

## **ABSTRACTS OF THE PUBLICATIONS**

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Functions) / State Gazette 52, 2 July 2019

### ***After the Invitation to Bessel Functions,***

Softtraid, Sofia, 2019 (monograph, 116 pp.);

ISBN 978-954-334-216-7

Jordanka Paneva-Konovska

#### **Abstract**

The topics of this book include enumerable families of special functions, namely Bessel functions and their generalizations with two, three and four indices, briefly called Bessel type functions in the book. The present book has to be considered as a monograph, since it treats a specialized topic as well as since it is mostly based on the Authors own works. The book consists of introduction, six chapters and bibliography. Various properties and asymptotic formulae, integration operators and some of their applications are given in the book. The discussed properties allow us to study the convergence of series in such families of special functions, obtaining results as Cauchy-Hadamard, Abel and Tauber theorems. Using Computer algebra system 'Maple', different examples of 3-dimensional graphs of generalized Bessel-Maitland functions are provided. Further, zeros of entire functions represented by integrals, involving Bessel functions are also studied. The distribution of zeros of finite Hankel transforms is obtained. The asymptotic behaviour of zeros of finite Hankel transforms of special kind is established. The investigations are based on a theorem by Hurwitz, who consider appropriate meromorphic functions instead of the entire functions. Finally, using the finite integral Hankel transform helps us to solve a practical problem.

*Comptes rendus de l'Académie bulgare des Sciences* **62** (2009),  
No 2, 161-166

### **Tauberian theorem of Littlewood type for series in Bessel functions of first kind**

Jordanka Paneva-Konovska

#### **Abstract**

A Tauberian theorem of Littlewood type for the summation of divergent series defined by means of Bessel functions of first kind is proved.

Key words and phrases: Bessel functions of first kind, Abelian and Tauberian theorems, summation of divergent series

2000 Mathematics Subject Classification: 33C10, 33C60, 30B30

*Mathematica Macedonica* **6** (2008), 55-60  
**Tauberian theorem of Littlewood type for series  
in Bessel-Maitland functions**  
Jordanka Paneva-Konovska

Abstract

A Tauberian theorem of Littlewood type for the summation of divergent series defined by means of the Bessel-Maitland functions is proved.

Key words and phrases. Bessel-Maitland functions, Abelian and Tauberian theorems, summation of divergent series.

2000 Mathematics Subject Classification. 30B10, 30B30, 33C10, 33C20, 33C60.

*Comptes rendus de l'Académie bulgare des Sciences* **63** (2010),  
No 6, 815-822  
**Convergence of series in Mittag-Leffler functions**  
Jordanka Paneva-Konovska

Abstract

In this paper Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems for series in Mittag-Leffler functions are given. There are also provided asymptotic formulae for “large” values of indices of these functions, used in the proofs of the convergence theorems for the considered series.

Key words and phrases: Mittag-Leffler functions, Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems, summation of divergent series, asymptotic formula

2000 Mathematics Subject Classification: 30A10, 30B10, 30B30, 30B50, 30D15, 33C47, 33E12

*Fractional Calculus & Applied Analysis* **13** (2010), No 4, 403-414  
**Series in Mittag-Leffler functions: inequalities and convergent theorems**  
Jordanka Paneva-Konovska

Abstract

In studying the behaviour of series, defined by means of the Mittag-Leffler functions, on the boundary of its domain of convergence in the complex plane, we prove Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems. Asymptotic formulae are also provided for the Mittag-Leffler functions in the case of “large” values of indices that are used in the proofs of the convergence theorems for the considered series.

Key Words and Phrases: Mittag-Leffler functions, inequalities, asymptotic formula, Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems, summation of divergent series

MSC 2010: 30A10, 30B10, 30B30, 30B50, 30D15, 33E12

*Amer. Inst. of Physics (AIP)– Conf. Proc. 1293 (2010), 157-164;*  
*doi:10.1063/1.3515580*

**Inequalities and asymptotic formulae related to  
generalizations of the Bessel Functions**

Jordanka Paneva-Konovska

**Abstract**

We consider some families of 3-index generalizations of the Bessel functions of first kind and study the behaviour of such families in domains of the complex plane. We also prove asymptotic formulae for "large" values of indices of these functions. Similar theorems have also been obtained by the author for the Bessel and Bessel-Maitland functions.

