

REPORT

by Prof. PhD Georgi Venkov, FAMI, TU – Sofia

on the dissertation “Subordination principle for generalized fractional evolution equations”,

with author Assoc. Prof. PhD Emilia Grigorova Bazhlekova

submitted for acquisition of the Academic Degree “Doctor of Sciences”

in direction of higher education 4 Natural sciences, mathematics and informatics

Professional field 4.5. Mathematics, Specialty Mathematical Analysis, for the needs of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

I present my report on this procedure as a member of the Scientific Jury, determined by the Order № 216/20.07.2022 of the Director of Institute of Mathematics and Informatics (IMI), Bulgarian Academy of Sciences (BAS).

The report was prepared in accordance with the requirements of:

- the Law for the Development of Academic Staff in the Republic of Bulgaria (ZRASRB),
- the Rules for the Implementation of the ZRASRB,
- The Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at BAS and at IMI of BAS.

1. General information about the applicant

According to the documents submitted for the competition, Assoc. Prof. Emilia Bazhlekova acquired the Master's degree at the Faculty of Mathematics and Informatics of Sofia University “St. Kliment Ohridski”. Until 2001 she was a doctoral student at the Technical University of Eindhoven, the Netherlands, in the scientific field 4.5 Mathematics, specialty "Mathematical Analysis" and defended her dissertation on "Fractional evolution equations in Banach spaces" to obtain the degree of PhD. The teaching experience of Assoc. Bazhlekova began in 1991 as an assistant professor at the Faculty of Physics of Sofia University “St. Kliment Ohridski”, as a teacher in mathematics at the high schools “Hr. Botev” and “Hr. Smirnenski”, city of Septemvri (1993-1994) and as an associate professor in the Section of Analysis, Geometry and Topology at IMI-BAS (since 2014). Assoc. Prof. Emilia Bazhlekova is distinguished by her active research activity. She participates in the editorial board of the international scientific journal “Fract. Calc. Appl. Anal.”, she is a reviewer in more than ten international scientific journals, participates as a member of international contracts, contracts to the FSR-MES, IMI-BAN and OP “Science and Education for Smart Growth”.

2. General characteristics of the submitted scientific works

For participation in the competition Assoc. Prof. Bazhlekova has submitted a dissertation of 192 pages, consisting of an Introduction, eight chapters, scientific contributions and a Bibliography in which 110 literature sources are cited. The dissertation is based on the results obtained by the author, published in 11 articles, which were not used to obtain the PhD degree, and to hold the AP “Associate Professor”. All the articles have been published in international

refereed and indexed journals, 6 of them are single-authored and the rest are co-authored, 8 have an impact factor (IF) and 4 have an impact rank (SJR). Therefore, according to the Annex 1 of the Regulations of the BAS for the Implementation of ZRASRB, giving the minimum scientific metrics to the candidates participating in the procedure for acquisition the degree “Doctor of Sciences” in the professional field 4.5 Mathematics, the following table can be given:

	Q1	Q2	Q3	SJR	other	sum
Number of publications	5	1	2	2	1	11
points	250	40	60	40	12	402

It is clear, that the presented publications of the candidate far exceed the minimum requirement of 100 points in Group of indicators G (402 points) of the Annex 1 to Rules of BAS for the Implementation of the ZRASRB. Moreover, the total IF and SJR of Assoc. Prof. Bazhlekova's articles are respectively 16,447 and 0,45, indicating a high level of results presented in them.

According to the attached list, the candidate's papers for this procedure are cited in 88 scientific publications in international journals, referenced and indexed in Web of Science and /or Scopus. According to Indicator D.11 of the Annex 1 to Rules of BAS for the Implementation of the ZRASRB, the citation index of Assoc. Prof. Bazhlekova equals 528 points, which far exceeds the minimum requirement of 100 points.

3. Main scientific contributions of the dissertation

This dissertation is built on the results obtained in papers [10 - 15], [18 - 20], [22] and [25] from the Bibliography. It is devoted to the study of the subordination principle for generalized fractional evolution equations. In general, the subordination principle makes it possible to represent solutions of complicated equations by solutions of simpler classical equations and is a useful tool for proving solvability of the mathematical problems, finding estimates for the solution, establishing its asymptotic behavior and other properties. Along with that, by the subordination principle, a hierarchy can be established among the set of generalized fractional evolution equations, which is important for the proper classification and evaluation of the physical meaning of the corresponding mathematical model.

In this dissertation, a methodology is developed that allows the establishment of a subordination relation between two equations and thus helps to classify these equations into two main groups: equations describing subdiffusion and diffusion-wave equations. A number of specific equations found in the scientific literature have been investigated.

The main mathematical tools used in the present studies are the theory of operators and special functions of the fractional calculus, the Laplace transform, the Bernstein theory of functions and special classes of functions related to them.

The dissertation of Assoc. Emilia Bazhlekova is organized as follows. The introduction

describes the motivation for the research and examples of different types of subordination principles are given. Chapter 1 contains notations, definitions and basic properties of fractional integration and differentiation operators, Laplace transform, Mittag-Leffler functions and some Wright-type functions. Chapter 2 introduces the theory of Bernstein functions as well as Volterra's integral equations. Two general subordination theorems are proved here. Chapter 3 is devoted to a detailed study of the subordination principle for evolution equations with fractional derivatives in time and in space, and the results obtained in [14] and [20] are used. As an application, integral representations for the fundamental solution and some explicit representations by special functions are obtained. In Chapter 4, following the results in [22], is considered the fractional Jeffreys' heat conduction equation by applying the subordination principle. In Chapter 5, based on publications [10], [11] and [12], subordination results are obtained for subdiffusion equations of distributed order in time and for more general equations with memory kernels. Useful estimates are derived in the scalar case. In Chapter 6, the multinomial generalization of Mittag-Leffler function is studied, which is related to relaxation equations with multiple time derivatives of different fractional order [15].

The last two chapters study equations describing phenomena that are intermediate between diffusion and wave propagation. In particular, Chapter 7 discusses and partially solves an open problem concerning the interpretation of the fundamental solution of distributed-order diffusion-wave equations as a probability density [13] and [18]. This property of the fundamental solution is important both for the physical meaning of the model and for establishing subordination with respect to the wave equation. In Chapter 8 are studied equations describing wave propagation in viscoelastic media with completely monotone relaxation moduli. Generalized fractional Maxwell and Zener models are considered, as well as a new model with a relaxation modulus that is represented by a fully monotone Mittag-Leffler binomial function. The special case of Jeffrey's fractional model is studied in detail and the physical meaning of the subordination formula is discussed.

After using the free platform “Plagiarism checker” (<https://plagiarismdetector.net/>) I can confirm the absence of plagiarism in the scientific works presented by the candidate.

4. Critical comments and recommendations

I have no critical remarks to the dissertation of Assoc. Emilia Bazhlekova for participation in this competition.

5. Conclusion

In conclusion, I consider that the submitted dissertation of Assoc. Prof. Dr. Emilia Grigorova Bazhlekova for the present competition fully meets the requirements of the Law for the Development of Academic Staff in the Republic of Bulgaria (ZRASRB), the Rules for the Implementation of the ZRASRB and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at BAS and at IMI of BAS.

Therefore, I strongly propose to the respected scientific jury to positively evaluate the candidature of Assoc. Prof. Dr. Emilia Bazhlekova and to recommend to the Scientific Council of IMI-BAS her election for the award of the degree “Doctor of Sciences” in the field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5. Mathematics, scientific specialty "Mathematical Analysis".

23.09.2022 г.

София

(проф. д-р Георги Венков)