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Dedicated to the 70th anniversary of Yuri Bahturin

ABSTRACTS

Sofia, 2016
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PREFACE

The Conference is organized by:

- Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,
- Department of Mechanics and Mathematics, Moscow State University,
- Department of Mathematics and Statistics, Memorial University of Newfoundland,
- Institute of Exact Sciences, University of Brasilia.

The purpose of the event is to present the current state of the art in group theory and ring theory and their applications. In particular, we emphasize combinatorial and geometric group theory, combinatorial and computational ring theory, the theory of PI-algebras, commutative and noncommutative invariant theory, automorphisms of polynomial and other free algebras, graded associative, Lie, and Jordan algebras and superalgebras. The applications include, but are not limited to, scientific computing, coding theory, cryptography, and statistics.

The Conference is dedicated to the 70th birthday of Yuri Bahturin, University Research Professor and Director of Atlantic Algebra Centre at Memorial University of Newfoundland (St. John’s, Canada), and Professor at Moscow State University (Russian Federation). Yuri Bahturin graduated from the Department of Mechanics and Mathematics of Moscow State University in 1969, obtained his Ph.D. degree in 1972 (advisor: Professor Alfred Shmelkin) and started his career at the Chair of Higher Algebra of Moscow State University. In 1999, Yuri Bahturin accepted a professorship at Memorial University of Newfoundland, where he continues to work now. At the same time, he keeps close ties with Department of Algebra of Moscow State University.

The original research field of Yuri Bahturin was the theory of varieties of Lie algebras. He was among the authors of the first research papers and a monograph in this field. For more than 40 years, Yuri Bahturin, his students, and the students of his students continue to obtain important results on polynomial identities of Lie algebras, Lie superalgebras and their enveloping algebras. Later in his career, Yuri Bahturin contributed to the foundation of several research areas including:
• Locally finite Lie algebras,
• Graded algebras and superalgebras,
• Algebras with action of Hopf algebras,
• Combinatorial theory of groups and algebras.

Yuri Bahturin generously shares his ideas, knowledge, and mathematical experience with young mathematicians. He has a long list of Ph.D. students who have successfully defended their theses and continue their successful scientific career in many countries. His monograph "Basic Structures of Modern Algebra" is widely used in university education. The scientific program of the Conference includes more than 50 invited and ordinary talks presented by mathematicians from 16 countries in Europe, Asia, North and South America. Many of the participants are recognized leaders in their fields. We are very glad that also young people from several countries participate at the meeting with their own scientific contributions which is a good promise for the future of Algebra.

Sofia, July 2016

The Programme and Organizing Committee
Contents

PREFACE ................................................................................................................. 1

MAIN TALKS

Aljadeff, E. On the codimension growth of affine PI-algebras –
the polynomial part ................................................................. 9
Dzhumadil’daev, A. S. Algebraic structures constructed by
Baxter operators ................................................................. 10
Elduque, A. Graded modules over simple Lie algebras .............. 13
Gateva-Ivanova, T. Braces, symmetric groups and the Yang–Baxter
equation ................................................................. 14
Grigorchuk, R. Automatically generated sequences and
representations of self-similar groups ....................................... 16
Horozov, E. Automorphisms of algebras and vector orthogonal
polynomials with Bochner’s property ....................................... 17
Kassabov, M. Almost commuting permutations and quantified
soficity ................................................................................. 19
Kochloukova, D. H. Homological properties of soluble groups ...... 20
Kolesnikov, P. On finite-dimensional double Lie algebras .......... 22
Koshlukov, P. Graded polynomial identities for Lie algebras ...... 25
Krasilnikov, A. Lie nilpotent associative algebras .................... 26
La Scala, R. Monomial right ideals and the Hilbert series
of noncommutative modules ................................................. 27
Olshanskii, A. Yu. Constructing highly transitive actions
of groups ................................................................................. 29
Premet, A. The maximal Lie subalgebras of exceptional Lie
algebras over fields of good characteristics ................................ 30
Regev, A. Multiplicities of cocharacters of matrices ................. 31
Shestakov, I. Non-matrix varieties for some classes of non-associative
algebras ................................................................................. 32
Stöhr, R. Free centre-by-nilpotent-by-abelian groups
and Lie rings ............................................................................ 33
Zelmanov, E. Harish Chandra modules over superconformal algebras ..... 34
TALKS

