S T A N D P O I N T

for occupation of an Associate Professor position

in Professional Direction 4.5 Mathematics, Algebra and Number Theory

at the Institute of Mathematics and Informatics,

Bulgarian Academy of Sciences

Order N 217/14.12.2020 appoints me as a member of the Scientific Jury for the competition on obtaining an Associate Professor position in Mathematics, Algebra and Number Theory at the Institute of Mathematics and Informatics (IMI) at the Bulgarian Academy of Sciences (BAS), which is announced in issue 89/16.10.2020 of the State Newspaper. The only applicant for the aforementioned position is D.Sc. Peter Vasilev Danchev. As a Ph.D. in Mathematics from 2018 and a D. Sc. in Mathematics from 2020, who has worked at IMI-BAN at least two years, he is eligible for an Associate Professor position, according to the Law on Development of Academic Personnel of Republic Bulgaria (LDAPRB). D.Sc. Peter Danchev applies for the current competition with articles on group algebras RG of abelian groups G over commutative rings R with unity. Here is a more detailed discussion on the fulfillment of the specific criteria of LDAPRB, the Rules on its implementation, Decree 26/13.02.2019 on the amendments of the Rules of implementation of LDAPRB, as well as the Rules on the terms and conditions for acquisition of academic degrees and occupation of academic positions at BAS and IMI-BAS.

As a substitute of a Habilitation Thesis, D.Sc. Peter Danchev provides six articles, which earn him 104 points versus the required 100 ones. These articles derive necessary and sufficient conditions for various types of "cleanness"of group rings RG in terms of the properties of R and G. Four of them are in journals with SJR and two are in refereed and indexed journals. Four of the aforementioned papers have appeared in 2019 and two in 2020. That explains the lack of citations of these six papers. However, the remaining 17 papers, provided for the competition have 24 citations. Moreover, an article of D.Sc. Danchev and Mc Govern from Journal of Algebra, 2015, whose topic (weakly nil clean RG) is close to the area of the six substitutes of a Habilitation Thesis, has been cited four times - twice in 2016, once in 2017 and once in 2020. That is an evidence for the vitality of the study of the weakly nil clean commutative rings S with unity, i.e., of the ones with $S = N(S) + \mathrm{id}(S)$ for the nil-radical $N(S)$ of S and the set $\mathrm{id}(S) := \{e \in S \,|\, e^2 = e\}$ of the idempotents of S. It can be viewed as a promise for future citations of the substitutes of a Habilitation Thesis of D.Sc. Danchev.

The characterization of the various "cleanness"conditions on a group algebra RG in terms of R and G yields strong restrictions on the commutative ring R and the abelian group G. Namely, if for any $x \in RG$ there exist $n \in N(R)$ and $e, f \in \mathrm{id}(R)$ with $x = n + e - f$ and RG is called feebly nil clean then $G = G_2$ is 2-torsion or $G = G_3$ is 3-torsion or $G = G_3 \times G[2]$ for the 2-socle $G[2] := \{g \in G \,|\, g^2 = e_G\}$. A ring R is π-regular if for any $r \in R$ there exist $i \in \mathbb{N}$ and $a \in R$ with $r^i = r^i a r^i$. D.Sc. Danchev shows that RG is π-regular if and only if R is π-regular and $G = G_t := \coprod\limits_p G_p$ is torsion. Feebly invo-clean rings R are the ones, whose entries $r \in R$ can be written as $r = v + e - f$ for some involution $v \in U(R)$, $v^2 = 1_R$ and some idempotents $e, f \in \mathrm{id}(R)$. The applicant proves that if RG is feebly invo-clean then $G^4 = \{e_G\}$ for the subgroup $G^4 := \{g^4 \,|\, g \in G\}$ of G and the neutral element $e_G \in G$. If RG is weakly invo-clean, i.e., if any $x \in RG$ is of the form $x = v + e$ or $x = v - e$ for some involution v and some idempotent $e \in \mathrm{id}(R)$, then $G^2 = \{e_G\}$. A ring R is weakly tripotent if for any $r \in R$ there holds $r^3 = r$ or $(1 - r)^3 = 1 - r$. D.Sc. Danchev establishes that RG is weakly tripotent only when $G^2 = \{e_G\}$. A ring R is periodic if for any $r \in R$ there exist different natural numbers m, n

with $r^m = r^n$. A group algebra RG turns to be a periodic ring if and only if R is periodic and $G = G_t$ is torsion.

