ABSTRACTS OF THE PUBLICATIONS

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(1) Koppitz J., T. Musunthia. The rank of the inverse semigroup of partial automorphisms on a finite fence. Semigroup Forum, 102, 2, Springer, 2021, ISSN:0037-1912, DOI:https://doi.org/10.1007/s00233-020-10150-1, 437-455.
Impact factor: 0.768
Zbl 07333917

Abstract

In this paper, we consider a subsemigroup of the symmetric inverts semigroup of all partial transformations on a finite set X. equipped with a partial order, namely the zig-zag order, which is closed to the linear order in some sense. Such a partial ordered set is called fence. We consider the set FI_n of all partial injections on X which preserve the zig-zag order. The set FI_n forms a monoid under the composition of transformations and was already considered by several authors. For the case that the cardinality of X is even, the rank of FI_n was already known, where the rank is the minimal size of a generating set for FI_n. But for the case that the cardinality of X is odd, the rank of IF_n was not yet determined. In this paper, we fill that gap. In particular, we provide a minimal generating set for FI_n.

(2) Dimitrova I., **J. Koppitz**. On relative ranks of finite transformation semigroups with restricted range. Ukrainian Mathematical Journal, 73, 5, Springer Science+Business Media, 2021, ISSN:00415995, DOI:10.1007/s11253-021-01955-6, 718-730.
Impact factor: 0.446
Zbl 07441687

Abstract

In 1975, Symons introduced and studied a semigroup T (X,Y), which is called a semigroup of transformations on X with restricted range Y. We consider X as finite linearly ordered set. Then set O(X) of all order-preserving transformations on X as well as the OP(X) of all orientation-preserving transformations on X form semigroups. In this paper we determine the relative rank of the semigroup T(X, Y) modulo the intersection of T(X,Y) and OP(X), which is denoted by OP(X,Y), i.e. we calculate the minimal size of a set A that together with OP(X,Y) generates T(X,Y). Moreover, we determine the relative rank of the semigroup OP(X,Y) modulo the set O(X,Y). The latter is the intersection of T(X,Y) and O(X). In both cases, we characterize the minimal relative generating sets.

(3) Dimitrova I., **J. Koppitz**. On relative ranks of the semigroup of orientation-preserving transformations on infinite chains. Asian-European Journal of Mathematics, 14, 8, World Scientific, 2021, ISSN:1793-5571, DOI:10.1142/S1793557121501461, 2150146-1-2150146-
Impact factor: 0.294
Zbl 1491.20128

Abstract

The semigroups O(X) and OP(X) of all order-preserving and orientation-preserving, respectively, transformations on a finite chain X are well studied. But the situation is different if X is an infinite chain. If X is an infinite chain then rank of the monoid OP(X) is not more finite and we consider the concept of the relative rank, which was introduced by Ruskuc for such situations. It is already that known, that the relative rank of OP(X) modulo O(X) is two if X has neither minimum nor maximum. In that paper, we proof that the relative rank of OP(X) modulo O(X) is one if the set X has a maximum or a minimum. Moreover, we determine each possible element that we can add to O(X) in order to obtain a generating set for OP(X).

(4) Dimitrova I., V. H. Fernandes, **J. Koppitz**, T. M. Quinteiro. Partial automorphisms and injective partial endomorphisms of a finite undirected path. Semigroup Forum, 103, 1, Springer, 2021, ISSN:0037-1912, DOI:10.1007/s00233-021-10193-y, 87-105.
Impact factor: 0.768
Zbl 1467.05137