Artemovych, O. D. Derivation rings of semiprime Lie rings ................. 37
Bahturin, Yu. Group gradings on real algebras ............................... 39
Baranov, A. Lie structure of finite dimensional associative
algebras .................................................................................... 40
Bavula, V. Classical left regular left quotient ring of a ring
and its semisimplicity criteria .................................................. 42
Bazhenov, D. Graded prime Goldie rings ...................................... 43
Boumova, S. Colength of *-polynomial identities of simple
*-algebras ................................................................................ 45
Carini, L. On the multiplicity-free plethysms $p_2[s_\lambda]$ .................... 47
Chipchakov, I. D. Formulae and bounds for the Brauer
$p$-dimensions of Henselian valued fields .................................... 49
Dangovski, R. On the maximal containments of lower central
series ideals ............................................................................... 51
de la Concepción, D. On color analytic loops and their tangent
spaces ...................................................................................... 52
Drensky, V. Visualization in algebra .............................................. 53
Fındık, Ş. The Nowicki conjecture for free metabelian
Lie algebras ............................................................................... 54
Gordienko, A. On the analog of Amitsur’s conjecture for
polynomial $H$-identities .......................................................... 56
Greenfeld, B. Free subalgebras, graded algebras and nilpotent
elements .................................................................................... 58
Hristova, E. Invariants of symplectic and orthogonal groups acting
on GL-modules ........................................................................... 60
Hryniewicka, M. E. Nondistributive rings ...................................... 62
Incesu, M. The first fundamental theorem for similarity groups
and $G$-equivalence conditions of vectors given in $R^2$ ................... 63
Ioppolo, A. Standard identities on matrix algebras with
superinvolution .......................................................................... 65
Kasparian, A., I. Marinov. Riemann Hypothesis Analogue
for locally finite modules over the absolute Galois group of a finite
field .......................................................................................... 67
Kochetov, M. Gradings by abelian groups on Lie algebras .............. 70
La Mattina, D. Varieties of algebras with superinvolution of polynomial growth ................................................. 71
Malev, S. Evaluations of non-commutative polynomials on finite dimensional algebras ............................................. 73
Martino, F. Polynomial growth and star-varieties ...................... 74
Melikyan, H. Maximal subalgebras of modular simple Lie (super)algebras of Cartan type ........................................... 75
Mészáros, S. Maximal commutativity of a subalgebra in quantized coordinate rings ......................................................... 76
Montaner, F. Extending Johnson’s theorem on regularity of algebras of quotients to Jordan algebras .................................. 77
Petrogradsky, V. Nil Lie superalgebras of slow growth .................. 78
Pryszczepko, K. A simple solution of the ADS-problem ............... 80
Rashkova, Ts. A fourth order matrix superalgebra with involution and its PI-properties ..................................................... 82
Sabinina, L. On some Malcev algebras and their corresponding Moufang loops .............................................................. 84
Sommerhäuser, Y. Cores in Yetter-Drinfel’d Hopf algebras ............ 85
Stocka, A. On sets of pp-generators of finite groups ........................ 86
Stoytchev, O. Binary symmetry groups of polytopes as quotients of braid groups ......................................................... 87
Todorova, T. L. An estimate of exponential sums over primes ........ 89
Tomanov, G. Properly discontinuous group actions on affine homogeneous spaces ........................................................ 92
Tvalavadze, M. Lie algebras of maximal class ............................. 93
Valenti, A. Exponential growth of the codimensions of PI-algebras with involution .......................................................... 95
Yılmaz, R. The triadjoints of $d$-bimorphisms .............................. 96
Zaicev, M. Pauli grading on simple Lie superalgebras and graded codimension growth .................................................... 98

PARTICIPANTS ........................................................................ 99
MAIN TALKS
On the codimension growth of affine PI-algebras – the polynomial part

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Keywords: Polynomial identity, growth, codimension.

2010 Mathematics Subject Classification: 16R50, 16P90, 16R10.

Kemer’s representability theorem (from mid 80’s) for affine PI-algebras over a field of characteristic zero $F$ says that if $W$ is such algebra, then it is PI-equivalent to a finite dimensional algebra $A$ over $F$. For his proof, Kemer introduced certain remarkable finite dimensional algebras, called “fundamental” or “basic”. On one hand they are special enough so that much of its structure (one may assume $F$ is algebraically closed) can be determined from its polynomial identities (or rather “its polynomial nonidentities”) and on the other hand, they are abundant enough so that any affine PI-algebra is PI-equivalent to the direct product of finitely many fundamental algebras. The representability theorem for affine PI-algebras was the key for Kemer’s solution of the famous Specht problem.

Earlier, in 1972, Regev introduced the codimension sequence $\{c_n\}$ of a PI-algebra $W$ and showed it is exponentially bounded. In 1998, Giambruno and Zaicev showed (as conjectured by Amitsur) the sequence $\{c_n^{1/n}\}$ has a limit and its limit is a nonnegative integer, called the exponent of $W$. Giambruno and Zaicev’s result (for affine PI-algebras) can be interpreted in terms of fundamental algebras, namely $\exp(A) = \dim_F(A_{ss})$ (the semisimple part of $A$). It turns out (based on results of Berele, Regev, Giambruno and Zaicev) that the asymptotic of the codimension sequence has a more subtle structure, namely up to a constant it has the form $n^b \times \exp(W)^n$, where $b$ is an integer or half integer. The goal of this lecture is to present an interpretation (conjectured by Giambruno) for $b$ for any fundamental algebra and hence, by Kemer theory, for any affine PI-algebra. Joint work with Janssens and Karasik.
Algebraic structures constructed by Baxter operators

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Keywords: Novikov algebras, Zinbiel algebras, Baxter operators, polynomial identities.

