# High-order Artificial Compressibility for the Navier-Stokes Equations

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We introduce a generalization of the artificial compressibility method for approximation of the incompressible Navier-Stokes equations. It allows for the construction of schemes of any order in time that require the solution of a fixed number of vectorial parabolic problems, depending only on the desired order of the scheme. These problems have a condition number that scales like $O(\delta t h^{-2})$, with $\delta t$ being the time step and $h$ being the spatial grid size. This approach has several advantages in comparison to the traditional projection schemes widely used for the unsteady Navier-Stokes equations. First, it allows for the construction of schemes of any order for both, the velocity and pressure, while the best proven accuracy achievable by a projection scheme is second order on the velocity and 3/2 order on the pressure. Second, the projection schemes require the solution of an elliptic scalar problem for the pressure that has a condition number $O(h^{-2})$, in addition to a vectorial parabolic problem for the velocity. This makes them slower if iterative methods are used to solve the linear systems, and less efficient if implemented on a parallel cluster. Further, we will discuss the approach for solving the linear system resulting from the momentum equation and more particularly, we will present an unconditionally stable split version of the scheme that treats the mixed derivatives in the grad div operator fully explicitly. Finally, we will present a one step version of the high order scheme that opens the door to time step control algorithms.

The accuracy and stability of the resulting schemes will be demonstrated on examples with manufactured solutions.