Topological Methods
in Analysis and Optimization
June 10–13, 2013, Sofia, BULGARIA

A B S T R A C T S
Single-directional Properties of Quasi-monotone Operators

Didier Aussel, Marián Fabian
Mathematical Institute of Czech Academy of Sciences,
Žitná 25, 115 67 Praha 1, Czech Republic
email: fabian@math.cas.cz

AMS Subject Classification: Primary 47H05. Secondary 49J52, 49J53

In honor of Petar S. Kenderov at the occasion of his 70th birthday

A large supply of quasi-monotone multivalued mappings with values in a weak* fragmentable dual Banach space is shown to be generically single-directional. This is of some interest in analysis of quasi-convex functions. The paper extends/strengthens some results of a recent paper by D. Aussel and A. Eberhard [1].

References


Bornologies in Metrizable Spaces

Gerald Beer
Department of Math, CSULA, Los Angeles CA 90032 USA
e-mail: gbeer@cslanet.calstatela.edu

Bornologies capture the large structure of topological spaces, in particular topological vector spaces and metrizable spaces. In this survey talk on bornologies on metrizable spaces, we look at (1) those bornologies that can be realized as either the bounded or totally bounded subsets with respect to some compatible metric; (2) the structure of the family of subsets that can be approximated in Hausdorff distance by members of a prescribed bornology; (3) bornological convergence of nets of closed sets – a natural generalization of Attouch-Wets convergence.

References


There is a growing need to visualize large mathematical data sets. Motivated by our own requirements—especially in optimization and number theory—we describe and illustrate various tools for representing floating point numbers (or lists) as planar (or three dimensional) walks and for quantitatively measuring their “randomness” or “structure”, see [1]. This and much more material is available at the Walking on Numbers web page


I. The main part of this lecture with all animations embedded is at


and in lower resolution at


II. As time permits we will also illustrate dynamic geometry tools for analysis of iterative systems, especially projection algorithms [2,3]. See


and

References


Variations Principles, Topological Games and Properties of Spaces

M. M. Choban$^1$, P. S. Kenderov$^2$, J. P. Revalski$^2$

1 Department of Mathematics, Tiraspol State University, Republic of Moldova, Chişinău, G. Iablocikin street 5, MD-2069, Republic of Moldova

2 Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Acad. G. Boncev Str., Bl. 8, 1113 Sofia, Bulgaria

We present conditions under which the set of continuous perturbations of a given lower semi-continuous function, which attain minimum with certain properties, is "big" in a topological sense. In the sequel, by a space we understand a completely regular topological Hausdorff space. We continue the investigations from [4, 5] and we use the classes of spaces defined and examined in [1, 2, 3].

Fix a non-empty subspace $X$ of a compact space $Z$ and let $C(Z|X)$ be the Banach space of continuous functions on $Z$ with the topology generated by the sup-norm $\|g\| = \sup\{|g(x)| : x \in X\}$. Denote by $T_X$ the topology of the space $X$.

Fix now a lower semi-continuous bounded from below proper function $f : X \to \mathbb{R} \cup \{+\infty\}$. Denote by $X_f$ the set $X$ considered with the topology $T_f$ generated by the open base $T_X \cup \{U \cap f^{-1}(-\infty, t) : U \in T_X, t \in \mathbb{R}\}$. Let $Y = f^{-1}(\mathbb{R})$ (which we consider with the topology inherited by the space $X$) and let
$Y_f$ be the set $Y$ considered as a topological subspace of the space $X_f$. Let us observe that the set $Y$ is open in $X_f$. For each $g \in C(Z|X)$ we put $m_f(g) = \inf \{f(x) + g(x) : x \in X\}$ and $M_f(g) = \{x \in X : f(x) + g(x) = m_f(g)\}$.

The set-valued mapping $M_f : C(Z|X) \to Y_f$ is open. Other properties of the set-valued mapping $M_f$ will be presented as well.