Keywords: asymptotic formula, Bessel functions, Bessel-Maitland functions, multi-index Mittag-Leffler functions, Wright functions.

*Amer. Inst. of Physics (AIP)– Conf. Proc. 1301 (2010), 636-643;*  
*doi:10.1063/1.3526665*

**Convergence of series in Mittag-Leffler type functions**

J. Paneva-Konovska

**Abstract**

Explicit solutions of some kinds of fractional order (or multi-order) differential and integral equations involving Erdelyi-Kober (E-K) operators are representable by means of series in Mittag-Leffler type functions like those considered in this work. The domains of convergence of such expansions are found. The series behaviour on the boundary of these domains are studied. Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems for such series are given. Asymptotic formulae for "large" values of indices of these functions, used in the proofs of the convergence theorems for the considered series, are also provided.

Keywords: Multi-index Mittag-Leffler functions, Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems, summation of divergent series, asymptotic formula.

PACS: 02.30.Gp, 02.30.Lt

*Proc. of International Symposium 'Geometric Function Theory and  
Applications' 2010, Sofia, IMI – BAS' (2010), 223-228*

**Series in some Mittag-Leffler type functions:  
theorems for their convergence in complex domain**

Jordanka Paneva-Konovska

**Abstract**

In this paper Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems for series in some multi-index Mittag-Leffler functions are given.

Key Words and Phrases: multi-index Mittag-Leffler functions, Cauchy-Hadamard,

Abel, Tauber and Littlewood type theorems, summation of divergent series  
MSC 2010: 30B10, 30B30, 30B50, 33C47, 33E12

*Proceedings of the 4th IFAC Workshop 'Fractional Differentiation and its  
Applications, Badajoz, Spain, October 18-20, 2010',  
(Eds: I. Podlubny, B. M. Vinagre Jara, YQ. Chen,  
V. Feliu Batlle, I. Tejado Balsera),  
(2010), Article no. FDA10-147, 1-4*

**Convergence of series in some multi-index Mittag-Leffler functions**  
Jordanka Paneva-Konovska

Abstract

In this paper Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems for series in some multi-index Mittag-Leffler functions are given. We provided also asymptotic formulae for 'large' values of indices of these functions, used in the proofs of the convergence theorems for the considered series.

Keywords: multi-index Mittag-Leffler functions, Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems, summation of divergent series, asymptotic formula

*Mathematica Balkanica New Series Vol. 26 (2012), Fasc. 1-2, 203-210*

**Inequalities and asymptotic formulae for  
the three-parametric Mittag-Leffler functions**  
Jordanka Paneva-Konovska

Abstract

We consider some families of 3-index generalizations of the classical Mittag-Leffler functions and study the behaviour of these functions in domains of the complex plane. First, some inequalities in the complex plane and on its compact subsets are obtained. We also prove an asymptotic formula for the case of "large" values of the indices of these functions. Similar results have also been obtained by the author for the classical Bessel functions and their Wright's generalizations with 2, 3 and 4 parameters, as well as for the classical and multi-index Mittag-Leffler functions.

Key Words: special functions, Mittag-Leffler function and its generalizations, entire functions, inequalities, asymptotic formulae

MSC 2010: 33E12, 30A10, 30D15, 30E15

*Proceedings of the Fifth Symposium on Fractional Differentiation and Its Applications, May 14-17 2012, Hohai University, Nanjing, China*  
(Editors: Wen Chen, HongGuang Sun and Dumitru Baleanu)  
(2012), Paper number: #284, 1-7

**Three-multi-index Mittag-Leffler functions,  
series and convergence theorems**  
Jordanka Paneva-Konovska

Abstract:

In this paper we consider a new class of special functions, namely, the so-called three-multi-index Mittag-Leffler functions. They are  $3m$ -index generalizations of the classical Mittag-Leffler functions and of the Prabhakar function. We study the basic properties of these entire functions: we find their order and type, an asymptotic estimation, represent them as Wright's generalized hypergeometric functions and Fox's  $H$ -functions. Formulae for integer and fractional order integration and differentiation are provided. We present also some interesting particular cases of the three-multi-index Mittag-Leffler functions. Series in such kind of functions are then studied in the complex plane. More precisely, their domains of convergence are found and the behaviour of such expansions on the boundary of their domains is studied. In this way, we find analogues of the Cauchy-Hadamard, Abel, Tauber and Hardy-Littlewood theorems for the power series.

