 1107 2020, LEBANON
e-mail: msalameh@aub.edu.lb

[3] Mathematics Department - Beirut Arab University
P.O. Box 11-5020, Beirut - LEBANON
e-mail: mokarram_h@hotmail.com

The classical economic production model (EPQ) is based on simplifying assumptions that ignore many factors encountered in real-life situations. Recently, this model has been extended in many directions by relaxing the underlying assumptions. In this paper, we examine an EPQ model that accounts for the cost of raw material needed for the production. In addition, we consider the case where the raw material acquired from the supplier contains a percentage of imperfect quality items. It is assume that the raw material is ordered and received instantaneously at the beginning of the inventory cycle. A 100 % screening process for detecting the imperfect quality items of raw material is conducted at a rate greater than the production rate.

Three different scenarios are considered. In the first, the imperfect quality items of the raw material are sold at a discounted price at the end of the screening period. The second scenario is to keep the imperfect quality items in stock until the end of the inventory cycle, and return them to the supplier when the next order of is received. The third scenario is to use the imperfect quality items of raw material in the production process, which results in a percentage of imperfect quality finished products.

MSC 2010: 90B05; **Key Words and Phrases**: economic production model, cost of raw material, imperfect quality items

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THE SPECIAL FUNCTIONS – CLASSICAL AND NEW, AND GENERALIZED FRACTIONAL CALCULUS

Virginia KIRYAKOVA

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bontchev Str., Block 8, 1113 – Sofia, BULGARIA
e-mail: virginia@diogenes.bg

Recently, there is an increasing interest and practice in using differential equations and systems of fractional order (that is, arbitrary one, not necessarily integer), as the better mathematical models of phenomena of various physical, engineering, automatization, biological and biomedical, chemical, earth, economics, etc. nature. The so-called "*Special Functions of Fractional Calculus*" (SF of FC) are the necessary tools, in terms of which the solutions to such problems can be provided in explicit form. Their theory is closely related to the theory of differentiation and integration of arbitrary (fractional) order known as classical Fractional Calculus (FC) and yet more, to the *Generalized Fractional Calculus* (GFC), in the sense of [1].

Under SF of FC, practically we mean the numerous, but still not widely popular and studied, examples of the Wright generalized hypergeometric function ${}_p\Psi_q$ and the *Fox H-function*, while the (subclass of) *classical Special Functions* (known as SF of Mathematical Physics) fall in the scheme of the generalized hypergeometric function ${}_pF_q$ and the *Meijer G-function.*

In this survey we provide an unexpectedly long *list of SF of FC*, already applied to variety of mathematical models and problems from sciences and practice. We propose an unified approach to these functions, their classification and new integral and differential representations, based on the GFC techniques and generalizing the Poisson and Euler integral formulas and the Rodrigues differential formulas. Extending our approach from [2], we prove that the *SF of FC can be represented as GFC operators of three basic simpler functions.* Part of these new results are given in [5].

Most of the SF of FC are particular cases of the so-called "*multi-index Mittag-Leffler* $(M-L)$ *functions*" $E^{(m)}_{(\alpha_1,\dots,\alpha_m),(\beta_1,\dots,\beta_m)}(z)$, introduced by the author and simultaneously by Yu. Luchko and his co-authors, and studied in a series of recent papers, as: [3], [4] and [6]. The same can be considered as fractional indices analogues of the Delerue hyper-Bessel functions and generate some Gelfond-Leontiev operators of generalized integration and differentiation, and a corresponding Laplace-type integral transform that generalizes the Obrechkoff transform.

Other *more popular examples* are: the classical M-L function $E_{\alpha,\beta}$ and its generalizations, Wright function, Bessel-Maitland, Wright-Lommel, Struve, Lommel and Airy functions, Rabotnov and Mainardi functions, Lorenzo-Hartley R-function, Dzrbashjan function with 2×2 indices, the hyper-Bessel functions, etc. We discuss also some *open problems.*

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MSC 2010: 33C60, 33E12, 26A33, 34A08

Key Words and Phrases: classical special functions, Wright generalized hypergeometric, G- and H-functions; integral representations, Rodrigues type differential formulas; generalized fractional calculus

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WEAKENED CONDITION FOR THE STABILITY TO SOLUTIONS OF PARABOLIC EQUATIONS WITH "MAXIMA"

Dimitar KOLEV [1], **Tzanko DONCHEV** [2], **Kiyokazu NAKAGAWA** [3,§]

[1] Department of Mathematics
University of Chemical Technology and Metallurgy
Kliment Ohridsky 8, Blvd., Sofia 1756 , BULGARIA
e-mail: mkolev@math.uctm.edu, kolev@uctm.edu

[2] Abdus Salam School of Mathematical Sciences
68-B, New Muslim Town, Lahore – PAKISTAN
e-mail: tzankodd@gmail.com

[3] Department of Information Science
Tohoku Gakuin University
Sendai 981-3193, JAPAN
e-mail: nakagawa@math.tohoku-gakuin.ac.jp

A class of reaction-diffusion equations with nonlinear reaction terms perturbed with a term containing "maxima" under initial and boundary conditions is studied. The similar problems that have no "maxima" have been studied during the last decade by many authors. It would be of interest the standard conditions for the reaction function to be weakened in the sense that the partial derivative of the reaction function, w.r.t. the unknown, to be bounded from above by a rational function containing $(1 + t)^{-1}$, where t is the time. When we slightly weaken the standard condition imposed on the reaction function then the solution still decays to zero not necessarily in exponential order. Then we have no exponential stability for the solution of the considered problem. We establish a criterion for the nonexponential stability. The asymptotic behavior of the solutions when $t \to +\infty$ is discussed. The parabolic problems with "maxima" arise in many areas as the theory of automation control, nuclear physics etc.

MSC 2010: 35R12, 35K50

Key Words and Phrases: reaction-diffusion equation, stability, equations with "maxima"

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ZERO DISTRIBUTION OF GEOMETRICALLY CONVERGENT SEQUENCES OF RATIONAL FUNCTIONS

Ralitza K. KOVACHEVA

Institute of Mathematics and Informatics, Bulg. Acad. Sci.
Acad. G. Bontchev str., Block 8, 1113 – Sofia, BULGARIA
e-mail: rkovach@math.bas.bg

The classical Montel's theorem says: If each function of a family $\mathcal{F}$ of functions holomorphic in some domain $B \in \mathbb{C}$ (analytic and single valued) omits some value a in B and takes the value of b, $a \neq b$ at not more than N points in B, then $\mathcal{F}$ is *normal* in B.

A natural question arises as to what happens if a family of analytic functions omits merely one finite value in B. This question is actual to sequences of rational functions armed additionally with approximating properties.

Theorem 1: *Given a domain D and a regular continuum $S \subset D$, suppose that the sequence $\{f_n\}$, $f_n \in \mathcal{R}_{n,n}$, $n = 1, 2, \cdots$ converges uniformly on ∂S to a function f with $f \not\equiv 0$ on some regular subset of ∂S in such a way that*

$$\limsup_{n\to\infty} \|f_n - f\|_{\partial S}^{1/n} < 1. \tag{1}$$

Assume that each $f_n \in \mathcal{A}(D)$ and, in addition,

$$\nu(f_n, K) = o(n) \ \ as \quad n \to \infty \tag{2}$$

on compact subsets K of D. Then the sequence $\{f_n\}$ forms a normal family in D; herewith f admits a holomorphic continuation into D.

We pose the question whether the normality remains on the sphere in case of meromorphic functions under preserving conditions (1) and (2). The answer is negative. However, a continuation as a meromorphic function is achieved.

Theorem 2: *Let* $\{f_n\}$, $f_n \in \mathcal{R}_n$, $n = 1, 2, \cdots$ *be a sequence of meromorphic functions in* D *with not more than* m *poles (poles are counted with regard to multiplicities). Under the same conditions on* S *and* D, *suppose that (1) and (2) hold. Then the sequence* $\{f_n\}$ *converges* m_1- *almost uniformly inside* D*; herewith* f *admits a continuation into* D *as a* $m-$ *meromorphic function.*

Both theorems have large applications to questions connected with holomorphic/meromorphic continuation of functions, as well as to results dealing with the behavior of the zeros of approximation sequences. The basis examples are the Pade approximants and the best rational Chebyshev aproximants.

The main idea of the talk is the presentation of these results. Results of Picard's type will be announced.

Part of the results are join with H.P. Blatt and R. Grothmann.

MSC 2010: 41A20, 30C15, 30D35, 31A05

Key Words and Phrases: rational approximation, meromorphic function, zeros and poles, Pade approximant, α-values, Picard theorem

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SOME COMPLETELY MONOTONIC PROPERTIES FOR THE $\Gamma_{p,q}$-FUNCTION

Valmir KRASNIQI [1] **, Faton MEROVCI** [2,§]

[1,2] Department of Mathematics – University of Prishtina
Mother Teresa No 5, Prishtinë - 10000, Republic of KOSOVO

e-mails: [1] vali.99@hotmail.com , [2] fmerovci@yahoo.com

For the $\Gamma_{p,q}$-function, we give some properties related to convexity, log-convexity and completely monotonic functions. Also, some properties of the $\psi_{p,q}-$analog of the ψ function have been established. As an application, when $p \to \infty, \quad q \to 1$, we obtain the results from [1] and [2].