2010 Mathematics Subject Classification: 17D99.

Let $A$ be an associative commutative algebra with a linear map $R : A \to A$. Let us endow $A$ by multiplication $\circ$ defined by $a \circ b = aR(b)$. Denote the obtained algebra $AR = (A, \circ)$. This algebra is right-commutative,

$$(a \circ b) \circ c = (a \circ c) \circ b$$

for any $a, b, c \in A$. We consider the case when $R$ is constructed by differentiation and integration operators and find additional identities of the algebra $AR$.

If $R$ is a derivation operator, $R(ab) = R(a)b + aR(b)$ for any $a, b \in A$, then $AR$ is a Novikov algebra,

$$(a, b, c) = (b, a, c)$$

for any $a, b, c \in A$, where $(a, b, c) = a \circ (b \circ c) - (a \circ b) \circ c$ is associator.

We now consider the case when $R$ is an inverse of a derivation operator, i.e., an integration operator. A linear map $R : A \to A$ is called a Baxter operator with weight $\lambda$ if it satisfies the following identity

$$R(a)R(b) = R(aR(b) + bR(a) + \lambda ab),$$

for any $a, b \in A$. Say that $A$ is a Baxter algebra if it has a Baxter operator.

**Theorem 1.** Let $A$ be Baxter algebra with Baxter operator $R$. Then the algebra $AR = (A, \circ)$ satisfies the following identities

$$(x \circ a) \circ b = (x \circ b) \circ a, \quad (1)$$
\[ a \circ ([x, y] \circ b) - (a, x, y \circ b) + (a, y, x \circ b) = 0 \] (2)

for any \(a, b, x, y \in A\), where \([a, b] = a \circ b - b \circ a\).

**Corollary.** If \(A\) is a Baxter algebra, then the algebra \(AR = (A, \circ)\) satisfies the identity

\[ (x, a, [b, c]) + (x, b, [c, a]) + (x, c, [a, b]) = 0 \] (3)

for any \(a, b, c, x \in A\).

**Theorem 2.** Let \(A\) be a Baxter algebra with Baxter operator \(R\) and \(AR^+ = (A, [ , ]\) be the algebra on the vector space \(A\) with multiplication \([a, b] = a \circ b - b \circ a\). Then \(AR^+\) satisfies the following identity of degree 4

\[ [[a, x], [b, y]] + [[a, y], [b, x]] = [\text{jac}(a, x, b), y] + [\text{jac}(a, y, b), x], \] (4)

where \(a, b, x, y \in A\) and \(\text{jac}(a, b, c) = [[a, b], c] + [[b, c], a] + [[c, a], b]\) be Jacobian of the multiplication \([ , , ]\).

If \(\lambda = 0\), then the algebra \(AR^+\) is associative and commutative. If \(\lambda \neq 0\), then it has no identity up to degree 4 except the commutativity and satisfies one identity degree 5 given below.

**Theorem 3.** Let \(A\) be a Baxter algebra with Baxter operator \(R\) and \(AR^+ = (A, \{ , \})\) be algebra on the vector space \(A\) and multiplication \(\{a, b\} = a \circ b + b \circ a\). Then \(AR^+\) satisfies the following identity of degree 5

\[ -\{a, y, \{b, x, c\}\} + \{b, x, \{a, y, c\}\} - \{b, y, \{a, x, c\}\} + \{a, x, \{b, y, c\}\} = 0, \] (5)

where \(a, b, c, x, y \in A\) and \(\{a, b, c\} = \{a, \{b, c\}\} - \{\{a, b\}, c\}\) is associator for the Jordan multiplication \(\{ , \}\).

**Example.** Let \(A = R[x]\) and \(R(a) = \int_0^x a \, dx\) be the integration operator. Then \(R\) is a Baxter operator with weight 0. The algebra \(AR = (A, \circ)\) satisfies the identities (1) and (2). In particular it satisfies the identities (3), (4), (5).

**Example.** Let \(A = R[x]\) and \(R(a) = \int_0^x (\int_0^x a \, dx) \, dx\) be the double integration operator. Then \(R\) is not a Baxter operator. The algebra \(AR = (A, \circ)\) satisfies the identities (1), (3) and (4), but not (2) and (5). So, the identities (1), (2) imply the identity (3), but the converse is not true. The identity (5) is a consequence of the identities (1), (2) but it does not follow from the identities (1) and (3).

**Example.** Let \(A = \{e_1, e_2, \ldots\}\) be the infinite-dimensional algebra with multiplication \(e_i \circ e_j = 0\) if \(i \leq j\) and \(e_i \circ e_j = e_i\) if \(i > j\). Then \(A\) satisfies the identities (1), (2), (4) and (5).

In the case of \(\lambda = 0\) these results were obtained in [1].
References

Graded modules over simple Lie algebras

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Simple graded modules over graded semisimple Lie algebras over an algebraically closed field of characteristic zero are studied. The invariants appearing in this classification are computed for the simple Lie algebras.

