ASSESSMENT OF THE ACADEMIC STANDING OF PD DR. JÖRG KOPPITZ

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1. INTRODUCTION

I have known Dr. Koppitz as a colleague in Mathematics for at least 20 years because we have met in many algebra conferences. I feel honoured to be asked for a report by the Bulgarian Academy of Sciences, and it is a pleasure for me to give an assessment of his academic standing.

2. PUBLICATION ACTIVITY

During the 30 years since his doctoral degree at the university of Potsdam, Germany, Dr. Koppitz has authored and co-authored about 90 publications. More than 20 of these papers have appeared in journals of high reputation: 9 papers in Semigroup Forum, 3 in the International Journal of Algebra and Computation, 4 in Journal of Algebra and its Applications, 3 in Algebra Universalis, 2 in Communications in Algebra, 1 in Acta. Sci. Math. (Szeged).

3. DETAILED REVIEW OF SELECTED PUBLICATIONS

In this section, the reviewer comments on the main results contained in the 17 publications listed by the author for the competition. They will be grouped into sections according to their content. The numbering follows the **List of Scientific Publications**.

3.1. **Equational logic and varieties.**

(17) The authors study varieties of algebras that have the property that replacing any common subterms of s and t in the identity $s \approx t$ with equivalent subterms is a correct way of inferring new identities. It is shown that there exist exactly 10 varieties of semigroups with these properties, and

that the varieties of idempotent and of commutative groupoids are also stable.

(12) The theory of *hyperalgebras* seeks to generalize results for classical algebras to algebras with non-deterministic operations. The authors establish that there are 17 semihypergroups of order 2, and they characterize those semigroups of order 3 that can be constructed from these semihypergroups. The description uses a concept of interest in its own, a description of algebras using *disjunctive identities*, i.e., formulae of the type $\forall \boldsymbol{x} \bigvee_{i \in I} s_i(\boldsymbol{x}) \approx t_i(\boldsymbol{x})$.

(5) A *generalized hypersubstitution* replaces function symbols and relational symbols in a first order formula by terms and relational symbols, respectively. For a fixed first order language, these hypersubstitutions form a monoid. The authors describe idempotent and regular elements of this monoid.

3.2. Structure Theory of Semigroups.

(16) For $Y \subseteq X$ with $|Y| = 2$, the authors dermine the maximal idempotent subsemigroup (and the maximal semiband and the maximal regular subsemigroup) of the semigroup of subsets of Y^X with the operation $F * G = \{f \circ g \mid f \in F, g \in G\}$.

(13) For the semigroups studied in [16], Green's relations are determined.

(6) In addition to the results obtained in [13], left ideals, right ideals, and ideals of the semigroups studied in [16] are determined.

3.3. Algebras with several associative operations: doppelsemigroups, n-tuple semigroups, . . .

(11) The authors study n-tuple semigroups, which have n binary operations and satisfy the law $x * (y \circ z) = (x * y) \circ z$ for each of the n^2 pairs $(*, \circ)$ of these operations. The authors give an explicit construction of of the free product of an arbitrary set $(T_i)_{i \in I}$ of semigroups and of the free commutative n-tuple semigroup over an arbitrary set X.

(10) The authors provide a concrete construction of ordered doppelsemigroups on a set of binary relations and they prove a Cayley-like theorem that

every ordered doppelsemigroup can be embedded into such a doppelsemi-group, and as a corollary that every doppelsemigroup can be embedded into a doppelsemigroup of transformations.

(8) A doppelsemigroup is called *rectangular* when both of its semigroup operations satisfy $xyz \approx xz$. The authors give an explicit construction of the free rectangular doppelsemigroup. As a consequence, the equational theory of rectangular doppelsemigroups is decidable. It is proved that the free rectangular doppelsemigroup on n elements has $n + 4n^2$ elements. It is shown that the two semigroups of the free rectangular doppelsemigroup are isomorphic, and the least congruence of the free doppelsemigroup with rectangular quotient is determined. All subdoppelsemigroups and endomorphisms of the free rectangular doppelsemigroup are described.

3.4. **Semigroups of order preserving functions.**

(15) The author describe Green's relations and the rank of the semigroup of those partial injections on a finite set that preserve a "fence" ordering $1 < 2 > 3 < 4 > \cdots$.

(14) A generating set of the semigroup of partial mappings on $\mathbb{N}$ preserving a fence order is given.

(9) The authors provide an exact formula for the number of transformations needed to generate the full semigroup of order-preserving maps on an n-element fence.

(7) On an n-element path, one can define several monoids of transformations, e.g., endomorphisms, weak endomorphisms (= endomorphisms of the path with loops added) and automorphisms. The minimal number of generators of each of these monoids is determined. Theorem 2.11 provides a formula for the number of weak endomorphisms. (Incidentally, this formula produces exactly the sequence A081113 of the on-line encyclopedia of integer sequences. Is there a combinatorial reason for this fact?)