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MSC 2010: 33B15, 26A51, 26A48

Key Words and Phrases: completely monotonic function, logarithmically completely monotonic function, Gamma function, Psi function, inequality

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THE BICENTENNIAL ANNIVERSARY OF GALOIS

Ivan LANDJEV

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
8 Acad. G. Bonchev str. 1113 Sofia, BULGARIA
e-mail: ivan@math.bas.bg

Évariste Galois is considered rightly as one of the greatest mathematicians of all times. In this year, we celebrate his bicentennial anniversary. The aim of this talk is not to go over Galois' life but to take a glimpse at a beautiful part of his mathematical heritage – Proposition VIII from his *Memoire sur les conditions de resolvabilité des equations par radicaux.* This is the last theorem in his memoir and seems to be not that well-known as it deserves to be. The formulation in his memoir is the following:

Theorem 1. *In order for an irreducible equation of prime degree to be solvable in radicals it is necessary and sufficient that once any two roots are known, the others can be deduced from them rationally.*

A modern formulation of this result would be:

Theorem 2. *Let $f(x)$ be an irreducible polynomial of prime degree p over thee field k with roots $\alpha_1, \ldots, \alpha_p$. The equation $f(x) = 0$ is solvable in radicals if and only if for all $1 \le i < j \le p$, we have $k(\alpha_1, \ldots, \alpha_p) = k(\alpha_i, \alpha_j)$.*

A translation into group theoretical language looks as follows:

Theorem 3. *Let p be a prime, and $G \le S_p$ – a transitive subgroup. Then, G is solvable if and only if the only element of G which fixes two or more points is the identity permutation.*

MSC 2010: 01A55; 11S20; 12F10
Key Words and Phrases: Galois theory, solvability in radicals, solvable groups, irreducible polynomials

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PASSAGE OF A LONG WAVE OVER A VERTICAL BARRIER: CALCULATION OF THE LOCAL DISTURBANCES BY A COMPLEX VARIABLE METHOD

Abdelhamid LAOUAR [1,§], **Allaoua GUERZIZ** [3], **S. BOUGHABA** [3]

[1] Dept. of Mathematics, [2] Dept. of Physics
University of Annaba, P.O. Box 12, 23000 – Annaba, ALGERIA

e-mails: [1] laouar.abdelhamid@univ-annaba.org, [2] a_guerzizw@yahoo.fr,
[3] sboughaba23@yahoo.fr

This paper deals with the study of a long gravity wave profile in a bidimensional rectangular basin and the free surface wave determination. This is performed by the placement of a piston wave maker at the basin extremity upstream. For $t \geq 0$, the latter starts to move according to a certain law and generates a movement. The equation of motion of such a problem with its boundaries conditions are reduced to a system of nonlinear equations, which is to be solved by applying the shallow water theory. The solution is expanded asymptotically in terms of small parameter ?. Numerical simulations are presented to support the theory.

MSC 2010: Primary 76B07, 37N10; Secondary 35Q35.
Key Words and Phrases: approximation, free surface, long gravity wave, shallow water theory

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INFORMATION TRANSMISSION IN A QUOTIENT SPACE

Gregory L. LIGHT
Department of Management, Providence College
Providence, Rhode Island 02918 – USA
e-mail: glight@providence.edu

First, information transmission is the sending and receiving electromagnetic waves in our recognized physical space-time; i.e., $\mathbb{E}(t,\mathbf{x}) = \mathbb{E}_o \cdot \cos(\omega t - \mathbf{k}\cdot\mathbf{x} + \theta)$, with $\mathbb{B}(t,\mathbf{x}) = \mathbb{B}_o \cdot \cos(\omega t - \mathbf{k}\cdot\mathbf{x} + \theta)$.

Second, the above equations can be re-expressed as $\mathbb{E}(t,\mathbf{x}) = \mathbb{E}_o \cdot e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x}+\theta)}$, with $\mathbb{B}(t,\mathbf{x}) = \mathbb{B}_o \cdot e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x}+\theta)}$, where the motion takes place in an "imaginary space-time" with the unit i.

Third, the above two alternative presentations are related by the standard covering map $p : \mathbb{R} \longrightarrow S^1$, with $p(x) = (\cos 2\pi x, \sin 2\pi x)$; that is, while the wave travels along $\mathbb{E}\times\mathbb{B} \equiv \mathbb{R}$, it keeps rotating around S^1, the quotient space of $\mathbb{R}$.

Fourth, we will explain the "double-slit paradox" by asserting that any photon γ knows if a slit on the barrier is open by the geometry of S^1, not by $\mathbb{R}$. That is, $\forall(t,\mathbf{x})$ in a neighborhook U of the ejection of γ**,** we have

$$t \equiv t_0 \left(\mathrm{mod}\ \frac{2\pi}{\omega} \equiv \frac{1}{\nu}\right) \text{ for some } t_0 \in \left[0, \frac{1}{\nu}\right], \tag{1}$$

and

$$\mathbf{x}\equiv\mathbf{x_0} \left(\mathrm{mod}\ \left(\frac{2\pi}{k}\right)\left(\frac{\mathbf{x}}{\|\mathbf{x}\|}\right) \equiv \lambda\left(\frac{\mathbf{x}}{\|\mathbf{x}\|}\right)\right) \tag{2}$$

for some $\mathbf{x}_0$ with $\|\mathbf{x}_0\| \in [0,\lambda]$; as such, $\forall\lambda \gtrapprox 0$ we have $t \cong 0$ and $\mathbf{x}\cong\mathbf{0}$, resulting in "instantaneous communication" across U; i.e., to propagate γ along the direction of $(0,\mathbf{0})$ to only one slit, say, the "upper slit" situated at $y = |d|$ meter, $\left(\frac{\sqrt{1+d^2}}{c}, 1 \text{ meter}, |d|\ \ ,0\right)$, is nearly the same as from $(0,\mathbf{0})$ to $(0,0,y>0,0)$, with $\left|\left\|\mathbb{E}_o \cdot e^{-i(\omega t - ky)}\right\|\right|^2 = \|\mathbb{E}_o\|^2 > 0$, a nonzero (constant) probability density along the y-axis for γ to be observed. However, if both slits are open, then there exists a superposition of fields, $\cos(\omega t - ky) + \cos(\omega t + ky) = 2\cos\omega t\ \cos ky$, and the probability density of γ equals zero $\forall y$ such that $\cos ky = 0$.

Fifth, we will use our previously constructed "combined $4-manifold$ $\mathcal{M}^{[3]}$" of the *Universe* to account for particles in the covering space $\mathbb{R} \subset \mathcal{M}^{[1]}$ and waves in its quotient space $S^1 \subset \mathbf{B} \subset \mathcal{M}^{[2]}$.

Last, we will draw a summary by addressing some of the implications, in particular, quantum entanglement.

MSC 2010: 32Q20; 54B15; 81P45; 81P40

Key Words and Phrases: electromagnetic waves, particle-wave duality, double-slit paradox, quantum entanglement, combined-manifold

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SOME EXISTENCE AND UNIQUENESS RESULTS FOR THE TIME-FRACTIONAL DIFFUSION EQUATIONS

Yuri LUCHKO

Department of Mathematics
Beuth Technical University of Applied Sciences Berlin
Luxemburger Str. 10, Berlin - 13353, GERMANY
e-mail: luchko@bht-berlin.de

Within the last few decades, partial differential equations of fractional order have been successfully used for modeling of many different physical, technical, biological, chemical, economical etc. processes that stimulated development of their mathematical theory. In applications, relevant processes like e.g. anomalous diffusion or wave propagation in complex systems mainly run within some bounded domains in space that corresponds to initial-boundary-value problems for the partial differential equations of fractional order, which model the processes under consideration. Both analytical and numerical investigation of these problems thus become an important task in Fractional Calculus.

In a series of author's papers, the time-fractional diffusion equation

$$(D_t^\alpha u)(t) = \mathrm{div}(p(x)\,\mathrm{grad}\,u) - q(x)u \,+ F(x,t), \quad 0 < \alpha \le 1,$$

with the Caputo fractional derivative D_t^α with respect to the time variable t along with some of its important generalizations (multi-term equation and equation of distributed order) was considered in open bounded n-dimensional domains with the corresponding initial and boundary conditions. This equation corresponds to the continuous time random walk model, where the characteristic waiting time elapsing between two successive jumps diverges, but the jump length variance remains finite and is proportional to t^α. In particular, the maximum principle well-known for the partial differential equations of elliptic and parabolic types was extended for the time-fractional diffusion equation as well as for the multi-term time-fractional diffusion equation and for the time-fractional diffusion equation of distributed order. The validity of the maximum principle is based on an important extremum principle for the Caputo fractional derivative. In its turn, the maximum principle was used to show uniqueness of solution to some initial-boundary-value problems for the time-fractional diffusion equations. This solution - if it exists - continuously depends on the problem data, i.e., on the source function and on the initial and boundary conditions.

As to the existence of solution, in the author's papers the Fourier method of variables separation was used to obtain a formal solution in form of a Fourier series with respect to the eigenfunctions of a certain Sturm-Liouville eigenvalue problem. Under certain conditions, the formal solution was shown to be a generalized solution in sense of Vladimirov, i.e., a continuous function that satisfies the partial differential equation of fractional order in generalized sense. To prove that this generalized solution is a solution in the classical sense, i.e., it is at least twice differentiable function with respect to the spatial variables and α-differentiable with respect to the time variable turned out to be a difficult undertaking that is still not resolved in general case.