In particular, some criteria are obtained to determine when a finite-dimensional irreducible module admits a compatible grading.

This is a joint work with Mikhail Kochetov (Memorial University of Newfoundland, Canada).
Braces, symmetric groups
and the Yang–Baxter equation

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Keywords: Yang–Baxter, quantum groups, multipermutation solutions, involutive braided groups, permutation groups, braces.

2010 Mathematics Subject Classification: 16T25, 16T20, 16N80, 20F45, 16N40, 16W22, 81R50.

Set-theoretic solutions of the Yang–Baxter equation form a meeting-ground of mathematical physics, algebra and combinatorics. Such a solution \((X, r)\) consists of a set \(X\) and a bijective map \(r : X \times X \to X \times X\) which satisfies the braid relations. It is known that set-theoretic solutions of YBE are closely related to braided groups. In particular, every braided group \((G, r)\) is a solution of YBE, moreover, every set-theoretic solution \((X, r)\) generates canonically a braided group \(G(X, r)\).

Recently we proved the equivalence of the two notions “a symmetric group” \((G, r)\) (an involutive braided group) and “a left brace” \((G, +, \cdot)\) (a generalization of a radical ring). We explore simultaneously the two parallel structures on \(G\) to study various natural conditions imposed on \(G\) and the impact of these on the corresponding solution \((G, r)\). Especially interesting are symmetric groups and braces, associated canonically with nondegenerate involutive set-theoretic solutions \((X, r)\) of YBE. These are the YB group \(G = G(X, r)\) generated by \(X\) and with quadratic relations defined by the map \(r\), and the associated permutation group \(G(X, r)\), determined by the natural (left) action of \(G\) on \(X\). For

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every solution \((X, r)\) we find an increasing chain of ideals in the brace \(G\) which is an invariant of the solution and reflects explicitly its recursive retractability and its multipermutation level. We prove that \((X, r)\) is a multipermutation solution iff its symmetric group \((G, r_G)\) has a finite multipermutation level, and in the special case when \((X, r)\) is a square-free solution there is an equality of the two multipermutation levels \(mpl_X = mpl_G\). Moreover, a square-free solution \((X, r)\) has multipermutation level 2 iff the corresponding symmetric group \((G, r)\) satisfies condition \(lri\). We initiate the study of braces which satisfy certain identities. This approach has interesting applications for braces related to solutions \((X, r)\), and we shall continue its further exploration.
Automatically generated sequences
and representations of self-similar groups

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Keywords: Self-similar groups, finite automata, automatic representations.

2010 Mathematics Subject Classification: 20E08, 28A80, 68Q70.

We will explain how finite automata and automatically generated sequences can be used in algebra. In particular we will show that self-similar groups have faithful automatic representations and some of them have Jacobi type representations over finite fields.

This is a joint work with Y. Leonov, V. Nekrashevych, and V. Sushchanskii.
Automorphisms of algebras
and vector orthogonal polynomials
with Bochner’s property

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Keywords: Vector orthogonal polynomials, finite recurrence relations, bispectral problem

2010 Mathematics Subject Classification: Primary: 34L20. Secondary: 30C15, 33E05.

Classical orthogonal polynomial systems of Jacobi, Hermite, Laguerre and Bessel have the property that the polynomials of each system are eigenfunctions of a second order ordinary differential operator. According to a famous theorem by Bochner they are the only systems with this property. Similarly, the polynomials of Charlier, Meixner, Kravchuk and Hahn are both orthogonal and are eigenfunctions of suitable difference operator of second order. We recall that according to the famous theorem of Favard-Shohah, the condition of orthogonality is equivalent to the 3-term recurrence relation. Vector orthogonal polynomials (VOP) satisfy finite-term recurrence relation with more terms, according to a theorem by P. Maroni and this characterizes them.

We are looking for VOP that are also eigenfunctions of differential (difference) operator. Motivated by Bochner’s theorem we call these simultaneous conditions Bochner’s property.
In this talk we introduce methods for construction of VOP with Bochner’s property. The methods are purely algebraic and are based on automorphisms of non-commutative associative algebras. Due to this fact the field over which we construct the VOP could be arbitrary. Applications of the abstract methods include broad generalizations of the classical orthogonal polynomials, both continuous and discrete. Other results connect different families of VOP, including the classical ones, by linear transforms of purely algebraic origin. The construction of VOP comes automatically with a number of their properties analogous to the properties of the classical orthogonal polynomials.
Almost commuting permutations and quantified soficity

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Keywords: Amenabe groups, soficity, sofic approximations.
2010 Mathematics Subject Classification: 20E07, 20E26.

Suppose we are given two permutations in a large symmetric group which almost commute, i.e., the support of the commutator is relatively small. The soficity and the amenability of the group $Z^2$ implies that it is possible to modify these permutations and make them commute. In this talk I will explain what is the relative size of the modification of the permutations needed to achieve that, the argument is almost entirely geometric and almost does not use any group theory. (Joint work with Harald Helfgott.)

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Homological properties of soluble groups

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2010 Mathematics Subject Classification: 20F16, 20J05.