For any $g \in C(Z|X)$ the function $f + g$ is a lower semi-continuous bounded from below proper function on $X$ and the minimization problem $(X, f + g)$ is considered as a perturbation of the minimization problem $(X, f)$. Let $S_Z(f) = \{g \in C(Z|X) : M_f(g) \neq \emptyset\}$ and $S_{(Z,k)}(f)$ be the collection of all $g \in C(Z|X)$ for which the minimization problem $(X, f + g)$ has the properties: the set $M_f(g)$ is compact and each minimizing sequence of $(X, f + g)$ has the properties: the set $M_f(g)$ is compact and each minimizing sequence of $(X, f + g)$ has an accumulation point in $X$. The set $S_{(Z,TW)}(f) := \{g \in S_{(Z,k)}(f) : M_f(g) \text{ is a singleton}\}$ is the set of all $g \in C(Z|X)$ for which the minimization problem $(X, f + g)$ is Tychonoff well-posed, i.e., has a unique strong minimum.

The set $S_Z(f)$ is dense in $C(Z|X)$. The conditions under which the sets $S_{(Z,k)}(f)$ and $S_{(Z,TW)}(f)$ are dense are will be given as well.

On the space $Y_f$ consider the Banach-Mazur game $BM(Y_f)$ with the players $\alpha$ and $\beta$: every play in this game is a sequence $\{A_n, B_n : n \geq 1\}$ of open subsets of the space $Y_f$, $\beta$ (who starts the game) chooses the open sets $A_n$, $\alpha$ responses by choosing the open sets $B_n$ and $A_{n+1} \subset B_n \subset A_n$ for each $n$. The winning rule for $\alpha$ is $\cap_n A_n = \cap_n B_n \neq \emptyset$. Otherwise $\beta$ wins.

**Theorem 1.** The player $\alpha$ has a winning strategy in the game $BM(Y_f)$ if and only if the set $S_Z(f)$ contains a dense $G_\delta$-subset of $C(Z|X)$.

**Theorem 2.** The set $S_{(Z,k)}(f)$ contains a dense $G_\delta$-subset of $C(Z|X)$ if and only if the space $Y_f$ contains a dense Čech-complete subspace.

**Theorem 3.** The set $S_{(Z,TW)}(f)$ contains a dense $G_\delta$-subset of $C(Z|X)$ if and only if the space $Y_f$ contains a dense completely metrizable subspace.

Other properties of the sets $S_Z(f)$, $S_{(Z,k)}(f)$ and $S_{(Z,TW)}(f)$ in $C(Z|X)$ will be determined as well.

**References**


**On differential inclusions with nonconvex right-hand side**

Bogdan Georgiev, Mikhail Krastanov, Nadezhda Ribarska

Department of Mathematics and Informatics, Sofia University and Institute of mathematics and Informatics, Bulgarian Academy of Science

The talk is a survey of some recent results on the existence of local solutions of a differential inclusion with upper semicontinuous possibly nonconvex right-hand side. Some sufficient conditions will be presented. Under the lack of convexity of the right-hand side, some unexpected properties of the weak invariance of a closed set with respect to the trajectories of the differential inclusion will be discussed. An application to Moreau’s sweeping process with the cone of limiting normals will be presented.

**On the continuity phenomenon of Petar Kenderov**

Pando G. Georgiev

Center for Applied Optimization, University of Florida, Gainesville

In 1983 P. Kenderov [4] proved a general result stating that an arbitrary set-valued mapping from a topological space $X$ to a set $Y$ has some properties resembling continuity at every point of a subset $X_0 \subset X$, which is residual in $X$ (i.e. its complement $X \setminus X_0$ is of first Baire category). This statement has far-reaching consequences and can be called a “continuity phenomenon”, since it
proves and unifies in a general approach several different results in topology and functional analysis, as theorems of Asplund [1], Namioka [8], Fort [3], Deutsch and Lambert [2], Kenderov [5], [6], etc.