In this paper, this problem is solved for the initial-boundary-value problem

$$u|_{t=0} = u_0(x),\ 0 \le x \le l,$$
$$u(0,t) = \phi_1(t),\ u(l,t) = \phi_2(t),\ 0 \le t \le T$$

for the one-dimensional time-fractional diffusion equation

$$(D_t^\alpha u)(t) = \frac{\partial}{\partial x}\left(p(x)\frac{\partial u}{\partial x}\right) - q(x)\,u + F(x,t)$$

over an open bounded domain $(0,l) \times (0,T)$.

Under certain conditions, the unique solution to this problem in the classical sense is represented via a Fourier series with respect to the eigenfunctions of a corresponding Sturm-Liouville eigenvalue problem. Properties of the solution including its asymptotics are investigated for some special cases of the source function F.

MSC 2010: 26A33; 33E12; 35B45; 35B50; 35K99; 45K05

Key Words and Phrases: time-fractional diffusion equation, extremum principle, Caputo fractional derivative, initial-boundary-value problems, maximum principle, Fourier series, asymptotics

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ON THE DISTINGUISHED ROLE OF THE MITTAG-LEFFLER AND WRIGHT FUNCTIONS IN FRACTIONAL CALCULUS

Francesco MAINARDI [1,§], **Rudolf GORENFLO** [2]

[1] Department of Physics, University of Bologna
Via Irnerio 46, Bologna, I – 40126, ITALY
e-mail: francesco.mainardi@unibo.it

[2] Department of Mathematics an Informatics
Free University of Berlin
Arnimallee 3, Berlin, D –14195, GERMANY
e-mail: gorenflo@mi.fu-berlin.de

Fractional calculus, in allowing integrals and derivatives of any positive real order (the term "fractional" is kept only for historical reasons), can be considered a branch of mathematical analysis which deals with integro-differential equations where the integrals are of convolution type and exhibit (weakly singular) kernels of power-law type. As a matter of fact fractional calculus can be considered a "laboratory" for special functions and integral transforms. Indeed many problems dealt with fractional calculus can be solved by using Laplace and Fourier transforms and lead to analytical solutions expressed in terms of functions of Mittag-Leffler and Wright type. We outline these problems in order to single out the role of these functions.

MSC 2010: 26A33, 33E12, 33C40, 44A10, 45K05, 60G52
Key Words and Phrases: integral transforms, special functions, fractional calculus, Mittag-Leffler functions, Wright functions, stable probability densities

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THE DEFORMED TRIGONOMETRIC FUNCTIONS OF TWO VARIABLES

Sladjana MARINKOVIĆ [1,§],
Miomir STANKOVIĆ [2], **Edin MULALIĆ** [3]

[1,3] University of Niš, Faculty of Electronic Engineering
Aleksandra Medvedeva 14, Niš - 18000, SERBIA
e-mail: sladjana.marinkovic@elfak.ni.ac.rs, edinmulalic@yahoo.com

[2] University of Niš, Faculty of Occupational Safety
Čarnojevića 10A, Niš - 18000, SERBIA
e-mail: miomir.stankovic@gmail.com

In the recent research papers, various generalizations and deformations of elementary functions were introduced. Since a lot of natural phenomena have both discrete and continual aspects, deformations which are able to express both them are of particular interest.

In this paper, we consider the trigonometry induced by one parameter deformation of exponential function of two variables $e_h(x,y) = (1+hx)^{y/h}$ ($h \in \mathbb{R} \setminus \{0\}$, $x \in \mathbb{C} \setminus \{-1/h\}$, $y \in \mathbb{R}$), [1]. In that manner, we define the functions

$$\cos_h(x,y) = \frac{e_h(ix,y) + e_h(-ix,y)}{2}, \quad \sin_h(x,y) = \frac{e_h(ix,y) - e_h(-ix,y)}{2i},$$

and analyze their various properties. We give series expansions of these functions, formulas which have their similar counterparts in regular trigonometry, and interesting difference and differential properties.

References:

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MSC 2010: 33B10, 33E20

Key Words and Phrases: exponential function, trigonometric functions, deformed exponential

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CONFLICT-CONTROLLED PROCESSES INVOLVING FRACTIONAL DIFFERENTIAL EQUATIONS WITH IMPULSES

Ivan MATYCHYN [1,§], **Arkadii CHIKRII** [1], **Viktoriia ONYSHCHENKO** [2]

[1] Department of Optimization of Controlled Processes
– Glushkov Institute of Cybernetics
40 Glushkov Prsp., Kiev – 03187, UKRAINE
e-mail: Matychyn@gmail.com , Chikrii@gmail.com

[2] State University of Information and Communication Technologies
7 Solomyanska st., Kiev – 03110, UKRAINE
e-mail: oviva@gmx.com

Here we investigate a problem of approaching terminal (target) set by a system of impulse differential equations of fractional order in the sense of Caputo. The system is under control of two players pursuing opposite goals. The first player tries to bring the trajectory of the system to the terminal set in the shortest time, whereas the second player tries to maximally put off the instant when the trajectory hits the set, or even avoid this meeting at all.

We derive analytical solution to the initial value problem for a fractional-order system involving impulse effects. As the main tool for investigation serves the Method of Resolving Functions based on the technique of inverse Minkowski functionals. By constructing and investigating special set-valued mappings and their selections, we obtain sufficient conditions for the game termination in a finite time. In so doing, we substantially apply the technique of $\mathcal{L} \times \mathcal{B}$-measurable set-valued mappings and their selections to ensure, as a result, superpositional measurability of the first player's controls.

MSC 2010: 34A08, 34A37, 49N70

Key Words and Phrases: fractional calculus, fractional differential equations with impulses, differential games

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SPECIAL CLASSES OF ORTHOGONAL POLYNOMIALS AND CORRESPONDING QUADRATURES OF GAUSSIAN TYPE

Gradimir V. MILOVANOVIĆ [1,§],
Aleksandar S. CVETKOVIĆ [2]

[1] Serbian Academy of Sciences and Arts
Knez Mihailova 35, 11000 - Belgrade, SERBIA
and
Faculty of Computer Sciences – Megatrend University Belgrade
Bulevar umetnosti 29, 11070 - Belgrade, SERBIA
e-mail: gvm@megatrend.edu.rs

[2] Faculty of Mechanical Engineering – University of Belgrade
11000 – Belgrade, SERBIA
e-mail: acvetkovic@mas.bg.ac.rs

In this lecture we present first a short account on some important properties of special classes of orthogonal polynomials, including a package of routines Orthpol, given by Walter Guatschi, for generating orthogonal polynomials and the corresponding quadratures of Gaussian type, as well as our The Mathematica Package "OrthogonalPolynomials" (Cvetković & Milovanović (2004)). Using symbolic calculations today we can implement several procedures and solve very easy many difficult problems from the past. We discuss polynomials orthogonal with respect to the measure $\exp(-|x|^p)$ supported on the real line $\mathbb{R}$ or on $\mathbb{R}_+$, for some specific values of the parameter $p > 0$. Also, we give results for some specific weight functions (e.g. hyperbolic weights, etc.), which were used earlier for summation of the slowly convergent series (Gautschi & Milovanović (1985), Gautschi (1991), Milovanović (1994, 1995), Dahlquist (1997, 1999), Milovanović & Cvetković (2003, 2004), Engblom (2006), Monien (2010)).

The second part of this lecture is dedicated to orthogonal polynomials for various oscillatory measures, which are important in numerical integration of highly oscillatory functions (cf. Milovanović & Cvetković, *J. Comp. Appl. Math.*, **179** (2005), 263–287; Milovanović, Cvetković, Stanić, *Appl. Math. Letters*, **22** (2009), 1189–1194). Also, we propose some stable numerical algorithms for constructing the corresponding quadrature formulae. In particular, for some special classes of linear operators we obtain interesting explicit results connected with the theory of orthogonal polynomials, as well as some nonstandard Gaussian quadrature formulae based on operator values (cf. Milovanović & Cvetković, *Adv. Comput. Math.*, **32** (2010), 431–486).

Finally, the theory of distributions, which is very modern, and associated with theory of orthogonal polynomials is discussed.

MSC 2010: 30E20; 33C47; 33C90; 40A25; 41A55; 65D30; 65D32

Key Words and Phrases: orthogonal polynomials, three-term recurrence relation, Gaussian quadrature formula, weight function, measure, slowly convergent series, software

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SOME FRACTIONAL ORDER ANALOGUES OF THE LAGUERRE, KUMMER AND GAUSS FUNCTIONS

Stefan MIREVSKI [1], Lyubomir BOYADJIEV [2]

[1] Hewlett-Packard Global Delivery Center Bulgaria, BULGARIA
e-mail: stefan.mirevski@gmail.com

[2] Faculty of Applied Mathematics and Informatics
Technical University of Sofia, BULGARIA
e-mail: boyadjievl@yahoo.com

We discuss two approaches for fractional order generalizations of some classical orthogonal polynomials and special functions. The paper presents some generalizations of the Laguerre and Jacobi polynomials.

As a first approach, the generalized Laguerre and Jacobi functions are derived by means of the Riemann–Liouville operator of fractional calculus and Rodrigues' type representation formula of fractional order.

As a second approach, the generalized Kummer and Gauss functions are obtained by a modified power series method as solutions of fractional extensions of the Kummer and Gauss differential equations. The generalized Laguerre function is introduced in a similar manner.

The Laguerre polynomials and functions are thus presented as special cases of the generalized Laguerre and Kummer functions. The relation between the Laguerre polynomials and the Kummer function is extended to their fractional counterparts.