Let $G$ be a soluble group of homological type $FP_{2m}$, i.e., there is a projective resolution of the trivial $\mathbb{Z}G$-module $\mathbb{Z}$ with all projectives finitely generated in dimension $\leq 2m$. For example every finitely presented group has type $FP_2$ but the converse is not true [1]. It is known that for metabelian groups the properties $FP_2$ and finite presentability are equivalent [2] and it is an open problem whether the same holds for all soluble groups.

We study the behavior of $\dim H_i(U, \mathbb{Q})$, where $U$ runs through the subgroups of finite index in $G$ for $i \leq m$, $G$ of type $FP_{2m}$ and show that:

1) if $G$ is nilpotent-by-abelian-by-finite, of type $FP_2$ (i.e., $m = 1$) then

$$\sup_U \dim H_1(U, \mathbb{Q}) = \sup_U (U/[U,U] \otimes \mathbb{Q}) < \infty,$$

see [3];

2) If $G$ is abelian-by-(nilpotent of class 2), of type $FP_{2m}$ and $m$ is arbitrary then

$$\sup_U \dim H_i(U, \mathbb{Q}) < \infty \text{ for } i \leq m,$$

see [4].

References


On finite-dimensional double Lie algebras

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Keywords: Double Poisson algebra, Rota–Baxter operator, averaging operator.

2010 Mathematics Subject Classification: 15A24, 17B62.

The idea of a double algebra comes from [1], where the notion of a double Poisson bracket was introduced in order to construct noncommutative Poisson algebras. The latter have their application in the theory of noncommutative integrable systems and in noncommutative differential geometry.

In general, a double algebra is a linear space \( V \) equipped with a linear map \( (\cdot, \cdot) : V \otimes V \to V \otimes V \) called double bracket. There is a natural way to extend a double bracket to the following four linear maps (see [2]) \( V^3 \to V^3 \):

\[
\begin{align*}
    a \otimes b \otimes c & \mapsto (a, b \otimes c)_L, & a \otimes b \otimes c & \mapsto (a, b \otimes c)_R, \\
    a \otimes b \otimes c & \mapsto (a \otimes b, c)_L, & a \otimes b \otimes c & \mapsto (a \otimes b, c)_R
\end{align*}
\]

for \( a, b, c \in V \).

A double algebra \( \langle V, (\cdot, \cdot) \rangle \) is said to be DLA (double Lie algebra) [2] if

\[
(a, b) = -(b, a)^{(12)}, \quad (a, (b, c))_L - (b, (a, c))_R = ((a, b), c)_L.
\]

In this talk we present a solution of a problem stated by V. Kac in his talk on the conference “Lie and Jordan Algebras, Their Representations and Applications-VT”, Dec. 2015: prove that there are no simple finite-dimensional double Lie algebras. We also introduce the notion of a double associative algebra and prove that they are naturally related to double Lie algebras.

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**Definition.** A double algebra $\langle V, (\cdot, \cdot) \rangle$ is said to be associative if
\[
(a, (b, c))_L = ((a, b), c)_L, \quad (a, (b, c))_R = ((a, b), c)_R
\]
for all $a, b, c \in V$.

**Proposition 1.** A double associative algebra $\langle V, (\cdot, \cdot) \rangle$ turns into a double Lie algebra relative to the new double bracket
\[
[a, b] = (a, b) - (b, a)^{(12)}.
\]

For a finite-dimensional space $V$ a structure of a double algebra on $V$ is completely described by a linear operator $R : L(V) \to L(V)$, where $L(V)$ is the space of all linear transformations of $V$. Namely, we use the well-known series of isomorphisms
\[
L(V \otimes V) \simeq L(V) \otimes L(V) \simeq L(V)^* \otimes L(V) \simeq L(L(V)),
\]
where the isomorphism $L(V) \simeq L(V)^*$ corresponds to the trace form on $L(V)$. To be precise, if $\{e_{ij}\}$ is the basis of matrix units in $L(V)$ and $R : L(V) \to L(V)$ then
\[
(a, b) = \sum_{i,j} e_{ij}(a) \otimes R(e_{ji})(b), \quad a, b \in V.
\]

It is natural to expect that if a double bracket $(\cdot, \cdot)$ satisfies some conditions then so is the corresponding operator $R$. Indeed, we have the following

**Proposition 2.** Let $\langle V, (\cdot, \cdot) \rangle$ be a double algebra, and let $R : L(V) \to L(V)$ be the corresponding operator, $R^*$ stands for its conjugate operator relative to the trace form. Then

1. $\langle V, (\cdot, \cdot) \rangle$ is a DLA if and only if
\[
R = -R^*, \quad R(x)R(y) = R(R(x)y + xR(y))
\]
for all $x, y \in L(V)$;
2. $\langle V, (\cdot, \cdot) \rangle$ is a double associative algebra if and only if
\[
R(x)R(y) = R(R(x)y), \quad R^*(x)R^*(y) = R^*(R^*(x)y)
\]
for all $x, y \in L(V)$.