Abstract

As well as automorphisms of graphs allow one to establish natural connections between Graph Theory and Group Theory, endomorphisms of graphs allow similar connections between Graph Theory and Semigroup Theory. Likewise, in particular, partial automorphisms of graphs relate Graph Theory with Inverse Semigroup Theory. In this paper, we study partial automorphisms and, more generally, injective partial endomorphisms of a finite undirected path from Semigroup Theory perspective. Our main objective is to give formulas for the ranks of the monoids $\mathrm{IEnd}(P_n)$ and $\mathrm{PAut}(P_n)$ of all injective partial endomorphisms and of all partial automorphisms of the undirected path P_n with n vertices. We also describe Green's relations of $\mathrm{PAut}(P_n)$ and $\mathrm{IEnd}(P_n)$ and calculate their cardinals. First, we calculate the rank of the monoid $\mathrm{IEnd}(P_n)$ and provide a generating set of minimal size. Based on the fact that $\mathrm{PAut}(P_n)$ is a submonoid of $\mathrm{IEnd}(P_n)$, using our results for $\mathrm{IEnd}(P_n)$, we calculate the rank of $\mathrm{PAut}(P_n)$ and provide also a generating set of minimal size for $\mathrm{PAut}(P_n)$.

(5) Phusanga D., J. Joomwong, S. Jino, **J. Koppitz**. All idempotent and regular elements in the monoid of generalized hypersubstitutions for algebraic systems of type (2; 2). Asian-European Journal of Mathematics, 14, 02, World Scientific, 2021, ISSN:1793-5571, DOI:10.1142/S1793557121500157, 2150015-1-2150015-9.
Impact factor: 0.294
Zbl 1477.08007

Abstract

We consider algebraic system in sense of Malcev as a triple (A;F,R), where A is a nonempty set, F a set of operations on A, and R a set of relations on A. The concept of hypersubstitutions was introduced for the classification of varieties. We want to transfer this idea from universal algebras to algebraic system. There are two different concepts for hypersubstitutions for algebraic systems. In this paper, we follow the more natural and

practicable one. On the other hand, we have the concept of generalized hypersubstitutions. The set of all hypersubstitutions for algebraic systems forms a monoid under to composition of functions. Following both ideas, one obtains the concept of a monoid of generalized hypersubstitutions for algebraic systems in a canonical way. The purpose of this paper is the study of that monoid. We characterize the idempotent as well as regular elements in this monoid.

(6) Anantayasethi A., **J. Koppitz**. The Algebraic Structure of a Semigroup of Sets of Transformations with Restricted Range. Thai Journal of Mathematics, 18, 4, 2020, ISSN:1686-0209, 1701-1713.
Impact factor: 0.179
Zbl 1491.20125

Abstract

We study a semigroup, which represents a semigroup of sets of Boolean functions on a finite set using the concept of transformations with restricted range. We consider the semigroup T(X,Y) of all transformations with image in Y, where Y is an two-element set. The set of all non-empty subsets of T(X,Y) forms a semigroup under the complex-product of sets. For this semigroup, we determine the algebraic structure. In particular, we characterize the (left, right, and two-sided) ideals and the Green's relations. Moreover, for each of the Green's relations, we provide the greatest included congruence.

(7) Dimitrova I., V. H. Fernandes, **J. Koppitz**, T. M Quinteiro. Ranks of Monoids of Endomorphisms of a Finite Undirected Path. Bulletin of the Malaysian Mathematical Sciences Society, 43, 2, Springer, 2020, ISSN:0126-6705, DOI:https://doi.org/10.1007/s40840-019-00762-4, 1623-1645.
Impact factor: 0.856
Zbl 1434.05081

Abstract

In this paper, we focus our attention on a very important invariant of a semigroup or a monoid, which has been the subject of intensive research in semigroup theory. We are referring to the rank, i.e., to the least number of generators of a semigroup or a monoid S. The main aim is to determine the ranks of the both monoids wEnd P_n and End P_n of all weak endomorphisms and all endomorphisms, respectively, of the undirected path P_n with n vertices. We also calculate the cardinality of wEnd P_n as well as End P_n and characterize the regular elements in these both monoids. Moreover, we determine the set Aut(P_n) of all automorphisms on P_n and show that Aut(P_n) has rank 1. Similar results, we obtain for the monoids of strong and strong weak, respectively, endomorphisms.