(4) The cardinality and the rank of the monoid of *partial* automorphisms of an n-element path, and of the monoid of injective partial endomorphisms of this path are determined. Green's relations are determined for these semigroups.

(3) Orientation-preserving mappings are a supersemigroup of order preserving mappings. The authors show that at most 2 orientation-preserving

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mappings need to be added to the order preserving mappings on an infinite densely ordered chain in order to generate all orientation preserving mappings, and describe when one such mapping suffices.

(2) For $Y \subseteq X \subseteq \{1, \ldots, n\}$, the authors determine how many transformations need to be added to the semigroup of order preserving maps from X to Y in order to generate all orientation preserving mappings, and minimal generating sets are determined. It is also shown (with $m := |Y|$, $n := |X|$) that $S(n, m) - \binom{n}{m}$ elements are needed to generate all transformations from the orientation preserving ones.

(1) The authors provide an exact formula for the number of transformations needed to generate the semigroup of *partial automorphisms* on an n-element fence.

3.5. **Assessment of the quality of these publications.** All presented papers contain new mathematical results. The research and presentation are done lege artis. At least 5 of these papers have appeared in journals of high reputation.

The three papers on equational logic and varieties address different questions. The first paper presents an extension of equational logic that allows additional inferences. The intuition behind these inferences is not discussed, but it is shown that in certain varieties of semigroups, the additional inference rules are correct. The paper (12) on hyperalgebras describes classes of algebras by disjunctive identities. The power of such identities is an interesting question; however, the scope of the paper (2 and 3 element semigroups and hypersemigroups on 2 elements) is still somewhat limited. *Hypersubstitutions*, which are addressed in the third paper (5), might be a tool for investigating Mal'cev conditions. In this paper, they are treated mainly as a source of semigroups, but it is not discussed which insights idempotency or regularity of a hypersubstitution can provide.

The papers on the Structure Theory of Semigroups determine the structure of certain semigroups of functions along the lines of general semigroup theory. One such class of semigroups is studied in the three papers (16),(13),(6), and many questions on their structure are answered in a satisfactory way. Eight of the presented papers investigate semigroups of order preserving functions. In addition to the standard semigroup theoretic properties (Green's relations), much work is done on finding minimal generating sets of these semigroups, their order and

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their rank. This includes a detailed study of the combinatorial structure of these mappings, and produces several beautiful formulae for rank and order.

Together with A.V.Zhuchok and Y.V.Zhuchok, Koppitz has investigated algebras with several associative operations and give concrete constructions of the free algebras in several varieties of this type. The authors' constructions are much more concrete than the general universal algebraic ones, and indeed often provide a way to decide the equational theory of such varieties. This research requires and provides substantial insight into the inner working of free structures and universal algebra in general, and in particular into the structure of doppelsemigroups and related structures.

In total, the quantity of Koppitz's research results is impressive. A main theme in his research questions is: take a concrete semigroup of mappings, and determine its structural properties (rank, order, Green); 12 of the 17 papers follow this program. These are very suitable problems for beginning researchers, and in fact, 13 of the presented papers are co-authored with scientists of much less experience, and it is safe to assume that Koppitz had the role of an advisor in this research. The quality of the research results is good, but the scope is not exceptionally broad: still, the 17 presented papers have been cited several times in papers in prominent journals such as, e.g., the Journal of Algebra and Semigroup Forum.

4. Scholar Activities

4.1. **PhD-students.** Dr. Koppitz has successfully advised 7 PhD-students coming from several different countries.

4.2. **Talks.** Many (> 50) talks at international conferences, **4 invited talks**. Invited lecturer in Blagoevgrad (Bulgaria), Brno (Czech Republic), Lisbon (Portugal), Luhansk (Ukraine), Szeged (Hungary).

4.3. **International scientific collaboration.** Research visits in Blagoevgrad (Bulgaria), Lethbridge (Canada), Lisbon (Portugal). Managing editor of 2 international journals.

4.4. **Service.** Co-organization and organization of 8 conferences. The reviewer has attended at least 2 of them and was pleased by Dr. Koppitz's preparation of these meetings.

5. Conclusion

Dr. Koppitz is a very active member of the algebraic community who successfully contributes to many aspects of scientific work. He has advised several international students, takes editorial duties, organizes conferences and is in good contact with many researchers all over the world. He produces a large amount of sound research, often in collaboration with much less experienced or younger researchers. I was particularly impressed by the results on finite transformations of algebraic and combinatorial nature in which he characterizes generators of certain order preserving functions and obtains closed formulae for their ranks, and by the skillful construction of concrete representations of free objects in his joint works with A.V. and Y.V.Zhuchok. It is to be expected that Koppitz will produce a continuous output of research publications of good quality also during the next years.

From these considerations, I can fully recommend Dr. Koppitz for a professorship in the Bulgarian Academy of Sciences.

Linz, March 20th, 2023 Assoz.Univ.-Prof. Dipl.-Ing. Dr.techn. Erhard Aichinger