MSC 2010: 26A33, 33C45

Key Words and Phrases: Riemann–Liouville fractional differentiation and integration operators, Jacobi polynomials, Rodrigues' type representation, Laguerre functions, Jacobi functions, Kummer function, Kummer differential equation, Gauss hypergeometric functions, Gauss hypergeometric differential equation

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APPLICATION OF FRACTIONAL OPERATORS IN INDUSTRIAL CONTROL SYSTEM (FRACTIONAL $\mathcal{ML} - \mathcal{DTC}$ CONTROL SYSTEMS – DESIGN AND ANALYSIS)

Emil Nikolov

Fac. of Automatization, Dept. "Industrial Automatization"
Technical University Sofia
8, Kliment Ohridski Bul., Sofia – 1000, BULGARIA
e-mail: nicoloff@tu-sofia.bg

One essentially new class of fractional $\mathcal{ML} - \mathcal{DTC}$ control systems is proposed in this work. It is configured through combinations of strategy repetitive control, dead-time compensation control and fractional control. Repetitive control is an effective strategy for periodic disturbances suppression by filtering their influence into the control system, assuming that the period of disturbances is known. The use of fractional dead-time compensators in the systems gives advantages in quality control of industrial plants with a variable delay. The control with operators of fractional order integration and differentiation enters the control systems in the class of robust control systems. In this work we give some methods, criteria and synthesis algorithms for fractional $\mathcal{ML} - \mathcal{DTC}$ control systems. Their application and analysis of the quality are examined.

MSC 2010: 93B51, 93D09, 26A33
Key Words and Phrases: fractional repetitive and dead-time compensation control – configuration, design, analysis and applications, robust stability and performance, robust margins

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ON THE BEHAVIOR OF ULTRASPHERICAL POLYNOMIALS IN THE COMPLEX PLANE

Geno NIKOLOV [1,§], **Alexander ALEXANDROV** [2]

[1,2] Department of Mathematics and Informatics
Sofia University "St. Kliment Ohridski"
5 James Bourchier Boulevard, Sofia - 1164, BULGARIA
e-mail: [1] geno@fmi.uni-sofia.bg

It is well-known that the squared modulus of every function f from the Laguerre–Pólya class $\mathcal{L}-\mathcal{P}$ obeys a MacLaurin series representation

$$|f(x+iy)|^2 = \sum_{k=0}^{\infty} L_k(f;x)y^{2k}, \quad (x,y) \in \mathbb{R}^2,$$

which reduces to a finite sum when f is a polynomial having only real zeros. The coefficients $\{L_k\}$ in this formula are representable as non-linear differential operators acting on f, and by a classical result of Jensen, $L_k(f;x) \geq 0$ for $f \in \mathcal{L}-\mathcal{P}$ and $x \in \mathbb{R}$. Here, we prove a conjecture formulated by the first-named author in 2005, which states that for $f = P_n^{(\lambda)}$, the n-th ultraspherical polynomial, the functions $\{L_k(f;x)\}_{k=1}^{n}$ are monotone decreasing on the negative semi-axis and monotone increasing on the positive semi-axis. We discuss the relation between this result and certain polynomial inequalities, which are close in spirit to the celebrated refinement of the classical Markov inequality, found by R. J. Duffin and A. C. Schaeffer in 1941.

MSC 2010: 33C45, 41A17
Key Words and Phrases: Laguerre–Pólya class, ultraspherical polynomials, Duffin–Schaeffer-type inequality

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α-MELLIN TRANSFORM AND EXAMPLE OF ITS APPLICATIONS

Yanka NIKOLOVA

Faculty of Applied Mathematic and Informatics
Technical University of Sofia
8, Kliment Ohridski St., Sofia – 1156, BULGARIA
e-mail: jvr@abv.bg

In this paper we consider a generalization of the classical Mellin transformation, called α-Mellin transformation, with an arbitrary (fractional) parameter $\alpha > 0$. Here we continue the presentation from the paper [1], where we have introduced the definition of the α-Mellin transform and some of its basic properties. Some examples of special cases are provided. Its operational properties as Theorem 1, Theorem 2 (Convolution theorem) and Theorem 3 (α-Mellin transform of fractional R-L derivatives) are presented, and the proofs can be found in [1].

Now we prove some further properties of this integral transform, that happen to be useful for its application to solving some fractional order differential equations. An example of such application is proposed for the fractional order Bessel differential equation of the form

$$t^{\beta+1}{}_0D_t^{\beta+1}\, y(t) + t^{\beta}{}_0D_t^{\beta}\, y(t) = f(t), \quad 0 < \beta < 1.$$

References: [1] Y. Nikolova, Definition of the α-Mellin transform and some of its properties, In: Proc. American Institute of Physics, 37th Intern. Conference "Applications of Mathematics in Engineering and Economics, AMEE-11", Sozopol, 8-13 June (2011), to appear.

MSC 2010: 35R11, 44A10, 44A20, 26A33, 33C45

Key Words and Phrases: Mellin transformation, integral transforms method, Riemann-Liouville fractional derivative, fractional Bessel differential equation

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FRACTIONAL VARIATIONAL CALCULUS OF VARIABLE ORDER

Tatiana ODZIJEWICZ [1,§],
Agnieszka B. MALINOWSKA [2], **Delfim F. M. TORRES** [3]

[1] Department of Mathematics - University of Aveiro
3810-193 Aveiro, PORTUGAL
e-mail: tatianao@ua.pt

[2] Faculty of Computer Science – Bialystok University of Technology
15-351 Bialystok, POLAND
e-mail: a.malinowska@pb.edu.pl

[3] Department of Mathematics - University of Aveiro
3810-193 Aveiro, PORTUGAL
e-mail: delfim@ua.pt

We show the boundedness of the variable order Riemann-Liouville fractional integral in the space of integrable functions and integration by parts formulas for variable order fractional operators. Moreover, we prove necessary optimality condition for fundamental problem of variable order calculus of variations. Fractional integrals are considered in the sense of Riemann-Liouville, while derivatives are of Caputo type.

MSC 2010: 26A33, 34A08, 49K05
Key Words and Phrases: fractional operators, fractional integration and differentiation of variable order, fractional variational analysis, Euler–Lagrange equations

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THREE MULTI-INDEX MITTAG-LEFFLER FUNCTIONS AND CONVERGENT SERIES IN THEM

Jordanka PANEVA-KONOVSKA

Faculty of Applied Mathematics and Informatics
Technical University of Sofia
8 Kliment Ohridski, Sofia - 1000, BULGARIA
e-mail: yorry77@mail.bg

In this paper we consider a new class of special functions. The so-called three multi-index Mittag-Leffler functions are $3m$-index generalizations of the classical Mittag-Leffler function $E_{\alpha,\beta}$ and of the Prabhakar function $E^{\gamma}_{\alpha,\beta}$. We study the main properties of these entire functions: we find their order and type, asymptotic formulae in different parts of the complex plane, represent them as Wright's generalized hypergeometric functions and Fox's H-functions. The latter yields Mellin-Barnes-type contour integral representation of these functions. Formulas for integer and fractional order integration and differentiation are provided.

Some interesting particular cases of the three multi-index Mittag-Leffler functions are illustrated and series in them are studied in the complex plane. More precisely, their domains of convergence are found and the behaviour of such expansions on the boundary of their domains is studied. In this way, we find analogues of the Cauchy-Hadamard, Abel, Tauber and Hardy-Littlewood theorems for the power series.

MSC 2010: 33E12, 30D15, 30B10, 30B30, 30B50, 30A10, 26A33

Key Words and Phrases: $3m$-parametric multi-index Mittag-Leffler functions, order and type of entire function, asymptotic formula, Mellin-Barnes-type integral representation, Riemann-Liouville fractional integral and derivative, convergent series, summation of divergent series

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CERTAIN CLASSES OF FUNCTIONS WITH NEGATIVE COEFFICIENTS

Donka PASHKOULEVA
Institute of Mathematics and Informatics – Bulg. Academy of Sci.
Acad. G. Bonchev Str., Block 8, Sofia – 1113, BULGARIA
e-mail: donka_zh_vasileva@abv.bg

Let S denote the class of functions of the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1}$$

which are analytic and univalent in the open unit disk $U = \{z : |z| < 1\}$.

For a function $f \in S$, we define ($n \in \mathbb{N} = \{1, 2, 3, \ldots\}$)

$$D^0 f(z) = f(z);\ D^1 f(z) = \frac{f(z) + zf'(z)}{2} = Df(z); D^n f(z) = D\left(D^{n-1} f(z)\right).$$

For $\beta \geq 0, -1 \leq \alpha \leq 1$ and $n \in \mathbb{N}_0$, denote the subclass of S consisting of functions $f(z)$ of the form (1) and satisfying the analytic condition

$$\Re\left\{\frac{z\left(D^n f(z)\right)'}{D^n f(z)}\right\} > \beta \left|\frac{z\left(D^n f(z)\right)'}{D^n f(z)} - 1\right|.$$

We denote by T the subclass of S consisting of functions of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k \quad a_k \geq 0. \tag{2}$$

Further we define the class $S_T(n, \alpha, \beta) = S(n, \alpha, \beta) \cap T$.

Theorem 1. *A necessary and sufficient condition for the function $f(z)$ of the form (2) to be in the class $S_T(n, \alpha, \beta)$ is that*

$$\sum_{k=2}^{\infty} [k(1+\beta) - (\alpha+\beta)] \left(\frac{k+1}{2}\right)^n a_k \leq 1 - \alpha,$$

where $-1 \leq \alpha \leq 1, \beta \geq 0$ and $n \in \mathbb{N}_0$.

