REPORT

By Prof. Jordanka Paneva-Konovska, DSc,
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Member of the Scientific Jury, appointed by order №: 216/20.07.2022 г.
of the Director of IMI – BAS

on the materials presented in defense of the Dissertation work
Assoc. Prof. Dr. Emilia Grigorova Bazhlekova
entitled: 'Subordination principle
for generalized fractional evolution equations'
for awarding of the scientific degree 'Doctor of Science'
Higher education area:
4. Natural Sciences, Mathematics and Informatics,
Professional field: 4.5. Mathematics,
Scientific specialty: 'Mathematical analysis'

With order No: 216/20.07.2022 of the Director of IMI – BAS I was appointed as a member of the Scientific Jury for the defence of this dissertation work. According to the decision of the first meeting of the jury, held on 21.07.2022, I present a report. IMI - BAS is accredited by NAOA for the doctoral program 'Mathematical Analysis'.

After examining the materials presented by the dissertant E. Bazhlekova, I am convinced that both they and the dissertant satisfy the requirements of Chapter 2 of the Act on Development of the Academic Staff in the Republic of Bulgaria (ZRASRB). E. Bazhlekova was awarded a PhD degree at Eindhoven University of Technology, the Netherlands on 01.10.2001 (the diploma is legalized in Bulgaria: diploma 000001 / 16.12.2011). All the recommended scientometric criteria of Chapter **2** of the Regulations on the Conditions and Order for Acquiring Scientific Degrees and Occupying Academic Positions at the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences are satisfied and significantly exceeded.

1. Brief biographical data

Mrs. Bazhlekova graduated from the "Geo Milev" Mathematics High School in the city of Pleven, then - higher education (master's) in mathematics in 1986 at the Faculty of Mathematics and Informatics of the Sofia University 'St. Kliment Ohridski' with a specialization in Complex Analysis and a subsequent one-year specialization in the period 1986-1987. In the period December 1997 -

December 2001, Bazhlekova was on a scientific specialization abroad (the Netherlands), Applied Analysis group, Eindhoven University of Technology, Eindhoven, where in 2001 she defended herPhD thesis. The topic of her PhD thesis is "Fractional evolution equations in Banach spaces" and is under the scientific supervision of prof.dr.ir. J.de Graaf and prof.dr. Ph. Clément. She speaks English (very well), Dutch (very well) and Russian.

The candidate worked as a teacher and research scientist in the specialty of mathematics. She began her scientific and teaching career as a part-time assistant in Mathematical analysis at the Faculty of Physics of Sofia University 'St. Kliment Ohridski' and teacher of Mathematics and English in the town of Septemvri. From February 1995 to October 2004 and then from March 2011 to October 2011, she worked as a mathematician in the "Complex Analysis" section (now AGT) of IMI. From 2011 to 2014, she was an assistant professor in the same section, and from 2014 - an associate professor. Her scientific interests are mainly in the field of Mathematical analysis, namely: fractional calculus, fractional evolutionary equations (including an operator-theoretic approach for abstract problems in Banach spaces, spectral expansions of solutions for boundary value problems, estimates, asymptotics, generalized Mittag-Leffler functions), applications of fractional calculus in mathematical modelling, integral transformations and special functions, convolutional calculus and its application to finding Duhamel representations of solutions of nonlocal linear boundary value problems.

General description of the dissertation work and the presented scientific publications:

- **Dissertation work for awarding of the scientific degree 'Doctor of Science'** – 200 pages in total, written in English. It consists of a Preface, Introduction, 8 chapters, Main Scientific Contributions, Bibliography (incl. 110 titles) and Alphabetical Index. The scientific specialty of this dissertation is Mathematical Analysis, more precisely - special functions, fractional calculus and their applications.
- **Abstract of dissertation** – in Bulgarian (34 pages) and in English (33 pages), presented in electronic form.
- **Contributions and Citations Reference** – 2 pages (in Bulgarian and in English).
- **Scientific works related to the results of the dissertation - 11 scientific articles.** They all are published in English. Of them, 6 are without coauthors and 5 co-authored. All presented works of the dissertant are from the period 2015-2021 after the legalization of her diploma for the PhD degree and the subsequent habilitation in January 2014. She had not participated with them in previous procedures.
- **Publications are in peer-reviewed and indexed journals in Scopus/WoS and are distributed as follows:**

  Scientific articles in journals with IF/SJR – 10 items
  Scientific articles in journals without IF/SJR – 1 item

- **Scientometric indicators of articles:**

  8 of the presented articles are in journals with an impact factor (Web of Science, Thomson Reuters), with a cumulative IF = 16.447. Of these, 5 publications are in the prestigious Q1 category, in Q2 – 1 work and in Q3 – 2 works.