In this talk we show that, in the case when \( X \) is a metric space, the set \( X_0 \) is even sigma-porous. It implies that the above (and other) “generic” results have “sigma-porous” versions, with unified proofs. For instance, in the case of monotone operators, the Preiss-Zajíček [7] sigma-porous generalization of Kenderov’s generic result follows immediately from the sigma-porous continuity phenomenon, as well as the similar result for submonotone operators. Several results in [7] can be obtained by this approach.

References


Baire theorems: old and new applications

Gilles Godefroy
University Paris 6, 4 Place Jussieu, 75252 Paris Cedex 05, France

Baire category theorem, and his characterization of pointwise limits of sequences of continuous functions, are among the basic tools and functionals analysis for more than a century. However, these tools are to this day surprisingly efficient and they can still lead to discoveries, and to natural problems. We will display some recent applications of Baire methods in geometry of Banach spaces.

A characterization of the Radon-Nikodym property

Robert Deville
Institut de Mathématiques de Bordeaux, Université Bordeaux 1, 351, cours de la libération, 33405, Talence, France
e-mail: Robert.Deville@math.u-bordeaux1.fr

We present a joint work with O. Madiedo [1]. Our goal is to state an analogue of the fact that every bounded below and non increasing sequence in the real line \( \mathbb{R} \) converges in the framework of a Banach space \( X \). This is not clear, even whenever \( X = \mathbb{R}^2 \). However, we shall see that it is indeed possible in Banach spaces with the Radon-Nikodym property.

**Theorem.** Let \( X \) be a Banach space with the Radon-Nikodym property. Let \( f \) in the unit sphere of the dual space \( X^* \) of \( X \) and \( \varepsilon \in (0,1) \) be fixed. There exists a function \( t : X \to X^* \) such that, for all \( x \in X \), \( \|t(x) - f\| < \varepsilon \), and, for all sequence \( (x_n) \), if the sequence \( (f(x_n) - \varepsilon \|x_n\|) \) is bounded below and if \( \langle t(x_n), x_{n+1} - x_n \rangle \leq 0 \) for all \( n \in \mathbb{N} \), then the sequence \( (x_n) \) converges in \( X \).

Let us notice that this result is actually a characterization of the Radon-Nikodym property, and can be reformulated in terms of games. Such games were introduced in [4], and studied in [2], [6] and [5]. Let us notice that a particular case of this result has been used in [4] and [2] to give a simple proof of Buchzolich’s solution.
of the Weil gradient problem, and also used in [3] to construct almost classical solutions of Hamilton-Jacobi equations.

References


Separable reduction in variational analysis

A. Ioffe

Department of Mathematics
Technion, Haifa, Israel

A property is *separably determined* if it is valid on the entire Banach space, provided it holds on all (or arbitrarily big) separable subspaces. By separable reduction we usually mean a demonstration that a certain property is separably determined. In the talk we shall discuss some new results concerning separable reduction of (a) Fréchet subdifferentiability (a joint work with M. Fabian)
and (b) the three equivalent metric regularity properties (linear openness, metric regularity proper and the pseudo-Lipschitz property)

Concepts of similarity for agents

R. Lucchetti
Politecnico di Milano, Milano, Italy
e-mail: robluc@mate.polimi.it

A central role in economical models is played by the preferences that agents have on some commodity metric space $X$. In fairly general cases these preference relations can be described by suitable utility functions. However in general it is impossible to ask the agent to express preferences on the whole space of commodities: he/she will be interested in expressing his/her taste only on a part of it. This implies that the existence of the utility function is guaranteed only on a subset of the space $X$. This motivated the introduction of the concept of /partial map/, i.e. a map usually defined only a subset of the reference space. Furthermore, it is of great interest to define when agents have similar preference systems, and to consider also the problem of approximating complex systems with simpler ones. All of this justifies the need to construct topologies on the partial maps. We present a general way to do it, extending former definitions working only in the locally compact case.

