Review

on a competition for an academic position "Associated Professor", in the field of higher education 4. Natural sciences, mathematics and informatics, professional advancement, 4.5 "Mathematics", specialty Differential equations.

by Prof. DSc. Tsvatko Rangelov, Member of the Scientific Jury, order N: 105/15.06.2021 of the Director of the Institute of Mathematics and Informatics, BAS.

1) The competition, with a term of 2 months, was announced in the State Gazette no. 30 of 13.04.2021 for the needs of the Institute of Mathematics and Informatics (IMI), BAS. Assistant Professor PhD. Borislav Tsonev Yordanov has submitted documents for participation in it. He graduated from the Faculty of Mathematics and Informatics, Sofia University, majoring in Mathematics in 1991. In 2002, after postgraduate studies he defended his PhD thesis "Global solutions of nonlinear wave equations with damping"in University of Wisconsin - Milwaukee, WI, USA. From 1994 to 1998 and since 2010 he works as an Assistant Professor at the Institute of Mathematics and Informatics, section "Differential Equations and Mathematical Physics". Since 2016 he works as Assistant Professor at Hokaido University, Japan.

2) Ass. Prof. B. Yordanov's scientific activity is in the field of partial differential equations, existence of solutions for Cauchy problem for hyperbolic equations, asymptotic behavior of solutions and smoothness for equations with linear dissipation, sharp estimates for the energy decay when $t \to \infty$. The result of this scientific activity is contemporary publications, some of which are submitted for participation in this competition.

In the last 20 years the above scientific subjects are investigated by many researchers as L. Hsiao, R. Ikehata, L. Wirth, V. Georgiev, G. Todorova, E. Zuazua, K. Wakasa, P. Radu, Q. Zhang, ect. and among them the works of Ass. Prof. B. Yordanov occupy a significant place. This is confirmed by more of 170 citations of the submitted for the competition papers. For example,

paper [3] published 2019 is cited 9 times, paper [6] published 2016 is cited 14 times, ect.

There are presented a total list of 24 publications, 18 of which for participation in the competition. Articles are published since 2005 in renowned mathematics journals, such as: Dicr. Contin. Dyn. Sis. - 1; J. Math. Anal. Appl. - 1; Nonlin. Anal. - 2; Trans. Am. Math.Soc. - 2; J. Diff. Eq. - 4; J. Math. Soc. Japan - 2; Funkcialaj Ekvacioj - 1; J. Non-Crist. Solids - 1, Ind. Univ. Math. J. - 1; J. Funct. Anal. - 1; SIAM J. Math. Anal. - 2. All publications have Impact Factor which means that an article 3(1),2. of the IMI rules for the competition is fulfilled.

All publications are co-authored with: G. Todorova, T. Phan, K. Wakasa , P. Radu, R. Ikehata, D. Ugurlu, R. Kirova, V. Georgiev, V. Rubino, B. Sampalmieri, Q. Zhang. I accept that the candidate's contribution is equal to that of his co-authors.

In connection with Art. 2 of the IMI Rules for the "minimum required score by set of indicators" for the candidate Ass. Prof. B. Yordanov is obtained the following: А - 50 points; Б - 100 points; В - 200 points; Г - 220 points; Д - 144 points; Е - 40 points, which means that this requirement is fulfilled.

3) The author's report correctly reflects the content and contributions in the works of Ass. Prof. B. Yordanov, papers [1, 8, 9, 15] are not commented there.

I will analyze shortly the presented for the competition works following the conditional division made in the author's report.

3a) Study of Cauchy problem for wave equation with linear dissipation, in this group are the papers [6, 8, 10, 11, 12, 14]. A model problem is

$$\begin{cases} u_{tt} - \operatorname{div}(b(x)\nabla u(x)) + a(x)u_t = f(x,u), \quad (x,t) \in \mathbb{R}^n \times (0,\infty), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \mathbb{R}^n, \end{cases} \tag{1}$$

In [11, 12] is used strengthen multipliers method to estimate the decay of the energy for large times. It is obtained power decay for weight estimates of solutions of (1) with $f = 0$ in dependance of the norm of initial data with

compact support. For the problem (1) with nonlinear function $f = \pm|u|^{p-1}u$ in [13, 14] are obtained results for global existence of solutions as well as blow-up for a finite time in dependance of p. Let us mention that the applied methods and also the new results in the above papers [11, 12, 13, 14] are cited more then 50 times and used by other researchers.

In [6, 8, 10] it is studied the following abstract Cauchy problem in Hilbert space H

$$\begin{cases} Cu_{tt} + Bu + u_t = g, \quad t > 0, \\ u(0) = u_0, \quad u_t(0) = u_1, \end{cases} \tag{2}$$

where $B : \mathfrak{D}(B) \to H$, $C : H \to H$ are two nonnegative, self-adjoint operators and $(u_0, u_1) \in \mathfrak{D}(B^{1/2}) \times H$, $g \in C(\mathbb{R}_+, H)$. By using the diffusion phenomenon, i.e., obtaining estimates by comparing the solutions of the wave equation with diffusion (2) with the corresponding parabolic equation, it is obtained an estimate of the kind $O(e^{-t/16})$ for $t \to \infty$ for the norm of $u(t) - e^{-tB}(u_0 + u_1)$. Note that, the results in the papers [6, 8, 10] are used and cited by other authors more then 40 times.