Except by the six substitutes of a Habilitation Thesis, D.Sc. Peter Danchev applies for the Associate Professor position by 17 other articles, which appeared between 1997 and 2012. Four of them are published in scientific journals with IF and 13 in refereed and indexed journals. The aforementioned 17 articles earn 238 points, while the rules of BAS and IMI-BAS require 220 ones.

Three of the aforementioned 17 articles concern the σ-summability and the summability of an abelian p-group. Two of these three articles are in journals with IF. In order to formulate precisely, let us denote by $U(RG)$ the multiplicative group of RG, put $\Delta_R G$ for the augmentation ideal of RG and consider the group $V(RG) := U(RG) \cap (1 + \Delta_R G)$ of the normalized units of RG. An abelian p-group G of length $l_p(G)$ is σ-summable if its p-socle can be represented as an increasing union $G[p] = \cup_{n \in \mathbb{N}} S_n$ of subgroups $S_n \leq S_{n+1}$, such that for any $n \in \mathbb{N}$ there is an ordinal $\alpha_n < l_p(G)$ with $S_n \cap G^{p^{\alpha_n}} = \{e_G\}$. An abelian p-group G of length $l_p(G)$ is reduced if $G^{p^{l_p(G)}} = \{e_G\}$. A p-group is p^α-projective for an ordinal α if for any abelian group K the group $\mathrm{Ext}^1(G/G^{p^\alpha}, K)$ of the equivalence classes of the extensions of G/G^{p^α} by K has $\left[\mathrm{Ext}^1(G/G^{p^\alpha}, K)\right]^{p^\alpha} = \{0\}$. A p-group G is totally projective if it is p^α-projective for all ordinals α. An article of D.Sc. Peter Danchev from the Proceedings of the American Mathematical Society, 1997 establishes that if R is of prime characteristic $\mathrm{char}(R) = p$ and $N(R) = \{0_R\}$ then the Sylow p-subgroup $V_p(RG)$ of $V(RG)$ is σ-summable if and only if the p-component G_p of G is σ-summable. When G_p is σ-summable and of limit p-length $l_p(G)$, the group $V_p(RG)/G_p$ is σ-summable. If the group ring $RG \simeq RH$ of G is isomorphic to the group ring RH of a group H as an R-algebra then for any ordinal α the group rings $R\left(G/G_p^{p^\alpha}\right) \simeq R\left(H/H_p^{p^\alpha}\right)$ are isomorphic as R-algebras. Moreover, if G has a totally projective p-component G_p of countable p-length $l_p(G_p) = \omega$ and the group rings $RG \simeq RH$ are isomorphic as R-algebras then the p-components $G_p \simeq H_p$ are isomorphic groups.