Therefore, the structure of a double Lie algebra on $V$ is determined by a skew-symmetric Rota–Baxter operator on $L(V)$, and the structure of a double associative algebra corresponds to a left averaging operator such that its conjugate is also left averaging.
It is not hard to show that for every Rota–Baxter operator $R$ on $L(V)$ its image $B = R(L(V))$ may not be an irreducible subalgebra of $L(V)$. This implies

**Corollary 1.** *Simple finite-dimensional double Lie algebras do not exist.*

However, simple double associative algebras exist. For example, so are spaces with double brackets $(a, b) = a \otimes b$ or $(a, b) = b \otimes a$. These brackets correspond to operators $R(e_{ij}) = \delta_{ij} \text{id}$ and $R = \text{id}$, respectively.

**Corollary 2.** *Let $V$ be a finite-dimensional space, and $T : L(V) \to L(V)$ be a left averaging operator such that $T^*$ is also left averaging. Then*

$$R = T - T^*$$

*is a skew-symmetric Rota–Baxter operator on $L(V)$.*

It is natural to ask whether every skew-symmetric Rota–Baxter operator on $L(V)$ may be obtained in this way as a “commutator” of a left averaging operator (i.e., whether a DLA structure on $V$ is induced by double associative structure). Let us call such Rota–Baxter operators of the form $T - T^*$ special.

**Theorem.** *Non-special Rota–Baxter operators on $L(V)$ exist for dim $V \geq 2$."

The latter result is obtained by Gröbner bases method.

The following problems remain open:
- Describe all simple finite-dimensional double associative algebras;
- What is a “double analogue” of the general linear Lie algebra $gl(V)$ (stated by V. Kac)?
- Whether every (finite-dimensional) double Lie algebra may be embedded into a (finite-dimensional) double associative algebra?

**References**


Graded polynomial identities for Lie algebras

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Keywords: Graded Lie algebras, graded identities, group gradings on algebras.

2010 Mathematics Subject Classification: 17B01, 17B70, 16R99.

We discuss group gradings on Lie algebras and the corresponding graded identities.

We collect known results about the graded identities of important Lie algebras such as $sl_2$, the Lie algebra of the $2 \times 2$ traceless matrices over an infinite field (results of the speaker), and $W_1$, the simple Lie algebra of the derivations of the polynomial ring in one variable (over a field of characteristic 0). The algebra $W_1$ is graded in a natural way by the integers. We describe the graded identities for this grading: these do not admit any finite basis. These are joint results of the speaker together with Krasilnikov and Freitas.

Finally we present new results concerning the gradings and the corresponding graded identities for the Lie algebra of the upper triangular matrices of any size. (Joint results with Yukihide.)

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Lie nilpotent associative algebras

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**Keywords:** Polynomial identity, $T$-ideal, commutator, nilpotent.

**2010 Mathematics Subject Classification:** 16R10, 16R40.

An associative algebra $A$ is called Lie nilpotent if its associated Lie algebra $A^{(-)}$ (with the Lie bracket defined by $[a, b] = ab - ba$) is nilpotent. The study of Lie nilpotent associative rings and algebras was started by Jennings in 1947. Since then such rings and algebras have been investigated in many papers from different points of view. Recent interest in Lie nilpotent associative algebras has been motivated by the study of the quotients $L_i/L_{i+1}$, where $A^{(-)} = L_1 \supset L_2 \supset L_3 \supset \cdots$ is the lower central series of $A^{(-)}$. This study was initiated in 2007 in the pioneering article of Feigin and Shoikhet and continued by other researchers.

In my talk I will discuss some recent results about Lie nilpotent associative algebras obtained by various authors. I will also present some new results about these algebras.
Monomial right ideals and the Hilbert series of noncommutative modules

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Keywords: Hilbert series, noncommutative modules, automata.

2010 Mathematics Subject Classification: Primary: 16Z05. Secondary: 16P90, 05A15.

In this talk we present a procedure for computing the rational sum of the Hilbert series of a finitely generated monomial right module $N$ over the free associative algebra $\mathbb{K}\langle x_1, \ldots, x_n \rangle$. We show that such procedure terminates, that is, the rational sum exists, when all the cyclic submodules decomposing $N$ are annihilated by monomial right ideals whose monomials define regular formal languages. The method is based on the iterative application of the colon right ideal operation to monomial ideals which are given by an eventual infinite basis. By using automata theory, we prove that the number of these iterations is a minimal one. In fact, we have experimented efficient computations with an implementation of the procedure in Maple which is the first general one for noncommutative Hilbert series.

References


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Constructing highly transitive actions of groups

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Keywords: Highly transitive action, coset graph, hyperbolic group.