Keywords:  $3m$ -parametric multi-index Mittag-Leffler functions, order and type of entire function, asymptotic formula, Mellin-Barnes-type integral representation, Riemann-Liouville fractional integral and derivative, convergent series, summation of divergent series

*Amer. Inst. of Physics (AIP)– Conf. Proc. 1497 (2012), 318-325;*  
*doi: 10.1063/1.4766800*

**Fatou type theorems for series in Mittag-Leffler functions**  
Jordanka Paneva-Konovska

Abstract

In studying the behaviour of series, defined by means of the Mittag-Leffler functions, on the boundary of its domain of convergence in the complex plane, we give analogues of the classical theorems for the power series like Cauchy-Hadamard, Abel, as well as Fatou theorems. The asymptotic formulae for the Mittag-Leffler functions in the cases of "large" values of indices that are used in the proofs of the convergence theorems for the considered series are also provided.

Keywords: Mittag-Leffler functions, inequalities, asymptotic formula, Cauchy-Hadamard, Abel and Fatou type theorems.

PACS: 02.30.Gp, 02.30.Lt

*Proceedings of 'BGSIAM'12, Sofia, Bulgaria', (2012), 111-115,*

**Comparison between the convergence of power  
and Mittag-Leffler series**

Jordanka Paneva-Konovska

**Abstract**

In this paper we consider series in functions of Mittag-Leffler type with one and two indices. We discuss their convergence, more precisely, we determine where these series converge and where they do not, where the convergence is uniform and where it is not. The radius of their disks of convergence is given and the behaviour on the boundaries of these domains is studied. Namely, we provide theorems of Cauchy-Hadamard, Abel and Fatou type and compare them with the classical results for the more popular power series.

*Amer. Inst. of Physics (AIP)– Conf. Proc. 1570 (2013), 383-392;*

*doi: 10.1063/1.4854780*

**Comparison between the convergence  
of power and Bessel series**

Jordanka Paneva-Konovska

**Abstract**

We consider series defined by means of the Bessel functions, find their domains of convergence and study the behaviour of such series on the boundaries of these domains. Analogues of the classical theorems for the power series like Cauchy-Hadamard, Abel, as well as Fatou type theorems are proposed.

Keywords: Series in Bessel functions, Cauchy-Hadamard, Abel and Fatou type theorems.

PACS: 02.30.Gp, 02.30.Lt

*Proc. of Intern. Conf. 'Complex Analysis and Applications '13',*

*Sofia (2013), 166 -173*

**Comparison between the convergence  
of power and generalized Mittag-Leffler series**

Jordanka Paneva-Konovska

**Abstract**

In this paper we consider a family of the three-index generalizations of the classical Mittag-Leffler functions, introduced by Prabhakar. We consider series in such type of functions in the complex plane and study their convergence. More precisely, we determine where the series converges and where it does not, where the convergence is uniform, which the domain of convergence is, what the behaviour of the series is "near" the boundary of the domain of convergence, and on itself. Along with this, we state analogues of the Cauchy-Hadamard, Abel and Fatou theorems for the power series.

Finally, we compare the obtained results with the classical ones for the widely used power series.

Key Words and Phrases: Mittag-Leffler functions and generalizations, series in generalized Mittag-Leffler functions, convergence and divergence, Cauchy-Hadamard, Abel and Fatou type theorems

MSC 2010: 40A30, 33E12, 31A20, 30D15, 30B30, 30B10

*Amer. Inst. of Physics (AIP)– Conf. Proc. **1631** (2014), 303-312;  
doi: 10.1063/1.4902491*

**Fatou theorems for multi-index Bessel series**  
Jordanka Paneva-Konovska

**Abstract**

We consider series defined by means of the generalized Bessel functions, find their domains of convergence and study the behaviour of such series on the boundaries of these domains. Analogues of the classical theorems for the power series like Cauchy-Hadamard, Abel, as well as Fatou type theorems are proposed.