Let us recall that a commutative ring R is perfect if any non-empty set of principal ideals has a minimal element. A p-primary group G of length $l_p(G)$ is summable if its p-socle $G[p] = \oplus_{\alpha < l_p(G)} S_\alpha$ is a direct sum of subgroups $S_\alpha \leq G^{p^\alpha}$, $\alpha < l_p(G)$ with $S_\alpha \cap G^{p^{\alpha+1}} = \{e_G\}$. An article from Communications in Algebra, 2007 shows that if R is a perfect commutative domain with unity and G is an abelian group with $l_p(G) = \omega$ then $V_p(RG)$ is summable if and only if G_p is summable. If G_p is summable and $l_p(G) = \omega$ then $V_p(RG)$ is shown to decompose into a direct sum of G_p and countable subgroups. Let F be a field of prime $\mathrm{char}(F) = p$ and G_p be a totally projective p-group of $l_p(G) = \omega$, which splits as a direct summand of G. When the quotient G_t/G_p is finite then the group rings $FG \simeq FH$ are isomorphic as F-algebras if and only if H_p splits as a direct summand of H, $H_p \simeq G_p$, $H/H_t \simeq G/G_t$, $|H_t/H_p| = |G_t/G_p|$ and $\left|(H_t)^{q^i}/H_p\right| = \left|(G_t)^{q^i}/G_p\right|$ for all the primes $q \neq p$ and all $i \in \mathbb{Z}^{\geq 0}$, for which different primitive q^i-th roots $\eta, \zeta \in \mathbb{C}$ of $1 \in \mathbb{C}$ provide different extensions $F(\eta) \neq F(\zeta)$.

Recall that an abelian p-group G is a C_λ-group for some ordinal λ if G/G^{p^α} is totally projective for any ordinal $\alpha < \lambda$. An article from Annales Mathematiques Blaise Pascal, 2008 studies the abelian p-groups G, whose p-length $l_p(G) \leq \omega_1$ does not exceed the first uncountable ordinal ω_1. It shows that if G^{p^α} and G/G^{p^α} are summable for some ordinal α then G is summable. Moreover, if G is a $C_{l_p(G)}$-group and H is an unbounded fully invariant subgroup of G then H is summable if and only if G is summable.

Recall that a subgroup H of an abelian group G is pure if for any $h \in H$ and $n \in \mathbb{N}$ the solvability of the equation $x^n = h$ in G implies its solvability in H. A pure subgroup B of a group G is p-basic if B is a direct sum of cyclic groups of order p^n and infinite cyclic groups, whose quotient G/B is p-divisible. A fully invariant subgroup L of an abelian p-group G is large if $L + B = G$ for any p-basic subgroup B of G. An article of D.Sc. Peter Danchev from the Proceedings of Indian Academy of Sciences, 2004 establishes that certain properties $\mathcal{P}$ of abelian p-groups G hold exactly when they are true for a large subgroup L of G. The properties $\mathcal{P}$ include

$p^{\omega+1}$-projectivity for the minimal infinite ordinal ω, summability, being a C_λ-group and others.

An article of D.Sc. Peter Danchev from Radivi Matematicki, 2004 discusses the maximal divisible subgroup $[V_p(RG)/G_p]_d$ of $V_p(RG)/G_p$. More precisely, let R be a commutative ring with unity of prime characteristic $\mathrm{char}(R) = p$, R_{pd} be the p-divisible subring of R, G be an abelian group with p-primary torsion subgroup $G_t = G_p$, G_{pd} be the maximal p-divisible subgroup of G and G_d be the maximal divisible subgroup of G. The work shows that $[V_p(RG)/G_p]_d \simeq V_p(R_{pd}G_{pd})/(G_{pd})_p$. For a field F of prime characteristic $\mathrm{char}(F) = p$ with maximal perfect subfield F_d, there hold $[V(FG)/G]_d \simeq V_p(F_dG_{pd})/(G_{pd})_p$ and $V(FG)_d \simeq G_d \times [V(FG)/G]_d$.

Warfield invariants $W_{\alpha,p}(G)$ of an abelian group G with respect to an ordinal number α and a prime integer p are defined as $W_{\alpha,p}(G) := \log_p \left| G^{p^\alpha}/(G^{p^{\alpha+1}}G_p^{p^\alpha}) \right|$ for finite $G^{p^\alpha}/(G^{p^{\alpha+1}}G_p^{p^\alpha})$ and $W_{\alpha,p}(G) := \left| G^{p^\alpha}/(G^{p^{\alpha+1}}G_p^{p^\alpha}) \right|$ otherwise. An article from Extracta Mathematicae from 2005 relates the Warfield invariants $W_{\alpha,p}(V(RG))$ of the group $V(RG)$ of the normalized units of RG to the Warfield invariants $W_{\alpha,p}(G)$ of G and the cardinality $|R|$ of R, whenever R is a perfect commutative domain with unity of $\mathrm{char}(R) = p$. In the case of a p-primary torsion subgroup $G_t = G_p$ of G and $\chi_0 \leq |R| \leq W_{\alpha,p}(G)$, there follows $W_{\alpha,p}(V(RG)) = W_{\alpha,p}(G)$ for all ordinal numbers α.