3b) Asymptotic behavior and smoothness for solutions of quasilinear wave equations, in this group are the papers [4, 5, 7, 13, 16]. The interaction between nonlinear dissipation and nonlinear source is described by the following model equation

$$\Box u + a|u|^{p-1}u + b|u_t|^{m-1}u_t = 0, \quad (x,t) \in \mathbb{R}^3 \times (0,\infty), \tag{3}$$

here $\Box$ denotes the wave operator, $a \geq 0$, $b > 0$, $p > 1$, $m > 1$ and $(u, u_t) \in H^1(\mathbb{R}^3) \times L^2(\mathbb{R}^3)$. Studied are the behavior of solution of (3) for $t \to \infty$ as well as smoothness of the Cauchy problem with initial data $(u(x,0), u_t(x,0)) \in H^k(\mathbb{R}^3) \times H^{k-1}(\mathbb{R}^3), k \geq 3$. For the first time in [16] it is obtained a power rate of the energy decay $Ct^{-d(m,n)}$. This is in difference with previous works, where maximal decay rate is logarithmic one. The constant $d(m,n)$ is implicitly given and in the subsequent applicant's papers [4, 5, 7, 13] this constant is obtained for particular cases as $n = 1$ or $m = 3$. In these papers are studied also the smoothness of solutions.

3c) Finite time blow-up for solutions of Cauchy problem, in this group

are papers [2, 3, 9, 17, 18] where it is studied the problem

$$\begin{cases} u_{tt} - \Delta_g u + h_0 u + h_1 u_t = |u|^p, \quad (x, t_{\in}\mathbb{R}^n \times (0,\infty), \\ u(x,0) = \varepsilon u_0, u_t(x,0) = \varepsilon u_1 \quad x \in \mathbb{R}^n, \end{cases} \tag{4}$$

here $p > 1$, $\varepsilon > 0$, Δ_g is generalized Laplace operator and $(u_0, u_1) \in C_0^\infty(\mathbb{R}^n) \times C_0^\infty(\mathbb{R}^n)$.

With a construction of an appropriate interpolation function in [18] it is obtained a new result in [17] showing that positive solutions of (4) with small initial data in the case $p < p_0(n)$ where $p_0(n)$ is positive root of the equation $-(n-1)p_(^2n+1)p+2$ following the idea of W. Strauss, blow-up for a finite time. Results and methods of these two papers [17, 18] are successfully applied in papers [2, 3] for quasilinear equations with variable coefficients.

At the end of this short exposition I will mention papers [1,15]. In [1] for generalized Shrödinger operator basing on the previous applicants' results for diffusion phenomenon there are obtained results for existence, uniqueness and regularity. In [15] it is studied an asymptotic behavior for solutions of Cauchy problem for nonlinear viscosity equations, topic which is in the beginning of its studies.

4) Ass. Prof. B. Yordanov participate in the following scientific projects:

- Grant-in-Aid for Science Research No.16H06339 and No. 19H01795, JSPS, Saporo, Japan;

- IMI, BAS, Application for equations of mathematical physics and Differential equations, equations of mathematical physics and applications.

5) Educational and pedagogical activity of Ass. Prof. B. Yordanov is significant. He has been teaching several mathematical subjects: 1994-1996 Mathematical analysis, Numerical methods in Technical University, Sofia; 2002-2004 Mathematical analysis, Vector analysis, Partial differential equations, Introduction to boundary value problems in University of California, USA; 2004-2007 и 2011-2013 Analysis 1 and 2, Ordinary differential equations, Partial differential equations, Linear algebra in University of Tennessee, USA; 2017-2021 Linear algebra 1 and 2, Analysis 1 and 2, Introduction to Ordinary differential equations, Introduction to vector analysis in Hokkaido

University, Japan.

6) I have no critical remarks. I know Ass. Prof. PhD B. Yordanov since he starts to work in IMI. He is industrious and actively working in the current field of differential equations. Having in mind new and interesting results in the publications for the competition as well as numerous citations of these works I think that he has enough material to prepare DSc thesis.

7) Conclusion: I give a positive assessment of the works of Ass. Prof. PhD B. Yordanov and I believe that he fully satisfies the requirements of the ZRASRB for the competitive position, also in the submitted ones for the competition articles have no plagiarism.

I recommend the Scientific Jury to propose to the Scientific Council of the Institute of Mathematics and Informatics to select Ass. Prof. PhD B. Yordanov for Associate Professor in the professional field 4.5 "Mathematics", specialty Differential Equations.

August 3, 2021 Signature:

Ts. Rangelov