Keywords: Bessel functions and generalizations, Series in multi-index Bessel functions, Cauchy-Hadamard, Abel and Fatou type theorems.

PACS: 02.30.Gp, 02.30.Lt

*Amer. Inst. of Physics (AIP)– Conf. Proc. **1690** (2015),  
art. no 050004, 050004-1 – 050004-8;  
doi: 10.1063/1.4936734*

**Overconvergence of Bessel type series**  
Jordanka Paneva-Konovska

**Abstract**

We consider series defined by means of the two-, three- and four-index generalized Bessel functions and study the behaviour of such series on the boundaries of their convergence domains. Analogues of the classical theorems for the power series like overconvergence, as well as Hadamard type theorems are proposed.

*Advances in Mathematics: Scientific Journal, **4** (2015), No.2, 175-181*  
**On the convergence and overconvergence of Mittag-Leffler series**  
Jordanka Paneva-Konovska

**Abstract**

We consider series defined by means of the Mittag-Leffler functions and their Prabhakar generalizations and study the behaviour of such series on the peripheries of their convergence domains. Analogues of the classical theorems for the power series like overconvergence, as well as Hadamard type theorems are proposed.

*International Journal of Applied Mathematics* **29** (2016),  
No. 1, 69-78; doi: 10.12732/ijam.v29i1.6  
**Periphery behaviour of series in Mittag-Leffler type functions, I**  
Jordanka Paneva-Konovska

Abstract

This is a survey on part of author's recent results on the subject. Different families of the the Mittag-Leffler functions and their 3-parametric generalizations are considered. First, asymptotic formulae necessary for proving the main results, are provided. Series defined by means of these families are further studied. Starting with their domains of convergence, the behaviour of such series on the peripheries of their convergence domains is investigated and analogues of the classical results for the power series are proposed.

Key Words: power series, series in special functions, Mittag-Leffler function and generalizations, convergence, uniform convergence

AMS Subject Classification: 30B30, 31A20, 30B10, 40A30

*International Journal of Applied Mathematics* **29** (2016),  
No. 2, 175-187; doi: 10.12732/ijam.v29i2.2  
**Periphery behaviour of series in Mittag-Leffler type functions, II**  
Jordanka Paneva-Konovska

Abstract

This is a survey on a part of author's recent results on the subject. It is devoted to different systems of the Mittag-Leffler functions and their 3-parametric generalizations. First, asymptotic formulae necessary for obtaining the main results, are provided. Series defined by means of these systems are further studied. Starting with their domains of convergence, the behaviours of such series on the peripheries of their convergence domains are investigated and analogues of the classical results for the power series are proposed. This serves as Part II, of our previous paper.

Key Words: power series, series in special functions, Mittag-Leffler functions and generalizations, convergence, uniform convergence, Hadamard gaps, overconvergence

AMS Subject Classification: 30B30, 30B40, 31A20, 30B10, 40A30



*Proc. of Intern. Conf. on Fractional Differentiation  
and its Applications, Novi Sad, Serbia (2016)*  
**Series in 3-parameter Mittag-Leffler functions  
– various convergence theorems**  
Jordanka Paneva-Konovska

Abstract

Series in Mittag-Leffler type functions with 3 indices with complex coefficients are considered in the complex plane. Their convergence is discussed and various results, completely analogical to those for the classical power series, are proposed.

*Amer. Inst. of Physics (AIP)– Conf. Proc. 1789 (2016), art. no 050008,  
050008-1 – 050008-9; doi: 10.1063/1.4968492*  
**Laplace Transform Approach for Solving Integral  
Equations Using Computer Algebra System**  
Jordanka Paneva-Konovska and Yanka Nikolova

Abstract

The Laplace transform method, along with Computer Algebra Systems (CAS) “Maple” v. 13, are extremely successfully applied for solving a class of integral equations with an arbitrary order, including fractional order integral equations. The combining of both powerful approaches allows students more quickly, enjoyable and thoroughly to master the material.