The p-height of an element g of a p-group G is the maximal ordinal number $\alpha = H_p(g)$, for which G^{p^α} contains g. If such an ordinal α does not exist then $g \in G$ is of infinite p-height $H_p(g) = \infty$. An abelian p-group G is separable if all $g \in G$ are of finite p-height. An article of D.Sc. Peter Danchev from Archivum Mathematicum (Brno), 2006 shows that a separable p-primary group G is $p^{\omega+n}$-projective for some $n \in \mathbb{N}$ exactly when some pure subgroup H with countable quotient G/H is $p^{\omega+n}$-projective.

An article from Analele Universitatii Bucuresti, Matematica, 2008 is announced to establish necessary and sufficient conditions for $V(RG) = GV_p(RG)$ but the text of the article is not provided by the applicant.

Two of the supplementary articles prove necessary and sufficient conditions for the normalized units $V(RG) = G$ of RG to be depleted by G. More precisely, let $\mathrm{supp}(G)$ be the set of those prime integers p, for which the p-component $G_p \neq \{e_G\}$ of G is non-trivial. Denote by $\mathrm{inv}(R)$ the set of the primes p, for which $p.1_R \in U(R)$ is invertible in R and put $\mathrm{zd}(R)$ for the set of the primes p with $pr = 0_R$ for some $0_R \neq r \in R$. In an article from Extracta Mathematicae, 2008 D.Sc. Peter Danchev shows that if $\mathrm{supp}(G) \cap \mathrm{inv}(R) \neq \emptyset$ or if $\mathrm{char}(R) = p$ divides the orders of all the elements of G then $V(RG) = G \neq \{e_G\}$ exactly when $\mathrm{id}(R) = \{0_R, 1_R\}$ and either $|G| = |R| = 2$ or $|G| = |U(R)| = 2$. In a work from the same journal Extracta Mathematicae, 2009 is proved that the equality $V(RG) = G$ holds if and only if R has $N(R) = \{0_R\}$, $\mathrm{id}(R) = \{0_R, 1_R\}$, the torsion subgroup G_t of G satisfies $V(RG_t) = G_t$ and either $G = G_t$ is a torsion group or $\mathrm{supp}(G) \cap [\mathrm{inv}(R) \cup \mathrm{zd}(R)] = \emptyset$. In particular, if R is the integers ring of a number field L then $V(RG) = G \neq \{e_G\}$ is shown to hold only in the following four cases: (i) G is torsion free; (ii) the torsion subgroup G_t of G has exponent 2 and $L = \mathbb{Q}$ or $L = \mathbb{Q}(\sqrt{-d})$ for a square-free $d \in \mathbb{N}$; (iii) the torsion subgroup G_t of G has exponent 3 or 6 and $R = \mathbb{Z}$ or $R = \mathbb{Z}\left[e^{\frac{\pi i}{3}}\right]$; (iv) the torsion subgroup G_t of G has exponent 4 and $R = \mathbb{Z}$ or $R = \mathbb{Z}[\sqrt{-1}]$.

