REPORT

from: **Krasimira Stoyanova Prodanova, Ph.D., Professor,**
Faculty of Applied Mathematics and Informatics,
Technical University – Sofia.

Procedure for awarding of the scientific degree
"Doctor of Science"

Candidate: **Emilia Grigorova Bazhlekova**
Dissertation title: **"Subordination principle for generalized fractional evolution equations"**
Higher Education area:
4. Natural Sciences, Mathematics and Informatics,
Professional field: 4.5. Mathematics,
Scientific speciality: Mathematical analysis

By Order No. 216/20.07.2022 of the Director of IMI – BAS I was appointed as a member of the scientific jury for the procedure for obtaining of the scientific degree "**Doctor of Science**" for this dissertation. According to the decision of the first meeting of the scientific jury on 21.07.2022, I present an opinion on the dissertation work.

The presented dissertation contains 200 pages. It consists of an Introduction and eight chapters.

The Introduction contains the aims and objectives of the dissertation: the study of the subordination principle for generalized fractional evolution equations. Also given here is a list of the candidate's 11 dissertation-related publications, all visible in the global databases of scientific information. After the exposition, the contributions of the dissertation are listed. The list of references contains 110 titles.

The presented list of publications on the dissertation is directly related to the content of the respective chapters. The articles are published in authoritative scientific journals that guarantee visibility and evaluation in the scientific community. The number of their citations in Scopus is 90.

The dissertation is in compliance with the set of criteria and indicators for the acquisition of the relevant degree according to the Act on Development of the Academic Staff in the Republic of Bulgaria, its Regulations, and the Regulations on the Conditions and Order for Acquiring Scientific Degrees and Occupying Academic Positions at the Bulgarian Academy of Sciences and at the Institute of Mathematics and Informatics – BAS.

Before presenting all materials with my opinion on them, I shall show in a table formal information on the scientometric minimal national requirements for the scientific degree "Doctor of Science" according to the Act on Development of the Academic Staff in the Republic of Bulgaria, and the Regulations on the Conditions and Order for Acquiring Scientific Degrees and Occupying Academic Positions at the Institute of Mathematics and Informatics – BAS, last modified and supplemented on 25.03.2022 (the indicators and the corresponding points can be found in Appendix 1 of the Regulations of BAS for the implementation of the Act on Development of the Academic Staff in the Republic of Bulgaria):

Group of indicators	Contents	Minimal requirements	Points of the candidate
G	**Sum of indicators from 5 to 10** dissertation-related publications (11): 5 in Q1 (**250** points); 1 in Q2 (**40** points); 2 in Q3 (**60** points); 2 with SJR (**40** points);1 in WoS and Scopus without IF/SJR (**12** points)	**100 points**	**402 points**
	From the dissertation-related publications:		
	Publications with IF or SJR	**7**	**10**
	Publications with IF (8 with total IF: 16.447)	**4**	**8**

D	Indicator 11	100 points	540 points
	Citations in Scopus (without self-citations) of the dissertation-related publications: **90** (x **6 points**)		

From the above table it is seen that the candidate Assoc. Prof. PhD Emilia Bazhlekova not only satisfies the requirements, but also the candidate's points significantly exceed the minimum required ones. This applies especially to group D indicators - citations of the dissertation-related works, which convincingly proves international recognition.

Description, analysis and assessment of contributions of the dissertation and related scientific publications

In the dissertation a unified methodology is developed that allows establishing a subordination relation between two equations and in this way helps to classify these equations into two main groups: subdiffusion equations and diffusion-wave equations. A number of specific equations found in the scientific literature have been investigated. The main mathematical tools that are used in the conducted research are the theory of operators and special functions of fractional calculus, Laplace transform, and the theory of Bernstein functions, as well as special classes of functions related to them.

Capter 1 contains notations, definitions and basic properties of fractional integration and differentiation operators, Laplace transform, Mittag-Leffler functions and functions of Wright type. In Chapter 2, after an introduction to Bernstein functions and some background material on abstract Volterra equations, Bazhlekova proves two general subordination theorems. The first theorem reduces the question of subordination to a problem related to Bernstein functions, and in the second the sector of analyticity is determined when the subordinate solution operator is bounded analytic. Chapter 3 is devoted to a detailed study of subordination principle for space-time fractional evolution equations. As an application, a number of explicit expressions in terms of special functions and integral representations for the solutions of the multi-dimensional space-time fractional diffusion equations are derived. The results in this chapter are published in articles [14, 20]. Chapter 4 is devoted to the fractional heat conduction equation of the Jeffreys type. The proven theorems show that for values a<b and a>b of the model parameters, the equation satisfies two fundamentally different subordination

principles, corresponding to the diffusion regime and the wave propagation regime. An example is given in which integral representations for the Green function of the one-dimensional Cauchy problem are derived. It is shown that the Green function is a probability density with respect to the spatial variable, which is unimodal in the diffusion regime and bimodal in the wave propagation regime. The results are published in work [22]. Chapter 5 mainly presents results on subordination for a distributed-order subdiffusion equation with a continuous or discrete distribution of the orders of the fractional derivatives in time. As an application, useful estimates in the scalar case are derived. The results are published in works [10], [11] and [12]. In Chapter 6, a generalization of the multinomial Mittag-Leffler function is studied, which is related to multi-term relaxation equations. The results are published in work [15]. The last two chapters are devoted to the description of the subordination principle for generalized diffusion-wave equations with fractional time- derivatives. Chapter 7 deals with diffusion-wave equations with fractional derivatives in Caputo sense, whose orders are discretely or continuously distributed. An open problem concerning the interpretation of the fundamental solution as a probability density is addressed and the subordination principle for a multi-term diffusion-wave equation is studied in detail. The results are published in works [13] and [18]. Chapter 8 is devoted to the subordination principle for equations modeling wave propagation in linear viscoelastic media with completely monotone relaxation moduli. Problems of wave propagation in a viscoelastic fluid with the fractional Jeffreys' model are studied in more detail and some applications of the subordination principle as well as its physical interpretation are discussed. The results of Chapter 8 are published in works [13], [19] and [25].

The Abstract of the dissertation fully and exactly reflects the content of the dissertation.

THE GENERAL ASSESSMENT of the results in the dissertation and the related publications is that they have substantial theoretical and applied contributions. The candidate Assoc. Prof. PhD Emilia Bazhlekova demonstrates a profound knowledge of the investigated problems in the specific area of subordination principles for generalized fractional evolution equations. The problems considered undoubtedly yield new contributions in this area of mathematics, which is also confirmed by the **high citability** (**540 points achieved at required minimum of 100 points**).

I assess the publication and scientific-research activity of Assoc. Prof. PhD Emilia Bazhlekova as completely sufficient for the procedure in volume, scientific level, as well as in citations at an international level.

CONCLUSION

Based on the above, I confidently recommend the Honorable scientific jury to award Emilia Grigorova Bazhlekova the scientific degree „Doctor of Science" in the Higher education area: 4. Natural Sciences, Mathematics and Informatics, Professional field: 4.5. Mathematics, Scientific speciality: Mathematical analysis.

Sofia,   Jury member:

23. 09. 2022   (Prof. PhD K. Prodanova)