Mereotopology – static and dynamic.

Mereotopology very often is used as a short name of Region-based theory of space (RBTS). Its roots go back to Whitehead [39], De Laguna [6] and some other philosophers and logicians. It results from certain criticism to the classical Euclidean approach to the theory of space, which takes as primitives the notions of point, line and plane which are abstract entities having no separate existence in reality. Whitehead's approach is in a sense opposite to the Euclidean – he puts on the base of the theory the realistic notion of "spatial region" and some simple intuitive relations between regions as "part-of", "contact " and some others. Points, lines and planes are not disregarded, but are introduced in the new theory by definitions. Since the philosophical theory of objects based on the relation "part-of" is called **mereology,** the new idea is to put the theory of space on the base of mereology. However, the language of mereology is too weak to express contact relation, which has some topological nature, hence the extension of mereology with contact or some other topological relations was simply called **mereotopology** which is closely rlated to RBTS**.** Survey papers on mereotopology and RBTS are [4,21,28,30]. Mereotopology and RBTS influenced also a special subfield of logic called logic of space (or spatial logic). A handbook devoted to spatial logic is [1].

Independently from philosophy, a similar idea in topology leads to the so called **point-free topology**. The first result in this new field is the famous Stone topological representation theory of Boolean algebras [29] representing them in certain topological spaces. The definable notion of point in Boolean algebra is just the previously known notion of ultrafilter. In a sense mereotopology can be considered as a certain kind of point-free topology. A very interesting theoretical subfield which generalizes the Stone duality theory is the categorical approach to mereotopology (see [7-12,15,20]).

A third time where mereotopology has been in a sense reinvented, is in computer science, namely in its subfield Knowledge Representation and Reasoning (KRR). It was recognized in KRR that the methods of classical theory of space are not efficient for representing of qualitative spatial information while the language of mereotopology (RBTS) due to its simplicity fits very well. This fact stimulated very fast development of mereotopology and RBTS: both theoretical and practical. Survey papers on practical applications of mereotopology and RBTS are [5,21,22].

An old idea of Whitehead motivated by relativity theory is to study space and time together on the base of point-free approach. Time points (moments of time) are also abstract entities similar to space points which also have no separate existence in reality. Whitehead's idea is this integrated theory to be developed on the base of some natural spatio-temporal relations between changing regions and time points to be introduced by certain definitions. A theory of similar kind, called **dynamic mereotopology** was developed recently in [25,26,31-35]. Similar integrated theory of space and time in which, however, time points are assumed as primitives, was developed in [17,18]. In the presence of dynamic mereotopology, mereotopology which studies static (unchanging) regions will be called sometimes **static mereotopology.**

The contributions of Bulgarian authors to the field of mereotopology and RBTS is significant and in a sense they are on the front line in the investigations . An incomplete list is, for instance, the following: a chapter in [1], [2,3,7-16,19,23-27,30-38]. Here one can see the names (alphabetically listed) G. Dimov, E. Ivanova, T. Ivanova, V. Nenchev, Y. Nenov, T. Tinchev and D. Vakarelov. The field is rapidly developing and many new results are expecting, namely new duality types theorems, generalized mereotopologies (mereotopologies on weaker or stronger bases), studiing mereotopologies with new important predicates, further developments of dynamic mereotopology.

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