2010 Mathematics Subject Classification: 20E05, 20B22, 20E07, 20E15, 54H15, 20F05.

An action of a group on an infinite set is called *highly transitive* if it is $k$-transitive for every $k \geq 1$. The first examples of highly transitive actions of finitely generated groups go back to B. Neumann’s observations of 1950s. A new approach based on the inductive enlargement of the point stabilizer was suggested in a joint work of Yu.A. Bahturin and the speaker in 2010. Such an approach (1) makes possible to obtain group actions with exotic properties and (2) significantly extends the class of groups admitting faithful highly transitive actions. I will survey some recent results obtained by Vladimir Chaynikov, Michael Hull, Denis Osin and myself.
The maximal Lie subalgebras of exceptional Lie algebras over fields of good characteristics

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Keywords: Simple algebraic groups, simple Lie algebras, maximal subalgebras.

2010 Mathematics Subject Classification: 17B45, 17B50.

Let $G$ be a simple algebraic group of exceptional type defined over an algebraically closed field $k$ of characteristic $p > 0$ and $\mathfrak{g} = \text{Lie}(G)$. We assume that $p$ is a good prime for the root system of $G$, that is $p > 5$ if $G$ is of type $E_8$ and $p > 3$ otherwise. In my talk based on recent joint work with David Stewart I am going to describe all maximal Lie subalgebras of the Lie algebra $\mathfrak{g}$. Our approach is based on the classification theory of simple modular Lie algebras and the description of maximal connected subgroups of the algebraic group $G$ obtained earlier by Seitz, Testerman and Liebeck–Seitz.
Multiplicities of cocharacters of matrices

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Keywords: Algebras with polynomial identities, multiplicities of cocharacters, polynomial identities of matrices.

2010 Mathematics Subject Classification: 16R30, 20C30.

The multiplicities of the cocharacters of matrices are different polynomials at different regions. Together with results of Berele and of Drensky and Genov, this imply that computing these multiplicities will be extremely complicated. We suggest to restrict this computations to a single “basic” region thus, possibly, simplifying these calculations.
Non-matrix varieties for some classes of non-associative algebras

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*Keywords:* non-matrix polynomial identities, algebras close to associative, Jordan algebra, alternative algebras, non-commutative Jordan algebras.

2010 Mathematics Subject Classification: 16R10, 17A15, 17A50, 17C05, 17D05.

A variety \( \mathcal{M} \) of associative algebras (over a field \( F \)) is called “non-matrix” if \( F_2 \notin \mathcal{M} \), where \( F_2 \) is the matrix algebra of order 2 over \( F \). This notion was introduced by V.N. Latyshev in 1977. Since then, several other equivalent characterizations for a non-matrix variety were obtained, for instance, by considering algebraic (G. Chekanu, 1979) and nilpotent (A. Mishchenko, V. Petrogradsky, A. Regev et al, 2012) elements, or in terms of identities (A. Kemer, 1980).

It looks natural to consider analogues of non-matrix varieties in non-associative algebras, first of all, for varieties of algebras, “close to associative”, that is, for alternative and Jordan algebras.

In this talk we define and study the non-matrix varieties in the classes of Jordan, alternative, and non-commutative Jordan algebras. We extend or generalize some results, mentioned above, for non-matrix varieties in these classes of algebras.

A joint work with Vinicius Bittencourt.
Free centre-by-nilpotent-by-abelian groups and Lie rings

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Keywords: Relatively free groups, relatively free Lie rings.

2010 Mathematics Subject Classification: 17B01, 20E22.

Let $L$ be a free Lie ring. The quotient

$$L/[(L')^c, L],$$

where $(L')^c$ is the $c$th term of the lower central series of the derived ideal $L'$, is the free centre-by-(nilpotent of class $c - 1$)-by-abelian Lie ring. This is a free central extension of the quotient $L/(L')^p$. A peculiar feature of these free central extensions is the occurrence of elements of finite order in the additive group. A similar phenomenon appears in its group theoretic counterpart, the free centre-by-nilpotent-by-abelian groups, that is quotients of the form $F/\gamma_c(F'), F'$, where $F$ is a free group and $\gamma_c(F')$ is $c$th term of the lower central series of the commutator subgroup $F'$. This was first discovered by C. K. Gupta in 1973, who proved that, for $c = 2$, these groups contain elements of order 2. In the talk I will discuss the phenomenon of torsion in such free central extensions, both for groups and Lie rings, starting with Gupta’s result and ending up with some recent developments.
Harish Chandra modules over superconformal algebras

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Keywords: Superconformal algebras, Harish Chandra modules.

2010 Mathematics Subject Classification: 17B65, 17B69, 17B81, 81T40.

We will discuss a complete classification of all known (conjecturally all) superconformal algebras of series W, S, K, CK(6). This is joint work with C. Martinez and O. Mathieu.
TALKS
Derivation rings of semiprime Lie rings

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Keywords: Semisimple Lie ring, derivation, generalized derivation.