  2 articles are in journals with SJR (one of them was presented on an international conference)

  1 article is in a journal without IF/SJR.

3. Analysis of scientific results and contributions in the presented dissertation work and related publications

The presented dissertation work is devoted to the study of the subordination principle for generalized fractional evolution equations. A methodology has been developed that allows establishing a subordination relationship between a linear evolution equation of general type and a linear evolution equation of fractional or integer order. Based on this methodology, a number of specific generalized fractional evolution equations have been investigated, which have been the subject of study in the scientific literature in recent years.

The main mathematical tools used in the dissertation are the theory of operators and special functions of fractional calculus, the Laplace transform technique and the theory of Bernstein functions and special classes of functions related to them.

In the dissertation, two classes of generalized fractional evolution equations are considered: equations describing anomalous diffusion and diffusion-wave equations. The two classes are conveniently defined by the subordination principle: the first class consists of equations subordinate to the classical diffusion equation, and the second class is from equations subordinate to the classical wave equation that do not belong to the first class. According to the methodology developed in the dissertation, the question of whether a generalized fractional evolution equation belongs to one of the two classes comes down to establishing whether a certain characteristic function is a complete Bernstein function.

Subordination representations have been obtained for the solutions of a number of equations containing fractional time derivatives (Chapters 4, 5, 7, 8). These are integral representations that contain a subordination kernel (a function that is a probability density) and the solution of a simpler equation of integer or fractional order. Such representations are particularly useful for studying solutions behavior.

As the main example of an equation containing fractional time derivatives, the Jeffrey-type fractional evolution equation is studied (Chapter 4 and Section 8.3). It is shown that, depending on the parameter values, it satisfies two different subordination principles and, accordingly, the solution has two fundamentally different types of behavior.

The subordination principle for equations describing anomalous diffusion is studied in Chapter 5, paying special attention to distributed order equations. As one application of the subordination formula, a two-sided estimate for the solution of the generalized relaxation equation is derived, which is useful for further applications.

For solving subdiffusion equations with several fractional time derivatives, a Prabhakar-type multinomial function is introduced and investigated (Chapter 6). Conditions are found where the function is completely monotonic.

The subordination principle for diffusion-wave equations is considered in Chapters 7 and 8. First, results are given on an open problem on the conditions under which the one-dimensional fundamental solution of the distributed-order diffusion-wave equation is a probability density (Section 7.1). The principle of subordination of diffusion-wave equations with several time derivatives of different fractional order is investigated in Section 7.2. Chapter 8 considers diffusion-wave equations describing wave propagation in viscoelastic media. It is proved for the relaxation moduli of some generalized fractional viscoelastic models to be completely monotonic functions, which plays an important role in establishing a subordination principle for the corresponding wave equations.

In addition, the subordination principle for space-time fractional evolution equations is studied in detail (Chapter 3). A number of integral representations for the subordination kernel have been derived. By applying the subordination principle, integral representations for multidimensional fundamental solutions of such equations have been derived. For some particular values of the fractional parameters and the dimensionality of the space, explicit representations are obtained by means of special functions.

The proposed dissertation work is not a mechanical collection of new results from the presented publications, but contains a common theory developed by the author and reflected in the publications presented according to the procedure. Ten of the eleven articles presented in the dissertation work were published mainly in journals with a high impact factor or have an impact rank, and have already received a large number of citations from leading foreign authors in the field.

I think that the main scientific contributions of E. Bazhlekova in the dissertation work and in the publications related to it are correctly reflected in the presented Reference for contributions and citations and in the Author's abstract of the dissertation work.

4. General characteristics of the research activity of the dissertant:

In addition to the given characteristics of scientific publications and their international impact, I will also note some other data:

- Participation of Dr. Bazhlekova in the organizing committees of 6 international mathematical conferences, in 1 of them – as co-chair.

- she is a member of the editorial board of the international scientific journal *Fractional Calculus and Applied Analysis* (indexed in *Web of Science, Q1 category*) since 2014.

- Participation in research projects: 1 – international project under OP ``Science and education for intelligent growth", 4 – at FNI-MES; 3 – on bilateral international cooperation of BAS.