A survey on topological games and their applications in analysis

Warren B. Moors
Department of Mathematics, The University of Auckland,
Private Bag 92019, Auckland 1142, NEW ZEALAND

In this talk I will summarize some of the recent results that have occurred in the study of topological games, focussing particularly on those results that have applications to abstract analysis. The topics given in this talk do not necessarily represent the most important problems in the area of topological games, but rather, they represent a selection of problems that are of interest to the speaker.
References


Let $C$ be a convex, closed, bounded, but not weakly compact set in a separable Banach space $E$ with the origin outside of $C$. Is it possible to find a linear functional strictly positive on $C$ but not attaining its minimum on $C$?

Motivated by this question of Freddy Delbaen we will present in the talk a possible one side version of James’s sup theorem for weak compactness as well as an answer of Richard Haydon to this problem.

Results obtained with B. Cascales and M. Ruíz will be presented.

**Finite slicing and uniformly convex norms**

Matias Raja

Universidad de Murcia, Murcia, Spain, e-mail: matias@um.es

Pisier proved in 1975 that super-reflexive Banach spaces has an equivalent norm with modulus of convexity of power type improving the uniformly convex renorming done previously by Enflo. Years later, Lancien was able to deduce that result by a geometrical construction based on the slicing of the unit ball. Our aim is to describe a sort of slicing process that will provide more information about the moduli of convexity of all possible renormings of a super-reflexive space. The power type estimation for good renormings will be deduced from the self-similarity of such a process.

Maxim Ivanov Todorov
UDLAP, Puebla, Mexico and IMI–BAS, Sofia, Bulgaria
e-mail: maxim.todorov@udlap.mx

A set is called Motzkin decomposable when it can be expressed as the Minkowski sum of a compact convex set with a closed convex cone. In this talk we analyze the continuity properties of the set-valued mapping, associating to each couple formed by a compact convex set $C$ and a closed convex cone $D$ its Minkowski sum $C+D$. The continuity properties of other related mappings are also analyzed. We shall present some preliminary results of the so-called Motzkin predecomposable sets, i.e., sets which are sum of a compact convex set with a convex cone.

References


Polyhedrality in pieces

Stanimir Troyanski
Universidad de Murcia, Departamento de Matemáticas
Campus de Espinardo, 30100 Murcia, Spain
e-mail:stroya@um.es

The aim of this talk is to present a tool that make the task of finding equivalent norms on certain Banach spaces easier and more transparent. The hypothesis of the tool is based on countable decompositions on the unit sphere $S_X$ and corresponding sequences of subsets of the dual ball $B_{X^*}$ of a given Banach space $X$. The sufficient conditions for this tool in the separable case are also necessary. We provide some examples.

The results are from a joint work with V. Fonf, A. Pallarés and R. Smith.

Homogeneous Compasta and Generalized Manifolds

Vesko Valov
Department of Computer Science and Mathematics, Nipissing University
100 College Drive, P.O. Box 5002, North Bay, ON, P1B 8L7, Canada

We are going to discuss to what extent homogeneous metric compacta have properties which are possessed by Euclidean manifolds. This research is motivated by the famous Bing-Borsuk conjecture that every homogeneous compact metric neighborhood retract of dimension $n$ is an Euclidean $n$-manifold.
Small Sets in Approximation Problems

N. V. Zhivkov
Institute of Mathematics and Informatics, Bulgarian Academy of Science, Sofia, Bulgaria

Best approximation problems to nonempty closed sets in some classes of Banach spaces are considered. The aim of this talk is to sketch some recent results about the structure properties of metric projections loci. The locus $E(A)$ of points $x$ in the space $X$ having best approximations with respect to a closed set $A$, as well as the locus $Q(A)$ of points with not more than one best approximation, are examined. Under certain geometric conditions on the norms it turns out that these loci are “big” sets, as they have complements which are $\sigma$-porous.