Theorem 2. *Let the function $f(z)$ defined by (2) be in the class $S_T(n, \alpha, \beta)$. Then, for $z \in U$ and $0 \leq i \leq n$,*

$$|D^i f(z)| \geq |z| - \frac{1-\alpha}{2-\alpha+\beta}\left(\frac{2}{3}\right)^{n-1} |z|^2,\ |D^i f(z)| \leq |z| + \frac{1-\alpha}{2-\alpha+\beta}\left(\frac{2}{3}\right)^{n-1} |z|^2.$$

MSC 2010: 30C45
Key Words and Phrases: univalent, convex, starlike functions

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MATRIX APPROACH TO DISCRETE FRACTIONAL CALCULUS AND ITS IMPLEMENTATION IN MATLAB

Igor PODLUBNY [1,§], Ivo PETRAS [1], Tomas SKOVRANEK [1], Victor VERBITSKYI [2], Blas VINAGRE [3], YangQuan CHEN [4], Virginia KIRYAKOVA [5]

[1] BERG Faculty, Technical University of Kosice
B. Nemcovej 3, 04200 Kosice, SLOVAKIA
e-mail: {igor.podlubny, ivo.petras, tomas.skovranek}@tuke.sk

[2] Institute of Mathematics, Economics and Mechanics
Odessa National University
Dvoryanskaya 2, 65082 Odessa, UKRAINE
e-mail: vvverb@mail.ru

[3] School of Industrial Engineering, University of Extremadura
Campus Universitario, s/n. 06006 Badajoz, SPAIN
e-mail: bvinagre@unex.es

[4] Department of Electrical and Computer Engineering
Utah State University
4160 Old Main Hill, 84322 Logan, Utah, USA
e-mail: yangquan.chen@usu.edu

[5] Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
"Acad. G. Bonchev" Str., Block 8 Sofia – 1113, BULGARIA
email: virginia@diogenes.bg

The so-called "matrix approach" to discretization of operators of arbitrary real (integer and non-integer) order and to numerical solution of fractional differential and integral equations is presented, with the focus on its implementation in the form of a MATLAB toolbox. This method allows easy, uniform, and transparent solution of ordinary fractional differential

equations, partial fractional differential equations, and fractional integral equations. It also works for equations with a mixture of left-sided, right-sided, and symmetric fractional-order operators, and for equations with delayed fractional-order derivatives.

Recent results include extensions of the matrix approach to non-uniform grids, variable step lengths, variable-order derivatives, and distributed-order derivatives.

Examples of numerical solution of various standard types of differential equations with all aforementioned types of fractional derivatives are provided along with the corresponding MATLAB code.

MSC 2010: 26A33; 34A08; 65D25; 65D30
Key Words and Phrases: fractional derivatives, fractional differential equations, variable-order derivatives, distributed-order derivatives, discretization, numerical solution

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INTEGRAL FORMS OF NEUMANN–TYPE SERIES OF BESSEL FUNCTIONS

Tibor K. POGÁNY

Faculty of Maritime Studies – University of Rijeka
Studentska 2, 51000 Rijeka, CROATIA
e-mail: poganj@pfri.hr

The main aim of this talk is to expose various integral representation formulae for the Neumann series of Bessel functions

$$\mathfrak{N}_\nu(x) = \sum_{n\geq 1} \alpha_n B_{\nu+n}(x),$$

where B stands for the Bessel functions of the first and second kind J, Y, and for the modified Bessel functions of the first and second kind I, K over the set of real numbers, for real parameter ν, see [BJP; PS].

Integral expression is given also for the so–called Neumann series of the second kind:

$$\mathfrak{G}^{a,b}_{\mu,\nu}(x) := \sum_{n\geq 1} \theta_n J_{\mu+an}(x) J_{\nu+bn}(x), \qquad \mu, \nu, a, b \in \mathbb{R}\,.$$

This type of Neumann series appear in certain results by von Lommel, Al–Salam and Thiruvenkatachar and Nanjundiah, and take place in [BP].

MSC 2010: Primary: 40H05, 40A30, Secondary: 33C10, 33C15

Key Words and Phrases: Bessel and modified Bessel functions of the first and second kind, Neumann series of Bessel and modified Bessel functions of first and second kind

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GROWTH THEOREM AND THE RADIUS OF STARLIKENESS OF CLOSE-TO-SPIRALLIKE FUNCTIONS

Yaşar POLATOĞLU [1],
Melike AYDOĞAN [2,§] ,
Arzu YEMIŞCI [3]

[1,3] Department of Mathematics and Computer Sciences
İstanbul Kültür University
E 5 Road, Bakirkoy - 34156, TURKEY
e-mails: [1] y.polatoglu@iku.edu.tr , [3] a.sen@iku.edu.tr

[2] Department of Computer Engineering, Yeni Yuzyil University
Yilanli Ayazma Road, Number: 26, Topkapi - 34030, TURKEY
e-mail: melike.aydogan@yeniyuzyil.edu.tr

Let A be the class of all analytic functions in the open unit disc $D = \{z| \quad |z| < 1\}$ of the form $f(z) = z + a_2z^2 + a_3z^3 + \cdots$. Let $g(z)$ be an element of A and satisfy the condition $Re(e^{i\alpha}\frac{g'(z)}{g(z)}) > 0$ for some α, $|\alpha| < \frac{\pi}{2}$. Then $g(z)$ is said to be α-spirallike. Such functions are known to be univalent in $|z| < 1$. It was shown by L. Spacek, that the α-spirallike functions are univalent in $|z| < 1$.

Let S^*_α denote the class of all functions $g(z)$ satisfying the above condition for a given α. A function $f(z) \in A$ is called close-to-α spirallike, if there exists a function $g(z)$ in S^*_α such that $Re(\frac{f(z)}{g(z)}) > 0$. The class of such functions is denoted by $S^*_\alpha K$.

The aim of this paper is to give a growth theorem and the radius of starlikeness of the class $S^*_\alpha K$.

MSC 2010: 30C45

Key Words and Phrases: close-to-α spirallike functions, growth theorem, radius of starlikeness

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HANKEL DETERMINANTS OF THE NUMBER SEQUENCES IN AN INTEGRAL FORM

Predrag RAJKOVIĆ [1,§], Paul BARRY [2] and Nataša SAVIĆ [3]

[1] Faculty of Mechanical Engineering, University of Niš
A. Medvedeva 14, 18 000 Niš, SERBIA
e-mail: pedja.rajk@yahoo.com

[2] School of Science, Waterford Institute of Technology
Cork Road, Waterford, IRELAND
e-mail: pbarry@wit.ie

[3] High Technical School, A. Medvedeva 20, Niš, SERBIA
e-mail: natasa.savic.4@gmail.com

The *Hankel transform* of a number sequence $G = \{g_n\}$ is the sequence of the Hankel determinants $H = \{h_n\}$ given by $h_n = |g_{i+j-2}|_{i,j=1}^{n}$.

We consider the special number sequences in the following integral form:

$$g_n^{(p)} = \frac{1}{2\pi}\int_a^b x^{n-\delta_{1,p}}(b-x)^{\nu-1}(x-a)^{\mu-1}(b_1-x)^{\nu_1-1}(x-a_1)^{\mu_1-1}\,dx,$$

where $a_1 < b_1 < 0 < a < b;\ p \in \{0,1\};\ \mu,\nu,\mu_1,\nu_1 > 0;\ n \in \mathbb{N}$.

We are interested in the special cases of $\{g_n\}$ which satisfy the *generalized convolution property of an order* r, i.e. it is valid

$$g_n = \sum_{k=1}^{r}\alpha_k g_{n-k} + \beta\sum_{k=0}^{n-r} g_k g_{n-r-k},$$

and the Hankel determinants have the *generalized Somos-4 property*, i.e. it exists a pair (r,s) such that $g_n g_{n-4} = r\ g_{n-1}g_{n-3} + s\ g_{n-2}^2\ \ (n = 4,5,\ldots)$.

MSC 2010: 11B83, 05A19, 33C45
Key Words and Phrases: special numbers sequence, determinants, polynomials, recurrence relations

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CLASSICAL SUMMATION THEOREMS FOR THE SERIES ${}_2F_1$ AND ${}_3F_2$ AND THEIR APPLICATIONS

Medhat A. RAKHA

Department of Mathematics and Statistics
College of Science - Sultan Qaboos University
Muscat - Sultanate of OMAN
e-mail: medhat@squ.edu.om

The generalized hypergeometric functions with p numerator and q denominator parameters are defined by ([1], [2], [3])

$$
{}_pF_q\left[\begin{matrix}\alpha_1, & \ldots, & \alpha_p & & \\ & & & ; & z\\ \beta_1, & \ldots, & \beta_q & & \end{matrix}\right] = {}_pF_q\left[\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z\right] = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n\ldots(\alpha_p)_n}{(\beta_1)_n\ldots(\beta_q)_n}\frac{z^n}{n!}\,,
$$

where $(\alpha)_n$ denotes the Pochhammer symbol (or the shifted factorial, since $(1)_n = n!$):

$$
(\alpha)_n = \begin{cases} \alpha(\alpha+1)\ldots(\alpha+n-1),\ n\in\mathbb{N}\\ 1,\ n=0. \end{cases}
$$

Using the fundamental property $\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$, $(\alpha)_n$ can be written in the form

$$
(\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}\,,
$$

where Γ is the well known gamma function.

