OPINION

by Prof. Miglena Nikolaeva Koleva, DSc

of the dissertation for obtaining the scientific degree

"Doctor of Sciences"

in scientific field 4. Natural sciences, mathematics and informatics, professional field 4.5. Mathematics,

scientific specialty Mathematical analysis

Title: SUBORDINATION PRINCIPLE
FOR GENERALIZED FRACTIONAL EVOLUTION EQUATIONS

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1. Brief biographic information about the candidate

Assoc. Prof. E. Bazhlekova received her higher education in 1986 at the Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski", specialty "Mathematics". Scientific and educational degree "Doctor" in professional field 4.5. Mathematics, Scientific speciality Mathematical Analysis, acquired in 2001 at Eindhoven University of Technology, Netherlands. Assoc. Prof. E. Bazhlekova started her scientific career at IMI-BAS in 1989 in the department "Complex Analysis". Until now, with short interruptions, she has worked at IMI-BAS, sections "Complex Analysis", "Analysis, Geometry and Topology". Since 2014 she is Associate Professor at IMI-BAS.

2. General characteristic of the presented materials

As a member of the scientific jury, I received a European-style curriculum vitae, a dissertation in English, an Abstract of dissertation in Bulgarian and English, a list of publications on the dissertation, a list of the citations of the publications on the dissertation, a description of the scientific contributions in the dissertation in Bulgarian and English, copies of the publications on the dissertation, and other documents that I will not list.

The submitted documents are in accordance with Law for Development of Academic Staff in Republic of Bulgaria (LDASRB), the Regulations for the Application of LDASRB and the rules and conditions and specific requirements of IMI-BAS for acquisition scientific degrees and competitions for academic positions occupation.

3. Overview of the dissertation and analysis of the results

The dissertation is devoted to the subordination principle for fractional order evolution equations and establishing a methodology for subordination dependence between two equations that helps in their physically correct classification.

The topic of the dissertation is up-to-date. In recent years, a large part of scientific investigations has been directed to the analytical and numerical study of fractional order differential equations, because due to their non-local nature, they more accurately describe real physical processes and phenomena.

The dissertation consists of 8 chapters, introduction, conclusions, bibliography, including 110 references and index. The volume of the dissertation is 200 pages.

The abstract of the dissertation has a volume of 52 pages and fully corresponds to the content of the dissertation.

Briefly, the results in the dissertation are as follows.

Chapter 1 presents notations, definitions, fractional integration and differentiation operators, special functions closely related to fractional calculus and their properties.

In *Chapter 2* two general subordination theorems that are used in the dissertation are proved.

Time-space fractional order differential equations are considered in *Chapter 3*. Different representations of the subordination kernel are obtained and its properties are studied. As an application of the subordination principle, integral representations for the fundamental solutions of the multidimensional case and explicit representations of the solution by special functions for some particular cases are derived.

In *Chapter 4*, integral representations for the Green's function for a Cauchy problem for the Jeffrey-type heat conduction equation are derived. The relationship between the subordination principle and the physical character of an evolution equation is shown.

In *Chapter 5* are established subordination dependences for the solutions of equations modelling anomalous diffusion. As an application of the subordination formula. A useful two-sided estimate for the solution of the generalized relaxation equation is derived, which is applied in the study of an inverse problem.

Investigation of the properties of the Mittag-Leer multinomial function that appears in the representation of the solutions of evolution equations with several time derivatives of different orders is presented in *Chapter 6*. Въвежда се и се изследва Prabhakar type generalization is introduced and studied. Conditions guaranteeing that the function is completely monotone are established.

In *Chapter 7* is discussed an open problem concerning the interpretation of the fundamental solution of the Cauchy problem for diffusion-wave equations with Caputo fractional derivatives with distributed order, as a spatial probability density. The

subordination principle for the multi-term diffusion-wave equation is investigated in detail.

The last Chapter 8 is devoted to the equations modeling wave propagation in linear viscoelastic media with completely monotone relaxation moduls. Various constitutive laws are considered, which are fractional generalizations of some classical models. It is proved that the relaxation modulus is a completely monotone function. The particular case of a fractional Jffrey model is studied in detail and the physical meaning of the subordination formula is discussed.

The results of the dissertation were presented at 4 scientific conferences in Bulgaria and abroad, 3 international forums held in Novi Sad, Serbia, a joint seminar and an annual scientific session of the Section "Analysis, Geometry and Topology", IMI-BAS and three seminars on Mathematical Modeling at the FMI, Sofia University " "St. Kliment Ohridski".

4. Publications and citations of the publications on the dissertation

The dissertation is written on the base of 11 publications that were published in the period 2015 - 2021. Of them, 8 have an impact factor, with a total impact factor of 16.447 and distributed by quartiles as follows - 5 in Q1, 1 in Q2 and 2 in Q3. Two of the articles have SJR rank and one is indexed in Web of Science and Scopus. There are 6 independent articles, 5 of which have an impact factor, the remaining 5 have one co-author. Assoc. Prof. E. Bazhlekova has made a clear distinction between the results obtained by her in the articles in which she is a co-author.

The condition, according to the regulations for the terms and conditions for acquiring scientific degrees and occupying academic positions at IMI-BAN, that the candidate for acquiring the scientific degree "Doctor of Sciences" must present at least 4 articles with impact factor is fulfilled. The points in group G are 402, with 100 required.

Assoc. Prof. E. Bazhlekova has presented a list of 90 citations of the dissertation articles indexed in the SCOPUS database, 77 of them are in issues with Impact Factor. Thus, the points in group D are 540 and many times exceed the 100 points according to the minimum national requirements and the specific criteria of IMI-BAS.

5. Conclusion

The presented dissertation is written precisely, contains new and significant scientific results. It meets all the criteria and indicators for obtaining the scientific degree "Doctor of Sciences", according to the Law for development of the academic staff in the Republic of Bulgaria, the Regulations for application of LDASRB, the Regulations for academic positions at the BAS and the specific requirements of IMI-BAS.

In view of all the above, I give a positive assessment of the dissertation " Subordination principle for generalized fractional evolution equations" by Assoc. Prof. Emilia Bazhlekov.

I strongly recommend to the scientific jury to award Assoc. Prof. Emilia Grigorova Bazhlekov, PhD the scientific degree "Doctor of Sciences" in the professional field 4.5 Mathematics, Scientific specialty Mathematical analysis.

3.10.2022 Jury member:…….

(Prof. M. Koleva, DSc)