(8) Zhuchok A.V., Yu.V. Zhuchok, **J. Koppitz**. Free rectangular doppelsemigroups. Journal of Algebra and Its Applications, 19, 11, World Scientific Publishing Company, 2020, ISSN:0219-4988, DOI:doi.org/10.1142/S0219498820502059
Impact factor: 0.61
Zbl 1454.08010

Abstract

A doppelsemigroup is a nonempty set equipped with two binary associative operations satisfying certain identities. Doppelsemigroups are natural generalization of semigroups. In this paper, we consider the variety of rectangular doppelsemigroups which are analogues of rectangular semigroups. Every variety contains free algebras and free objects in any variety of algebras are important in the study of that variety. We construct the free rectangular doppelsemigroup and characterize the least rectangular congruence on the free doppelsemigroup. As a consequence, the free rectangular semigroup is presented. We also describe all (maximal) subdoppelsemigroups, all idempotents and all endomorphisms of the free rectangular doppelsemigroup, and give a criterion for an isomorphism of endomorphism semigroups of free rectangular doppelsemigroups. In addition, we show that the endomorphism semigroup of the free rectangular doppelsemigroup is not regular in general.

(9) Fernandes V., **J. Koppitz**, T. Musunthia. The Rank of the Semigroup of All Order Preserving Transformations on a Finite Fence. Bulletin of the Malaysian Mathematical Sciences Society, 42, 5, Springer, 2019, ISSN:0126-6705, DOI:10.1007/s40840-017-0598-1, 2191-2211. **Impact factor: 0.867**
Zbl 1454.20110

Abstract

A zig-zag order is a special partial order on a (finite) set. In this paper, we consider the semigroup TF_n of all order-preserving transformations on an n-element zig-zag-ordered set (or fence). Transformations preserving the zig-zag order were already considered in several papers by several authors. But none of these papers provides a characterization of the transformations in TF_n. This paper fills that gap. We determine the rank of TF_n and provide a minimal generating set for TF_n. We observe that the formula for the rank differs for n is even or odd. Moreover, a formula for the number of idempotents in TF_n is given.

(10) Zhuchok Y., **J. Koppitz**. Representations of ordered doppelsemigroups by binary relations. Algebra and Discrete Mathematics, 27, 1, Institute of Applied Mathematics And Mechanics of the National Academy of Sciences of Ukraine, 2019, ISSN:1726-3255, 144-154.
Impact factor: 0.241
Zbl 1448.08003

Abstract

We extend the study of doppelsemigroups and introduce the notion of an ordered doppelsemigroup. It is well-known that according to Cayley's theorem for semigroups, every semigroup can be embedded into a semigroup of transformations of a suitable set. For ordered semigroups, Zaretskiy has shown that every ordered semigroup can be embedded into the ordered semigroup of all binary relations on a suitable set. In the present paper, we consider that problem for ordered doppelsemigroups. We give examples of ordered doppelsemigroups and construct the ordered doppelsemigroup of binary relations on an arbitrary nonempty set. The main result of this paper is a representation theorem, which shows that every ordered doppelsemigroup can be embedded into the constructed ordered doppelsemigroup of binary relations on a suitable set. As a consequence, we obtain an analogue of Cayley's theorem for semigroups in the class of doppelsemigroups. We also describe the representations of ordered doppelsemigroups, by binary transitive relations, i.e. we characterize all ordered

doppelsemigroups being isomorphic to some ordered doppelsemigroup of binary transitive relations.