It is not out of place to mention here that whenever a hypergeometric function reduce to gamma functions, the results are very important from the applications point of view. Only a few summation theorems for the series ${}_2F_1$ and ${}_3F_2$ are available in the literature.

The classical summation theorems such as of Gauss, Gauss's second, Kummer and Bailey for the series ${}_2F_1$ and Watson, Dixon and Whipple for the series ${}_3F_2$ play an important role in the theory of hypergeometric and generalized hypergeometric series.

Bailey, in his well known and very interesting paper [4] applied the above mentioned classical summation theorems and obtained a large number of known and unknown results involving products of generalized hypergeometric series.

Also, Berndt [5] has pointed out that the interesting summations due to Ramanujan can be obtained quite simply by employing the above mentioned classical summation theorems.

The aim of this research paper is to obtain the explicit expressions of such summation theorems in the most general case. Our results obtained in this paper are quite simpler and different from those already available in the literature and obtained by a very different manner.

As an application of our main results, we mention certain generalizations of the summations due to the genius Indian mathematician Ramanujan.

AMS Subject Classification: 33C05, 33D15
Keywords: Gauss hypergeometric function, ${}_3F_2$ hypergeometric function; hypergeometric summation theorems

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CLASSICAL HERMITE AND LAGUERRE POLYNOMIALS AND THE ZERO-DISTRIBUTION OF RIEMANN'S ζ-FUNCTION

Peter RUSEV

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bontchev Str., Block 8, 1113 – Sofia, BULGARIA
e-mail: pkrusev@math.bas.bg

Necessary and sufficient conditions for absence of zeros of the function $\zeta(s), s = \sigma + it$, in the half-plane $\sigma > \theta, 1/2 \leq \theta < 1$, are proposed in terms of representations of holomorphic functions by series in the Hermite and Laguerre polynomials as well as in terms of the Fourier and Hankel integral transforms.

As it is well-known, the function $\zeta(s)$ has no zeros in the half-plane $\sigma \geq 1$. Hence, there exists a region containing this half-plane and such that the function

$$\Phi(s) = -\frac{\zeta'(s)}{\zeta(s)} - \frac{1}{s-1}$$

is holomorphic there. Moreover, it can be proved that this function is bounded on the closed half-plane $\sigma \geq 1$. Therefore, the function $\Phi(1 + iz)$ is holomorphic and bounded on the closed half-plane $\Im z \leq 0$. Hence, there exist the Fourier-Hermite coefficients of the function $\Phi(1+ix), -\infty<x<\infty$, namely

$$a_n(\Phi) = \int_{-\infty}^{\infty} \exp(-x^2)H_n(x)\Phi(1 + ix)\, dx \quad n = 0, 1, 2, \ldots.$$

Furthermore, let $\tau_0(\Phi) = -\limsup_{n\to\infty}(2n+1)^{-1/2}\log(2n/e)^{-n/2}|a_n(\Phi)|$, then:

The function $\zeta(s)$ has no zeros in the half-plane $\sigma > \theta, 1/2 \leq \theta < 1$ if and only if $\tau_0(\Phi) \geq 1 - \theta$.

Riemann's hypothesis is true if and only if $\tau_0(\Phi) = 1/2$.

Let $\mathcal{G}(\gamma), -\infty < \gamma \leq \infty$ be the class of entire functions G such that $\limsup_{|w|\to\infty}(2\sqrt{|w|})^{-1}(\log|G(w)| - |w|) \leq -\gamma$. Then:

The function $\zeta(s)$ *has no zeros in the half-plane* $\sigma \geq \theta, 1/2 \leq \theta > 1$*, if and only if the Fourier transform of the function* $\exp(-x^2/4)\Phi(1+ix), -\infty < x < \infty$ *is of the form* $\exp(-u^2)E(u)$ *with a function* $E \in \mathcal{G}(1-\theta)$.

Riemann's hypothesis is true if and only if the Fourier transform of the function $\exp(-x^2/4)\Phi(1+ix)$ *is of the form* $\exp(-u^2)E(u)$ *with a function* $E \in \mathcal{G}(1/2)$.

Analogous criteria, involving the function $\Phi(1+ix) + \Phi(1-ix)$, are given in the "language" of representations by means of series in Laguerre polynomials and by the Hankel integral transform.

MSC 2010: 11M06, 33C45, 42A38, 44A15

Key Words and Phrases: zero-distribution, Hermite and Laguerre polynomials, Riemann's ζ-function, Fourier transform, Hankel transform

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POLAR DERIVATIVES AND APOLARITY

Blagovest SENDOV

Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Block 25 A, 1113 Sofia – BULGARIA
e-mail: sendov2003@yahoo.com

Let $p(z) \in \mathcal{P}_n$ and $\zeta \in \mathcal{C}$ be a complex number. Then the linear operator $\mathcal{D}_\zeta(p; z) = np(z) - (z - \zeta)p'(z)$ is called the **polar derivative** of $p(z)$ with pole ζ and

$$\lim_{\zeta\to\infty} \left(\frac{1}{\zeta} \mathcal{D}_\zeta(p; z) \right) = p'(z).$$

The fundamental theorem for polar derivatives is:

Theorem 1 (Laguerre) *Let $p(z)$ be a complex polynomial of degree $n \geq 2$ and let $\zeta \in \mathcal{C}$. A circular domain containing all the zeros of $p(z)$, but not the point ζ, contains all the zeros of the polar derivative $\mathcal{D}_\zeta(p; z)$.*

The polar derivative of order $l > 1$ is defined by recursion

$$\mathcal{D}_{\zeta_1,\zeta_2,\ldots,\zeta_{l+1}}(p; z) = \mathcal{D}_{\zeta_{l+1}}\Big(\mathcal{D}_{\zeta_1,\zeta_2,\ldots,\zeta_l}(p; z) \Big).$$

Definition 1 *For every polynomial $p(z) \in \mathcal{P}_n$, define*

$$D(p) = D(c(p); r(p)) = \{z : \ |z - c(p)| \leq r(p)\}$$

to be the smallest closed disk containing all zeros of $p(z)$.

A closed point set U is **minimal** *with the property W if every closed point set $V \subset U$ with property W coincide with U.*

For every $p(z) \in \mathcal{P}_n$, call the closed, simply connected point set $\Psi \subset D(p)$, **polar locus holder** *of $p(z)$, if for every $\zeta_1, \zeta_2, \ldots, \zeta_{n-1} \notin \Psi \setminus \partial\Psi$, the only zero of $\mathcal{D}_{\zeta_1,\zeta_2,\ldots,\zeta_{n-1}}(p; z)$ is on Ψ. The minimal polar locus holder $\Psi(p)$ is called* **polar locus** *of $p(z)$.*

Definition 2 *Two monic polynomials* $p(z) = z^n + a_1 z^{n-1} + \cdots + a_n$ *and* $q(z) = z^n + b_1 z^{n-1} + \cdots + b_n$ *are* **apolar** *if*

$$\sum_{k=0}^{n} (-1)^k p^{(k)}(0) q^{(n-k)}(0) = n! \sum_{k=0}^{n} (-1)^k \frac{a_{n-k} b_k}{\binom{n}{k}} = 0.$$

The relation between the zeros of two apolar polynomials is given by the classical Theorem of Grace:

Theorem 2 (Grace) *Let* $p(z)$ *and* $q(z)$ *be apolar polynomials. Then every circular domain containing all the zeros of one of them contains at least one zero of the other.*

Definition 3 *Let* $p(z) \in \mathcal{P}_n$ *and* $q(z) = (z - z_1)(z - z_2) \cdots (z - z_n)$ *be an arbitrary polynomial, apolar to* $p(z)$*. Call a closed and simply connected point set* $\Omega \subset D(p)$ **apolar locus holder** *of* $p(z)$*, if* Ω *contains at least one of the points* $z_1, z_2, \ldots, z_n$*.*

The minimal apolar locus holder $\Omega(p)$ *is called* **apolar locus** *of* $p(z)$*.*

If $q(z) = (z - z_1)(z - z_2) \cdots (z - z_n)$ is apolar to $p(z)$, then at least one of the numbers $z_1, z_2, \ldots, z_n$ is on the disk $D(p)$. According to Theorem 2, if a closed disk contains all the zeros of $p(z)$, then this disk is an apolar locus holder of $p(z)$. Then $D(p)$ is the smallest disk which is an apolar locus holder of $p(z)$.

The main result in this paper is

Theorem 3 *For every polynomial* $p(z) \in \mathcal{P}_n$*, the equality* $\Psi(p) = \Omega(p)$ *holds.*

Hence, we have a sharp version of Laguerre's Theorem 1:

Theorem 4 *Let* $p(z) \in \mathcal{P}_n$ *and* $\zeta \notin \Omega(p) \setminus \partial\Omega(p)$*. Then all zeros of the polar derivative* $\mathcal{D}_\zeta(p; z)$ *are on* $\Omega(p)$*. The point set* $\Omega(p)$ *is minimal with this property.*

Theorem 4 make sense if we may find $\Omega(p)$ for a given $p(z) \in \mathcal{P}_n$. In a previous paper, we found some properties of an apolar locus, which may be used to determine $\Omega(p)$.