The next two articles under consideration characterize the group algebras RG, whose group $V(RG)$ of normalized units is depleted by its subgroup

$$\mathrm{Id}(RG) := \left\{ \sum_{i=1}^{k} e_i g_i \,\middle|\, e_i \in \mathrm{id}(R),\, g_i \in G,\, \sum_{i=1}^{k} e_i = 1_R,\, e_i e_j = 0, \forall 1 \leq i \neq j \leq k \right\}$$

of the idempotent units. In an article from Kochi Journal of Mathematics, 2009 is shown that if R is of prime $\mathrm{char}(R) = p$ then $V(RG) = \mathrm{Id}(RG)$ holds only in the following four cases: (1) G is torsion free; (2) char (R) = |G| =2 and R is a boolean ring, i.e., $r^2 = r$ for all $r \in R$;

(3) $|G| = 2$ and $2r - 1_R \in U(R)$ for some $r \in R$ is equivalent to $r^2 = r$; (4) $|G| = 3$ and $1 + 3r^2 + 3f^2 + 3rf - 3r - 3f \in U(R)$ for some $r, f \in R$ if and only if $r^2 = r$, $f^2 = f$, $rf = 0_R$. An article from Communications in Algebra, 2010 generalizes the aforementioned result to group algebras over commutative rings R of arbitrary characteristic. It establishes that $V(RG) = \mathrm{Id}(RG)$ if and only if $V(RG_t) = \mathrm{Id}(RG_t)$, $N(R) = \{0_R\}$ and either $G = G_t$ is a torsion abelian group or $\mathrm{supp}(G) \cap [\mathrm{inv}(R) \cup \mathrm{zd}(R)] = \emptyset$.

An article from Journal of the Calcutta Mathematical Society, 2010 proves that if R is of prime $\mathrm{char}(R) = p$ then $V(RG) = \mathrm{Id}(RG)V(RG_t)$ exactly when $G = G_t$ is a torsion abelian group or G is a non-trivial torsion free group and $N(R) = \{0_R\}$. In a similar vein, a work from the Bulletin of the Calcutta Mathematical Society, 2011 shows that if R is of $\mathrm{char}(R) = p$ then the condition $V(RG) = \mathrm{Id}(RG)V(RG)_t \neq V(RG)_t$ holds if and only if the torsion part $G_t = G_p$ of G is p-primary.

Two of the articles of D.Sc. Peter Danchev derive some properties of the quasi-complete abelian p-groups under the assumption that there holds Continuum Hypothesis $\chi_1 = 2^{\chi_0}$. A separable abelian p-group G is a Q-group if for any infinite subgroup $H \leq G$ the first Ulm group $(G/H)^{p^\omega}$ of G/H is of cardinality $\left|(G/H)^{p^\omega}\right| \leq |G|$. A reduced abelian p-group G is quasi-complete if for any pure subgroup $H \leq G$ the first Ulm group $(G/H)^{p^\omega}$ of G/H is divisible. An article from Владикавказкий математический журнал, 2008 shows that if $\chi_1 = 2^{\chi_0}$ and G is a quasi-complete abelian p-group and a Q-group then G is bounded. A subgroup F of an abelian p-group G is nice if $(G/F)^{p^\alpha} = \langle G^{p^\alpha}, F\rangle/F$ for all ordinals α. A separable abelian p-group G is weakly χ_1-separable if any countable subgroup $H \leq G$ can be embedded in a countable pure and nice subgroup F of G. A work of D.Sc. Peter Danchev from Владикавказкий математический журнал, 2009 establishes that if $\chi_1 = 2^{\chi_0}$ then any quasi-complete weakly χ_1-separable abelian p-group G is bounded.