*Amer. Inst. of Physics (AIP)– Conf. Proc. 1910 (2017), art. no 050002,  
050002-1 – 050002-8; doi: 10.1063/1.5013984*  
**On a family of Bessel type functions:  
Estimations, series, overconvergence**  
Jordanka Paneva-Konovska

Abstract

A family of the Bessel–Maitland functions are considered in this paper and some useful estimations are obtained for them. Series defined by means of these functions are considered and their behaviour on the boundaries of the convergence domains is discussed. Using the obtained estimations, necessary and sufficient conditions for the series overconvergence, as well as Hadamard type theorem are proposed.

*Fract. Calc. Appl. Anal.* **21** (2018), No1, 254–265;

DOI:10.1515/fca-2018-0016

**Differential and integral relations in the class  
of multi-index Mittag-Leffler functions**

Jordanka Paneva-Konovska

**Abstract**

As recently observed by Bazhlekova and Dimovski [1], the  $n$ -th derivative of the 2-parametric Mittag-Leffler function gives a 3-parametric Mittag-Leffler function, known as the Prabhakar function. Following this analogy, the  $n$ -th derivative of the  $(2m.\text{index})$  multi-index Mittag-Leffler functions is obtained, and it turns out that it is expressed in terms of the  $(3m.\text{index})$  Mittag-Leffler functions.

Further, some special cases of the fractional order Riemann–Liouville and Erdrelyi–Kober integrals of the Mittag-Leffler functions are calculated and interesting relations are proved. Analogous relations happen to connect the 3m-Mittag-Leffler functions with the integrals and derivatives of 2m- Mittag-Leffler functions. Finally, multiple Erdrelyi–Kober fractional integration operators, as operators of the generalized fractional calculus, are shown to relate the 2m- and 3m-parametric Mittag-Leffler functions.

Key Words and Phrases: fractional calculus, Mittag-Leffler functions, multi-index Mittag-Leffler functions, Riemann–Liouville and Erdrelyi–Kober fractional integral and derivative

MSC 2010: 26A33, 33E12

*Amer. Inst. of Physics (AIP)– Conf. Proc.* **2048** (2018),

art. no. 050015, 050015-1 – 050015-6; doi: 10.1063/1.4766800

**Bessel type functions: Relations with integrals  
and derivatives of arbitrary orders**

Jordanka Paneva-Konovska

**Abstract**

As recently observed by Bazhlekova and Dimovski [2], the  $n$ -th derivative of the 2-parametric Mittag-Leffler function gives a 3-parametric Mittag-Leffler function, known as the Prabhakar function. Following the analogy, the  $n$ -th derivatives of the Bessel–Maitland functions are obtained, and it turns out they are expressed in terms of the generalized Bessel–Maitland functions with 3 indices. Further, some special cases of the fractional order Riemann–Liouville derivatives and integrals of the Bessel–Maitland functions are calculated and interesting relations are proved.

*International Journal of Applied Mathematics*, **32** (2019),  
No. 3, 357-380; doi: 10.12732/ijam.v32i3.1

**A survey on Bessel type functions as multi-index  
Mittag-Leffler functions: Differential and integral relations**  
Jordanka Paneva-Konovska

Abstract

As recently observed by Bazhlekova and Dimovski, the  $n$ -th derivative of the 2-parametric Mittag-Leffler function gives a 3-parametric Mittag-Leffler function, known as the Prabhakar function. Following the analogy, the  $n$ -th derivatives of the Bessel type functions are obtained, and it turns out that usually they are expressed in terms of the Bessel type functions with the same or more number of indices, up to a matching power function. Further, some special cases of the fractional order Riemann–Liouville derivatives and integrals of the Bessel type functions are calculated and interesting relations are proved.

Key Words: Bessel functions and generalizations, Mittag-Leffler functions, multi-index Mittag-Leffler functions, fractional calculus, Riemann–Liouville fractional integral and derivative

AMS Subject Classification: 33E12, 33C10, 26A33