MSC 2010: 30C10

Key Words and Phrases: polar derivative, apolarity, polar locus, apolar locus

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$SO(2,1)$-INVARIANT DOUBLE INTEGRAL TRANSFORMS AND FORMULAS FOR WHITTAKER AND OTHER FUNCTIONS

Ilya SHILIN

Department of Mathematical Simulation – Moscow Aviation Institute
Volokolamskoe shosse 4, Moscow - 125993, RUSSIA

Department of High Mathematics – M. Sholokhov Moscow State University for the Humanities
Verhnya Radishevskaya 16 – 18, Moscow - 109240, RUSSIA

e-mail: ilyashilin@li.ru

Assume that the space $\mathbb{R}^3$ is endowed with the quadratic form $q(x) := x_0^2 - x_1^2 - x_2^2$. Let σ be an arbitrary complex number, $\mathfrak{D}_\sigma$ be the linear space consisting of all functions f, defined on the cone $\mathsf{C} : q(x) = 0$ and satisfying the following conditions: first, f is infinitely differentiable and, second, f is a σ-homogeneous function, i.e., for any $\alpha \in \mathbb{C}$, the equality $f(\alpha x) = \alpha^\sigma f(x)$ holds. The representation of $SO(2,1)$ in $\mathfrak{D}_\sigma$ is defined by formula $T_\sigma(g)[f(x)] := f(g^{-1}x)$.

Let us denote by $\gamma_1, \gamma_2, \gamma_3$ the circle $x_0 = 1$, parabola $x_0 + x_1 = 1$, and hyperbola $x_1 = \pm 1$ on C, respectively. Let H_i mean the subgroup, which acts transitively on γ_i. Let $\mathrm{d}x$ be an H_i-invariant measure on γ_i. For each $i \in \{1, 2, 3\}$, we consider the bilinear functional

$$\mathsf{D}_i : \mathfrak{D}_\sigma^2 \longrightarrow \mathbb{C},\ (u, v) \longmapsto \int\limits_{\gamma_i} \int\limits_{\gamma_i} k(x, \hat{x})\, u(x)\, v(\hat{x})\, \mathrm{d}x\, \mathrm{d}\hat{x}.$$

We obtain such restrictions of the kernel k to $\gamma_i \times \gamma_i$ that these functionals are invariant with respect to T_σ. Such kernel follows to $\mathsf{D}_1 = \mathsf{D}_2 = \mathsf{D}_3$. Using these properties of D_i, we derive some formulas involving the Whittaker and other functions.

MSC 2010: 33C15, 33C05, 33C45, 65R10, 20C40

Key Words and Phrases: group $SO(2,1)$, double integral transform, Whittaker functions

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CURVES PROVIDED BY RATIO DIVISION OF A FEW SPECIAL FUNCTIONS

Miomir STANKOVIĆ [1],
Predrag RAJKOVIĆ [2] **and Predrag STANKOVIĆ** [1,§]

[1] Faculty of Occupational Safety, University of Niš
Čarnojevićeva 10a, 18 000 Niš, SERBIA
e-mail: miomir.stankovic@gmail.com

[2] Faculty of Mechanical Engineering, University of Niš
A. Medvedeva 14, 18 000 Niš, SERBIA
e-mail: pedja.rajk@yahoo.com, predrag.stankovic@znrfak.ni.ac.rs

Let $\Psi(\mathbf{x})$ be a function defined by an integral of the form

$$\Psi(\mathbf{x}) = \int_a^b K(\mathbf{x},t)\ dt \qquad (a < b,\ \mathbf{x} \in \mathbb{R}^n), \tag{1}$$

where $K(\mathbf{x},t)$ is a given kernel function.

The function $d = d(\mathbf{x};r)$ $(0 < r < 1)$ defined implicitly as

$$\int_a^d K(\mathbf{x},t)\ dt = r \int_a^b K(\mathbf{x},t)\ dt \quad (r = \text{const}) \tag{2}$$

is called r*–ordered division function* of $\Psi(\mathbf{x})$.

Especially, in the one-dimensional case for $r = 1/2$, C. Berg denoted it by $m(x)$ and named it a *median* function.

Its evaluation at any point is based on the quadratures because the function is defined implicitly. We examine the properties of such functions: monotonicity, convexity, asymptotic behavior and others.

MSC 2010: 65D18, 33B15, 33E20

Key Words and Phrases: median curve, surface, gamma function, beta function, quadrature

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NONLINEAR TIME FRACTIONAL EVOLUTION EQUATIONS WITH SINGULARITIES

Mirjana STOJANOVIC

Department of Mathematics and Informatics
University of Novi Sad
Trg D. Obradovica 4, 21 000 Novi Sad, SERBIA
e-mail: stojanovic@dmi.uns.ac.rs

By means of a fractional analogue of the Duhamel principle, we give some existence-uniqueness results for linear and nonlinear time fractional evolution equations in corresponding norm in the extended Colombeau algebra of generalized functions.

Many of these equations can be solved in classical spaces useful for applications, if the initial data and the coefficients are smooth enough. In order to solve such equations with singularities, we employ the extended Colombeau algebra of generalized functions.

Since the classical Colombeau algebra does not allow fractional derivatives involved in such kind of equations, we are dealing with an extension of this algebra to fractional derivatives, i.e. to derivatives D^α of arbitrary order $\alpha \in \bar{\mathbf{R}}_+$.

MSC 2010: 46F30, 26A33, 34G20, 35D05

Key Words and Phrases: linear and nonlinear time fractional evolution equations, existence-uniqueness result, Duhamel principle, extended Colombeau algebra

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ON THE APPROXIMATE SOLUTIONS OF A FRACTIONAL INTEGRO-DIFFERENTIAL EQUATION

Arpad TAKAČI [1,§], **Djurdjica TAKAČI** [2]

[1], [2] Department of Mathematics and Informatics
Faculty of Sciences, University of Novi Sad
Trg D. Obradovića 4, 21000 – Novi Sad, SERBIA
e-mail: takaci@dmi.uns.ac.rs

We consider the following time-fractional partial integro-differential equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} + a\int_0^t k(t-\tau)\frac{\partial^2 u(x,\tau)}{\partial \tau^2}d\tau$$

$$= b\int_0^t k(t-\tau)\frac{\partial^3 u(x,\tau)}{\partial x^2 \partial \tau}d\tau + \frac{\partial^2 u(x,t)}{\partial x^2}, \quad t > 0, \;\; x \in (0,1),$$

where a and b are numerical constants, and

$$u(0,x) = u_0(x), \;\; u_t(0,x) = u_1(x), \quad u(t,0) = u(t,1) = 0, \quad t > 0,$$

$$k(t) = \frac{t^{-\alpha}}{\Gamma(1-\alpha)}, \quad 0 < \alpha < 1, \quad 0 < t < T.$$

We determine the exact and the approximate solution in the sense of Mikusinski operators. We analyze the nature of the obtained operator solution and give sufficient conditions for the solution of the considered problem.

MSC 2010: 26A33, 44A45, 44A40, 65J10
Key Words and Phrases: fractional calculus, operational calculus, diffusion equation, Mikusiński operators

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ON THE VISUALIZATION OF GENERALIZED FUNCTIONS

Djurdjica TAKAČI

Department of Mathematics and Informatics
Faculty of Sciences, University of Novi Sad
Trg D. Obradovića 4, 21000 – Novi Sad, SERBIA
e-mail: djtak@dmi.uns.ac.rs

We introduce new technology in analyzing the generalized functions. We use the dynamic properties of the free software *GeoGebra* to visualize the convolution, the fractional integral, fractional derivative and few delta sequences and to show some of their most important properties.

We consider the approximate solutions of differential equations and their approaches to the exact solutions.

MSC 2010: 26A33, 44A45, 44A40, 65J10
Key Words and Phrases: GeoGebra, visualization, fractional calculus, operational calculus

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PROBABILITY DISTRIBUTIONS ASSOCIATED WITH MATHIEU TYPE SERIES

Živorad TOMOVSKI, Ram K. SAXENA, Tibor POGÁNY

Institute of Mathematics – St. Cyril and Methodius University
91000 Skopje, Republic of MACEDONIA
e-mail: tomovski@pmf.ukim.mk

Department of Mathematics and Statistics – Jain Narain Vyas University
Jodhpur–342004, INDIA
email: ram.saxena@yahoo.com

Faculty of Maritime Studies – University of Rijeka
Studentska 2, 51000 Rijeka, Republic of CROATIA
e-mail: poganj@pfri.hr

The main object of this talk is to present a systematic study of probability density functions and distributions associated with Mathieu series and their generalizations. Characteristic functions and fractional moments related to the probability density functions of the considered distributions are derived by virtue of Mathieu type series and Hurwitz–Lerch Zeta function. Special attention will be given to the so–called Planck(p) distribution.

MSC 2010: Primary 33E20, 44A10; Secondary 33C10, 33C20, 44A20

Key Words and Phrases: Mathieu and Mathieu type series, Fourier sine and cosine transforms, Fox H-function, integral representations, probability density functions, characteristic function, fractional moments, Planck distribution

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TAUBERIAN CLASS ESTIMATES FOR WAVELET AND NON-WAVELET TRANSFORMS OF VECTOR-VALUED DISTRIBUTIONS

Jasson VINDAS

Department of Mathematics – Ghent University
Krijgslaan 281 Gebouw S22, Gent - B 9000, BELGIUM
e-mail: jvindas@cage.Ugent.be

We present several characterizations of the spaces of Banach space-valued tempered distributions in terms of integral transforms of the form $M_{\varphi}^{\mathbf{f}}(x,y) = (\mathbf{f} * \varphi_y)(x)$, where the kernel φ is a test function and $\varphi_y(\cdot) = y^{-n}\varphi(\cdot/y)$. If the zeroth moment of φ vanishes, it is a wavelet type transform; otherwise, we say it is a non-wavelet type transform.