- Participation with talks (for the years related to the development of the dissertation work) in more than 10 international scientific conferences. In addition, the obtained results were presented at the joint seminar `Analysis, Geometry and Topology' at IMI-BAS (2015), at the annual scientific sessions of the `Analysis, Geometry and Topology' section at IMI-BAS, as well as at the seminar on Mathematical Modeling of FMI- Sofia University in 2015, 2017 and 2019.

- Reviewer of a number of international mathematical journals.

5. **Reflection of the candidate's scientific publications in the literature:**

Dr. Bazhlekova presented a list of 90 citations she noticed (and verified) in scientific publications indexed in the SCOPUS database (of which 77 are in publications with an Impact Factor) of the scientific works submitted for the dissertation. Separately, over 1500 citations were found for

her complete list of 50 publications (without self-citations). Only her PhD Thesis has been cited over 480 times in Scopus. The author's personal Hirsch index is h=9 (according to Thomson Reuters' Web of Knowledge & Scopus), and h=14 (according to Google Scholar Profile). Most of the citations are from well-known foreign authors in the field of special functions and fractional calculus, and are in reputable international publications.

6 Approbation of the results:

The results of E. Bazhlekova in this dissertation have been published in prestigious international journals and proceedings of international conferences, indexed in the primary sources of the world bibliographic network. Moreover, as noted above, they have been noticed and cited many times. They have been presented at a number of international scientific forums abroad (Serbia, Greece), and at national ones. These results are also an essential part of the work programs and are spread over the already mentioned scientific research projects (to FNI, IMI-BAS, bilateral cooperation). In view of all this, I consider the approbation of the results to be very good.

7. Evaluation of the candidate's personal contribution:

As a rule, the dissertation work for the scientific degree 'Doctor of Science' is prepared by the candidates themselves and is the personal work of the candidate. In this case, this is an indisputable fact: of the scientific works presented, most are without co-authors, and the rest are co-authored with one other author - in them I consider that the dissertant has an equal contribution.

8. Critical Notes**:**

In technical terms, the dissertation and the abstract are well formed. I have no major criticisms.

9. Personal impressions:

I know Prof. Bazhlekova since the time of her postgraduate studies in the "Complex Analysis" section of IMI (1990). She works actively in our Research project teams at FNI-MON, on budget projects of IMI-BAS (2 prs.), on bilateral cooperation of the BAS with the academy of Serbia. Given this, I know closely her scientific and scientific-organizational activities on topics close to my scientific interests and searches, I also have impressions of her talks at international conferences and national forums and how they are received by specialists in the field. They are that the candidate is an established and recognized author, in our country and abroad, in the field of special functions and their numerous applications.

10. Conclusion:

The review of the presented dissertation work, the abstract, and related scientific works and documents shows that Assoc. Prof. Dr. Bazhlekova is an internationally recognized specialist in Mathematical analysis, and in particular in the field of special functions and their applications, with significant personal contributions on the topic of the dissertation.

I value the contributions in the dissertation as theoretical generalizations and theory development in both an old classical and a rapidly developing field of Mathematical analysis.

According to the Regulations of IMI-BAS for the application of the ZRASRB, Ch. 2, the quantitative criteria for awarding the degree 'Doctor of Sciences' at IMI and their implementation in this case are significantly exceeded:

1) at least 7 publications in publications with IF or SJR – 10 items presented;

2) at least 4 of them should be in journals with an impact factor – 8 of the presented articles have an impact factor (and 2 –an impact rank);

3) Sum of indicators from 5 to 10 for dissertation publications (distribution by category) – should be at least 100 points. The dissertant has 402 points.

4) Citations (without self-citations) of the dissertation publications in the Scopus database: the candidate has to have at least 100 points. The result of the dissertant is 540 points.

Based on the above, I confirm that Assoc. Prof. Dr. Emilia Grigorova Bazhekova satisfies all the requirements of the ZRASRB and the Regulations of IMI-BAS for this procedure and for the presented dissertation work, and I recommend the scientific jury to award her the scientific degree 'Doctor of Sciences' in the higher education area: 4. Natural sciences, Mathematics and Informatics, by professional field 4.5 Mathematics, scientific specialty 'Mathematical analysis'.

I give a positive conclusion for awarding the scientific degree.

28.09.2022 г.
Sofia

The report is prepared by:
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