(11) Zhuchok, A.V., **Koppitz, J.**. Free products of n-tuple semigroups (engl.). Ukrainian Mathematical Journal, 70, 11, Springer, 2019, ISSN:0041-5995, DOI:10.1007/s11253-019-01601-2, 1710-1726.
Impact Factor: 0.362
Zbl 1450.20017

Abstract

The concept of a n-tuple semigroup was introduced by Koreshkov as a set together with n binary operations defined on this set satisfying several axioms. It is clear that each semigroup is an n-tuple semigroup for n = 1. However, there are many examples of n-tuple semigroups that are not semigroups. This paper is mainly devoted the study of the free product of n-tuple semigroups. We construct a free product of arbitrary n-tuple semigroups. Further, we introduce the notion of n-bands of n-tuple semigroups and, in terms of this notion, we describe the structure of the free product. We also construct a free commutative n-tuple semigroup of any rank and characterize one-generated free commutative n-tuple semigroups. Moreover, we describe the least commutative congruence on a free n-tuple semigroup and prove that the semigroups of the constructed free commutative n-tuple semigroup are isomorphic and that its automorphism group is isomorphic to a symmetric group. The results in that paper generalize the study of free products of doppelsemigroups.

(12) Worawiset S., **Koppitz, J.**, S. Chotchaisthit. THE CLASS OF ALL SEMIGROUPS RELATED TO SEMIHYPERGROUPS OF ORDER 2. Mathematica Slovaca, 69, 2, De Gruyter, 2019, ISSN:1337-2211, DOI:10.1515/ms-2017-0229, 371-380.
Impact factor: 0.314
Zbl 07093112

Abstract

This paper deals with semihypergroups of order two from the point of view of the model theory. We show that there are exactly 17 non-isomorphic semihypergroups of order two. Each of them corresponds in a canonical way to a semigroup of order three. So we can consider these semihypergroups as semigroups and classify all of them by generalized identities a concept which based on an idea by Lyapin. For a given set I of such generalized identities, the model class Mod(I) is closed under subalgebras and homomorphic images, but not under direct products. We show that there is one generalized identity s such that Mod({s}) is the class all non-group semihypergroups of order three (considered as semigroups).

(13) Anantayasethi, A., **Koppitz, J.**. Relations on a Semigroup of Sets of Transformations with Restricted Range. Comptes rendus de l'Acad´emie bulgare des Sciences, 70, 12, Proceedings of BAS, 2017, ISSN:1310–1331, 1621-1626.
Impact factor: 0.27
Zbl 1399.20074

Abstract

A non-deterministic transformation on a finite set with two-element target can be regarded as a set of transformations with restricted two-element range. For a semigroup (of sets of transformations) having these non-deterministic transformations as subsemigroup, we determine the Green's relations. Moreover, for each of the Green's relations, we provide the greatest included congruence.

(14) Dimitrova, I, **Koppitz, J**, Lohapan, L. Generating Sets of Semigroups of Partial Transformations Preserving a Zig-Zag Order on N. International Journal of Pure and Applied Mathematics, 117, 2, Sofia : Academic Publications, 2017, ISSN:1311-8080, DOI:10.12732/ijpam.v117i2.4, 279-289.
Impact Factor: 0.299

Abstract

This paper contributes to the study of semigroups of transformations on infinite sets. There is only a small number of essential results about the structure of infinite transformation semigroups since the structure depends from the chosen set on which the transformations act. We consider partial transformations on the positive integers **N** preserving the zig-zag order. We study rank properties for the infinite monoid $PF_{\mathbf{N}}$ of all partial transformations on **N** preserving a zig-zag order on **N**. Since the rank $PF_{\mathbf{N}}$ is infinite, we use the concept of relative rank. We determine the relative rank of $PF_{\mathbf{N}}$ modulo a set containing all idempotents and all surjections in $PF_{\mathbf{N}}$. Moreover, we show that all transformations in $PF_{\mathbf{N}}$ with finite rank (i.e. with finite image) can be generated by the idempotents with finite rank and the full transformation γ with infinite rank defined by $\gamma(n)=n+2$ for all positive integer n.