We shall consider the following problem. Suppose that the vector-valued tempered distribution $\mathbf{f}$ a priori takes values in a "broad" locally convex space which contains as a continuously embedded subspace the narrower Banach space E, and $M_{\varphi}^{\mathbf{f}}(x,y) \in E$, for almost every value of (x,y). If it is a priori known that $\mathbf{f}$ takes values in E, then one can verify that it satisfies an estimate of the form

$$\left\| M_{\varphi}^{\mathbf{f}}(x,y) \right\|_E \leq C \frac{(1+y)^k \, (1+|x|)^l}{y^k}. \qquad (1)$$

We call (1) a (Tauberian) *class estimate*. The problem of interest is the converse one: Up to what extend does the class estimate (1) allow one to conclude that $\mathbf{f}$ actually takes values in E?

Our results establish that if (1) holds, where φ satisfies a certain non-degenerateness condition, then $\mathbf{f}$ takes values in E, up to some correction term that is totally controlled by φ.

The talk reports on joint work with Stevan PILIPOVIĆ.

MSC 2010: 42C40, 46F05, 46F10, 46F12

Key Words and Phrases: regularizing transforms, wavelet transform, $\phi-$transform, Tauberian class estimates, vector-valued distributions, non-degenerate wavelets

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ON SOME GENERALIZATIONS OF CLASSICAL INTEGRAL TRANSFORMS

Nina VIRCHENKO

National Technical University of Ukraine
Kyiv, UKRAINE e-mail: nvirchenko@hotmail.com

It is known that the method of integral transforms is an effective contemporary analytical method for solving of a large class of problems in applied mathematics, engineering, etc., see for example [1], [2].

In this talk we discuss some generalizations of known classical integral transforms. For example, we consider:

- The generalized Laplace integral transforms:

$$L_m\{f(x);y\} = \int_0^\infty x^{m-1} e^{-x^m y^m}\ f(x)dx; \tag{1}$$

$$\tilde{L}_m\{f(x);y\} = \int_0^\infty x^{m-1} e^{-x^m y^m}\ {}_1\Phi_1^{\tau,\beta}(a;c;-b(x^m y^m)^y) f(x)dx; \tag{2}$$

- The generalized integral transforms in the potential theory:

$$P_{\nu,m}\{f(x);y\} = \int_0^\infty \frac{x^{m-1} f(x)}{(x^m+y^m)^\gamma} dx; \tag{3}$$

$$P^{\nu}_{m,1}\{f(x);y\} \tag{4}$$

$$= \frac{\Gamma(c)}{\Gamma(a)\Gamma(\nu)} \int_0^\infty \frac{x^{m-1} f(x)}{(x^m+y^m)^\gamma}\ {}_2\Psi_1\left[\begin{array}{l} (a;\tau);(\nu,\gamma) \\ (c;\beta) \end{array} \middle| -b\left(\frac{x^m}{x^m+y^m}\right)^\gamma \right] dx;$$

- The generalization of the Glasser integral transform:

$$G_{m,1}\{f(x);y\} \tag{5}$$

$$= \frac{\Gamma(c)}{\Gamma(a)} \int_0^\infty \frac{f(x)}{(x^m+y^m)^{\frac{1}{m}}} \, {}_2\Psi_1 \left[\begin{matrix} (a;\tau); \left(\frac{1}{m},\gamma\right) \\ (c;\beta) \end{matrix} \middle| -b\left(\frac{x^m}{x^m+y^m}\right)^\gamma \right] dx;$$

etc. In the above, ${}_1\Phi_1^{\tau,\beta}(a;c;z)$ stands for the generalized confluent hypergeometric function from [3] and ${}_p\Psi_q$ is the generalized Wright function [1].

Some basic properties and some inversion formulae for these integral transforms are established.

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MSC 2010: 44A05, 44A20

Key Words and Phrases: generalized integral transforms

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INVERSE PROBLEM FOR FRACTIONAL DIFFUSION EQUATION

VU Kim Tuan

Department of Mathematics – University of West Georgia
1601 Maple Street, Carrollton - GA 30118, USA
e-mail: vu@westga.edu

Consider the one-dimensional fractional diffusion equation defined by

$$\begin{cases} {}_0^C\mathcal{D}_t^\alpha u\,(x,t) = u_{xx}(x,t) - q(x)u(x,t), \qquad 0 < x < \pi,\ t > 0, \\ u_x(0,t) - hu(0,t) = 0, \\ u_x(\pi,t) + Hu(\pi,t) = 0, \\ u(x,0) = f(x), \end{cases} \tag{1}$$

where $q \in L_1(0,\pi)$, $f \in L_2\,(0,\pi)$, and ${}_0^C\mathcal{D}_t^\alpha u(t)$, $0 < \alpha < 1$, is the Caputo fractional derivative

$${}_0^C\mathcal{D}_t^\alpha u(t) = \int_0^t \frac{(t-x)^{-\alpha}}{\Gamma(1-\alpha)}\, u'(x)\, dx.$$

To express the dependence of u on the initial distribution f, we sometimes use the notation $u = u^f$. The constants h and H in the boundary conditions correspond to the insulation parameters at both ends and are supposed to be known. In a series of papers, Professor R. Gorenflo and his co-authors have studied applications of fractional diffusion equations in probability, random walk, and finance. In this talk we are concerned with the inverse problem of (1), namely, the problem of recovery of the diffusion coefficient $q(x)$ from the measurements of lateral distribution $u^f(0,t)$ if $h \neq \infty$ (or diffusive flux $u_x^f(0,t)$ in case $h = \infty$) and lateral distribution $u^f(\pi,t)$ if $H \neq \infty$ (or diffusive flux $u_x^f(\pi,t)$ in case $H = \infty$), on a time interval (T_0,T_1), when initial distributions f are given.

For the heat equation (equation (1) with $u_t(x,t)$ instead of ${}_0^C\mathcal{D}_t^\alpha u\,(x,t)$) usually the heat coefficient q is recovered from readings of the heat flux $u_x(0,t)$ given the temperature $u(0,t)$ at the boundary $x=0$ under zero initial temperature condition $u(x,0)=0$. In other words, q is determined from infinitely many measurements obtained by the full lateral Dirichlet-to-Neumann map, $u(0,t)\to u_x\,(0,t)$ provided $f(x)=0$. The recovery problem in this setting is over-determined, in the sense that more data is collected than needed.

In this talk, instead of giving the full lateral Dirichlet-to-Neumann map, that requires infinitely many measurements, we are given a partial initial-to-boundary map. More precisely, we choose some initial distribution $u(x,0)=f(x)$ and then measure the lateral diffusions $u^f(0,t)$ and $u^f(\pi,t)$ if h and H are finite and diffusive fluxes $u_x^f(0,t)$ and $u_x^f(\pi,t)$ otherwise. The main emphasis in our approach is to provide initial distributions that require finitely many measurements only. The data processing involves extracting boundary spectral data from standard readings of $u(0,t)$ or $u_x(0,t)$ at $x=0$ and $u(\pi,t)$ or $u_x(\pi,t)$ at $x=\pi$. Once all boundary spectral data are recovered, we will show how spectral data and diffusion coefficient q can be uniquely reconstructed. In fact, we show that for any $q\in L_1(0,\pi)$, with a known lower bound, a suitable choice of two initial conditions only $u(x,0)=f_i(x),\ i=1,2$, a power and a step functions, is enough to recover the diffusion coefficient q,

$$f_i(x)\to\{u^{f_i}(0,t),\ u^{f_i}(\pi,t)\},\ t\in(T_0,T_1),\ i=1,2$$

(with obvious modifications in case $h=\infty$, or $H=\infty$, or both). If instead of a lower bound for q only an upper bound for $\|q\|_1$ is known, then we would know explicitly the maximum number of required measurements. The third case would be when no information about q is given, then the blind search will still recover a unique q after a finite number of measurements. However, this number of measurements is known only during the extracting data process.

MSC 2010: 26A33, 33E12, 34K29, 34L15, 35K57, 35R30

Key Words and Phrases: fractional diffusion equation, Mittag-Leffler function, inverse problem, boundary spectral data, eigenfunction expansion

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LIST OF PARTICIPANTS in "TMSF'2011"

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FOREIGN: 35

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Melike AYDOGAN - Turkey
Lubomir BOYADJIEV - Kuwait
Bohdan DATSKO - Ukraine
Rudolf GORENFLO - Germany
Alfaisal HASAN - Egypt
Shyam L KALLA - India
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Valmir KRASNIQI - Kosovo
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Tibor POGÁNY - Croatia
Predrag RAJKOVIC - Serbia
Medhat A. RAKHA - Oman
Natasa SAVIC - Serbia
Ilya SHILIN - Russia
Predrag STANKOVIC - Serbia
Arpad TAKACI - Serbia
Djurdjica TAKACI - Serbia
Jasson VINDAS - Belgium
VU Kim Tuan - USA
Arzu YEMISCI - Turkey
Nina VIRCHENKO - Ukraine
Allaoua GUERZIZ - Algeria
Abdelhamid LAOUAR - Algeria

BULGARIAN: 27

Lilia APOSTOLOVA
Emilia BAZHLEKOVA
Georgi BOYCHEV
Nikolaj BOZHINOV
Georgi CHOBANOV
Johann DAVIDOV
Georgi DIMKOV
Dessislava DIMKOVA
Ivan DIMOVSKI
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Jordanka PANEVA-KONOVSKA
Donka PASHKOULEVA
Petar RUSEV
Blagovest SENDOV
Svetla SPIROVA
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