The last article under discussion is a joint work with B. Goldsmith. It is published in Communications in Algebra, 2012 and discusses the projectively socle-regular abelian p-groups. Let us recall that an abelian p-group G is socle-regular if for any fully invariant subgroup $H \leq G$ there exists such an ordinal $\alpha = \alpha(H)$ that the p-socles $H[p] = G^{p^\alpha}[p]$ of H and G^{p^α} coincide. In a similar vein, G is strongly socle-regular if for any characteristic subgroup $H \leq G$ there exists an ordinal $\alpha = \alpha(H)$ with $H[p] = G^{p^\alpha}[p]$. A subgroup H of an abelian p-group G is projection invariant if $\pi(H) \leq H$ for all homomorphisms $\pi : G \to G$ with $\pi^2 = \pi$. An abelian p-group G is called projectively socle-regular if for any projection invariant subgroup $H \leq G$ there exists an ordinal $\alpha = \alpha(H)$ with $H[p] = G^{p^\alpha}[p]$. The article provides sufficient conditions on G^{p^α} and G/G^{p^α} for G to be projectively socle-regular. Namely, if G^{p^ω} is projectively socle-regular and G/G^{p^ω} is a direct sum of cyclic groups then G is projectively socle-regular. Let us assume that G/G^{p^α} is totally projective. In the case of $\alpha < \omega^2$ the projective socle-regularity of G^{p^α} implies the projective socle-regularity of G. In general, the separability of G^{p^α} suffices for the projective socle-regularity of G. The projective socle-regularity of an abelian p-group G is shown to be inherited by all large subgroups L of G, as well as by the projection invariant subgroups $H \leq G$ with the same first Ulm group $H^{p^\omega} = G^{p^\omega}$ as G. The socle-regularity of an abelian p-group G turns to be equivalent to the socle regularity of its direct powers $G^{(\kappa)}$, $\kappa > 1$, as well as to the strong socle-regularity of $G^{(\kappa)}$, $\kappa > 1$ and to the projective socle-regularity of $G^{(\kappa)}$, $\kappa > 1$.

Four of the six articles of D.Sc. Peter Danchev, substituting a Habilitation Thesis are in journals with SJR. Four of the remaining 17 articles are in journals with IF. In such a way, the applicant fulfills the requirement of IMI-BAS to have at least five articles in journals with IF or SJR.

The 17 papers of D.Sc. Peter Danchev except the substitutes of a Habilitation Thesis have 24 citations. They earn him 99 points, superseding the required 70 points from citations. Three of these citations appeared within an year after the publication of the corresponding article, two - after two years from publication, two - after four years, one after five years, four after seven years, one after eight years, two after nine years, three after ten years, three after eleven years and three after twelve years. The uniform distribution in time of the citations of D.Sc. Peter Danchev shows that his works were up to date at the time of their appearance and have preserved their significance over the time. Two of the citations of the applicant are in the 2015-edition of the celebrated monograph "Abelian Groups"of Laszlo Fuchs. During 2019 and 2020 the applicant has participated in two scientific projects in the area of his research. Putting this together with the scientometric criteria, met by the articles and the citations of D.Sc. Peter Vasilev Danchev, I conclude that he complies with all the requirements of LDAPRB, the Rules on its implementation, Decree 26/13.02.2019 on the amendments of the Rules of implementation of LDAPRB, as well as the Rules on the terms and conditions for acquisition of academic degrees and occupation of academic positions at BAS and IMI-BAS.

I do not know personally D.Sc. Peter Danchev. His application data for the current Associate Professorship has convinced me that he is an expert on the theory of the abelian groups and their group algebras over commutative rings. D.Sc. Peter Danchev has obtained a lot of interesting and valuable results in this area and has posed modern open problems and conjectures. He has mastered various mathematical techniques and used them successfully for attaining best results and an international recognition of his research. The aforementioned circumstances convinced me that D.Sc. Peter Vasilev Danchev meets all the requirements of the Law for Development of Academic Personnel of Republic Bulgaria, the Rules on its implementation, Decree 26/13.02.2019 on the amendments of the Rules of implementation of LDAPRB, as well as the Rules on the terms and conditions for acquisition of academic degrees and occupation of academic positions at BAS and IMI-BAS. That is why, I assess positively the applicant and strongly recommend the respected Scientific Jury to vote positively on a proposal to the Scientific Council of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences, for granting the academic position Associate Professor at IMI-BAN to D.Sc. Peter Vasilev Danchev.

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