(15) Dimitrova, I, **Koppitz, J**. On the semigroup of all partial fence-preserving injections on a finite set. Journal of Algebra and Its Applications, 18, 12, World Scientific Publishing Company, 2017, ISSN:0219-4988, DOI:10.1142/S0219498817502231, 1750223-1750236.
Impact factor: 0.489
Zbl 1429.20045

Abstract

In this paper, we study a subsemigroup of the symmetric inverse semigroup on an n-element partial ordered set X_n. The partial order under consideration is the so-called zig-zag order, i.e. $X_n = \{1 < 2 > 3 < 4 \ldots n\}$, which is also called fence. We say that a partial injective transformation f is fence-preserving if $x < y$ implies that $xf < yf$, for all x, y in the domain of f. In this paper, we study the semigroup of all partial fence-preserving injections f of X_n , such that f^{-1} is also a partial fence-preserving injections f of X_n. we observe that IF_n is an inverse semigroup containing all regular elements of the semigroup of all partial fence-preserving injections of X_n. We characterize the (remaining) Green's relation **J** for the semigroup IF_n. Further, we prove that the semigroup IF_n is generated by its elements with rank $\geq n-2$. If n is odd then we can observe that IF_n is not generated by the transformations with rank greater than or equal to $n-1$. Moreover, in this case, there is no least generating set for IF_n. But, in the case n is even the situation is different. Then there is a least generating set and all its elements have rank $\geq n-1$ Moreover, we have determined the least generating set, which contains n+1 elements. Finally, we verify that n+1 is the rank of IF_n.

(16) Anantayasethi, Ananya, **Koppitz, Jörg**. On a semigroup of sets of transformations with restricted range.. Thai J. Math., 14, 3, 2016, ISSN:1686-0209, 667-676.
Impact factor: 0.249
Zbl 1364.20043

Abstract

This paper bases on the well-studied semigroup T(X,Y) of all transformations on X with restricted range Y, where Y is a subset of X for the case that |Y | = 2. We introduce the semigroup $T_P(X,Y)$ of all non-empty subsets of T(X,Y) under the complex product of sets. First, we observe that an idempotent element A of $T_P(X,Y)$ is a subsemigroup of T(X,Y) since AA = A. But conversely, not each subsemigroup of T(X,Y) is an idempotent in $T_P(X,Y)$. We determine the idempotent elements as well as the regular elements in $T_P(X,Y)$. We show that there are exactly two a maximal idempotent subsemigroups of $T_P(X,Y)$. Moreover, we prove that there are exactly two maximal regular subsemigroups of $T_P(X,Y)$ and determine the largest subsemigroup of $T_P(X,Y)$ containing all regular elements. We also provide the largest semiband in $T_P(X,Y)$.

(17) Slavcho Shtrakov, **Jörg Koppitz**. Stable varieties of semigroups and groupoids. Algebra Universalis, 75, 1, Springer International Publishing, 2016, ISSN:0002-5240, DOI:10.1007/s00012-015-0359-7, 85-106
Impact factor: 0.536
Zbl 1339.20054

Abstract

The paper deals with Σ-composition and Σ-essential composition of terms which lead to stable and s-stable varieties of algebras. We introduce the inductive, positional, and Σ-composition of terms and apply the concept of Σ-composition of terms to study the stable varieties of semigroups. In this paper all stable varieties of semigroups are described in analogy to the solid varieties, using some fundamental results in semigroup theory. A full description of all stable varieties of semigroups, commutative and idempotent groupoids is obtained. We prove that the varieties of commutative and idempotent groupoids are stable. We present stronger conditions for stability of varieties which successfully work in the variety of groupoids. These conditions allow us to define and study the s-stable varieties of groupoids. We use an abstract reduction system which simplifies the presentations of terms of type $\tau = (2)$ to study the variety of idempotent groupoids and s-stable varieties of groupoids. S-stable varieties are a variation of stable varieties, used to highlight replacement of subterms of a term in a deductive system instead of the usual replacement of variables by terms.