R E V I E W

on a Dissertation

for obtaining the educational and scientific degree "Doctor"

Research area: 4. Natural Sciences, Mathematics, and Informatics,

Professional field: 4.5. Mathematics

Author: Paskal Nikolaev Piperkov

Title: Discrete Transforms and Their Applications in Coding Theory and Combinatorics

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I am a member of the scientific panel for this procedure according to order No. 159/27.06.2022 of the Director of Institute of Mathematics and Informatics, Bulgarian Academy of Sciences. As a member of the scientific panel, I have received all the administrative and scientific documents required by the Act on the Development of the Academic Staff in the Republic of Bulgaria (ADASRB), the Rules for its implementation and the Rules on the terms and conditions for awarding of academic degrees and occupying of academic positions at IMI and BAS.

1. General characterization of the materials.

The dissertation developed methods for efficient calculation of weight distribution and covering radius of linear codes through fast discrete transformations based on the characteristic vector of a generating (parity-check) matrix. The weight spectrum and covering radius are important parameters for linear codes, and finding them using a given generating matrix is an important and computationally difficult task. The approaches considered so far are based on exhaustive algorithms. The presented methods are of a different nature and can be considered as a supplement and extension of Karpovski's development from the 1980s.

One of the significant advantages of this approach is that it does not depend on the length of the code when a characteristic vector is available.

2. **Personal data.**

Paskal Piperkov graduated in mathematics at Sofia University "St. Kliment Ohridski" in 1997. Between 2003 and 2016 he was an assistant professor in Calculus in Veliko Tarnovo University. The 2018 he was enrolled in the PhD program in Algebra and Number Thory at the Department of Mathematical Foundations of Informatics of Institute of Mathematics and Informatics. Since 2016 he has been a mathematician in the Institute.

I know Paskal Piperkov from his work in Veliko Tarnovo University and the organization of mathematical Olympiads and competitions. In my opinion, his multifaceted interests, also indicated in his autobiography, were not always in favor of developing his outstanding mathematical abilities in research work.

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3. **General characterization of the dissertation.**

The dissertation is characterized by completeness, depth and comprehensiveness. The research presented in the dissertation can be conditionally divided into two groups. The first group concerns finding the weight spectrum of a linear code. The second group is for covering radius.

Below I will give a brief summary of the chapters.

The **first chapter** is introductory. Along with the introduction of the necessary concepts, it also presents additional overview information that motivates the research. Information is presented on the nature of fast discrete Fourier transforms, as well as the Walsh-Hadamard and Vilenkin-Chrestenson transforms themselves. The theoretical basis is also described, according to which each element of a recurrent series of matrices defined by a Kronecker product can be considered as a product of matrices with a small number of nonzero elements. Fast discrete transforms of the considered type are also based on this fact. They are a multiplication of a transformation vector by a matrix. This chapter also defines the objectives of the dissertation. They can be considered as an extension of some of the known fast discrete transforms to transforms in which the matrices in the recurrence series are not pure Kronecker products. This, on the one hand, complicates the implementation of the

transformations, but on the other hand, it drastically increases the efficiency due to a reduction in the size of the matrices.

The **second chapter** is devoted to a method for finding the weight spectrum of a linear code over a non-binary prime field. The product of the normalized matrix of the simplex code by the characteristic vector of an arbitrary linear code gives a vector with the weights of all codewords. The main question here is what recurrent series of dimension-dependent generating matrices of the simplex code should be chosen so that its multiplication by a characteristic vector can be represented by a butterfly diagram. Additional concepts such as characteristic distribution and shortened characteristic distribution are introduced here. Propositions that are the basis of the proof of the correctness of the developed algorithm are proved. The complexity of this algorithm is also calculated.

In the **third chapter**, the issue of weight spectrum of a linear code over a composite finite field is considered. Each such code can be uniquely mapped to a code above the corresponding prime subfield. In this code, there are codewords of equal weight that correspond to the proportional words in the original code. To avoid this repetition, an algorithm was developed which is based on the algorithm from Chapter 2 and the transformation of traces. In the last part, a detailed algorithm for codes over fields with characteristic 2 is presented and its complexity is calculated.

In the **fourth chapter**, an algorithm for calculating the covering radius of a linear code over a non-binary field is presented. On the one hand, it is an extension of the algorithm for binary codes, but on the other hand, it can be considered as an analytical representation of the well-known covering radius algorithm by using linear combinations of its coordinates. The advantage of this algorithm is similar to advantages of the algorithm from Chapter 2.

The dissertation is very substantial, precise, with significant scientific contributions, which are detailed and correctly described in the "Scientific Contributions" section.

I would also like to point out the following other positives: most algorithms have detailed examples that help the reading and understanding. At the end of each chapter there is a commentary that gives insight into the development and approbation of the problems discussed in the chapter.

4. Publications and citations

Paskal Piperkov's dissertation is based on four publications. All of them are written in English and published in international scientific journals and in proceedings of international conferences. One of the papers is published in *Cryptography and Communications* JCR-IF

1.376. Another two publications are included in books of the prestigious academic publisher *World Scientific*. Paskal Piperkov has presented a list of two citations.

All publications are co-authored, and the PhD student's contribution is undisputed and can be considered to be equal to that of the other authors.

5. The author's summary

The author's summary is made in accordance with the rules and accurately and completely assesses the main results obtained in the thesis.

6. Conclusion

The presented dissertation satisfies all the criteria and indicators of the law and the regulations in Bulgaria. After I familiarized myself with the presented dissertation, the importance of the research and the scientific and applied contributions contained therein, I give an overall positive assessment to the applicant **Paskal Nikolaev Piperkov** to obtain the scientific degree "Doctor" in

Research area: 4. Natural Sciences, Mathematics and Informatics,
Professional field: 4.5 Mathematics.

17.08.2022 г.   Signature:

/Prof. D.Sci. Iliya Bouyukliev/