2010 Mathematics Subject Classification: 17B40, 17B20.

Let $A$ be a Lie ring (with the addition “+” and the multiplication “$[-, -]$”). An additive mapping $d: A \to A$ is called a derivation of $A$ if
\[ d([x, y]) = [d(x), y] + [x, d(y)] \]
for any $x, y \in A$. The set of all derivations $\text{Der} A$ of $A$ is a Lie ring. The mapping
\[ \text{ad}_a : A \ni x \mapsto [a, x] \in A \]
determines a derivation $\text{ad}_a$ of $A$ (so-called an inner derivation of $A$ induced by $a \in A$). Then the set
\[ \text{ad} A = \{\text{ad}_a \mid a \in A\} \]
of all inner derivations of $A$ is an ideal in $\text{Der} A$. An additive mapping $F: A \to A$ is called a generalized derivation of $A$ associated with a derivation $\delta \in \text{Der} A$ (in the sense of Brešar [1]) if
\[ F([x, y]) = [F(x), y] + [x, \delta(y)] \]
for any $x, y \in A$. The set of all generalized derivations $G\text{Der} A$ of $A$ is a Lie ring.

We will present some connections between a semiprime Lie ring $A$, an inner derivation ring $\text{IDer} A$, a derivation ring $\text{Der} A$ and a generalized derivations ring $G\text{Der} A$. Recall that a Lie ring $A$ is called semiprime if, for any ideal $I$ of $A$, the condition $[I, I] = 0$ implies that $I = 0$. 
References

Group gradings on real algebras

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**Keywords:** Group graded algebras, real algebra, polynomial identities.

**2010 Mathematics Subject Classification:** 16W50, 16R50, 17B60, 17B70.

I would like to talk on some recent results concerning the structure and identities of algebras over the field of real numbers.
Lie structure of finite dimensional associative algebras

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Keywords: Lie algebra, Lie structure, associative algebra, inner ideal.

2010 Mathematics Subject Classification: 17B60.

Any associative ring $A$ becomes a Lie ring $A^{(-)}$ under $[x, y] = xy - yx$. Let $A^{(1)} = [A, A]$ be the derived subalgebra of $A^{(-)}$ and let $Z$ be its center. In the early 1950s Herstein initiated a study of Lie ideals of $A$ in case of a simple ring. In particular, he showed that in that case $A^{(1)}/Z$ is simple, $A^{(1)}$ generates $A$ and $A^{(1)}$ is perfect (i.e., $[A^{(1)}, A^{(1)}] = A^{(1)}$), except if $A$ is of characteristic 2 and is of dimension 4 over its center. Over the years, his work was generalized in various directions, on the one hand, to the setting of prime and semiprime rings, and, on the other hand, to Lie structures other than Lie ideals. However, little is known about the Lie structure of non-semiprime rings.

In this work we study the Lie structure of finite dimensional associative algebras over an algebraically closed field of characteristic $p \geq 0$. Let $A$ be such an algebra. It is natural to suppose that $A$ has no 1-dimensional quotients. Indeed, if $A$ has a chain of ideals with 1-dimensional subquotients then $A^{(-)}$ is solvable and there is little correlation between ideals of $A^{(-)}$ and those of $A$. We say that $A$ is $k$-perfect if $\dim(A/T) > k$ for all ideals $T$ of $A$. Suppose that $A$ is 1-perfect if $p \neq 2$ and 4-perfect if $p = 2$. Then we prove that the Lie algebra $A^{(1)}$ is perfect and $A = A^{(1)}A^{(1)} + A^{(1)}$. Moreover, we show that most Lie ideals of $A^{(-)}$ and $A^{(1)}$ are induced by the ideals of $A$. We also describe Jordan-Lie inner ideals of $A^{(-)}$. Recall that a subspace $B$ of a Lie algebra $L$ is an inner ideal of $L$ if $[B, [B, L]] \subseteq B$. An inner ideal $B$ of $A^{(-)}$ is said to be
Jordan-Lie if $B^2 = 0$ (in that case $B$ is also an inner ideal of the Jordan algebra $A^{(\pm)}$).

This is a joint work with H. Shlaka (University of Leicester).
Classical left regular left quotient ring of a ring and its semisimplicity criteria

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**Keywords:** Goldie’s Theorem, the classical left quotient ring, the classical left regular left quotient ring.

**2010 Mathematics Subject Classification:** 16P50, 16P60, 16P20, 16U20.

Let $R$ be a ring, $C_R$ and $'C_R$ be the set of regular and left regular elements of $R$ ($C_R \subseteq 'C_R$). Goldie’s Theorem is a semisimplicity criterion for the classical left quotient ring $Q_{l,cl}(R) := C_R^{-1}R$. Semisimplicity criteria are given for the classical left regular left quotient ring $'Q_{l,cl}(R) := 'C_R^{-1}R$. As a corollary, two new semisimplicity criteria for $Q_{l,cl}(R)$ are obtained (in the spirit of Goldie).
Graded prime Goldie rings

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**Keywords:** Graded Goldie rings, group graded rings, maximal graded quotient rings.

**2010 Mathematics Subject Classification:** 16W50, 16P60, 16S85, 16U20.

An associative ring with unit is called a left Goldie ring if it does not contain infinite direct sums of left ideals and any ascending chain of left annihilators stabilizes. The name is given after the English mathematician Alfred Goldie who proved the following theorem in the end of the 1950s.

**Theorem.** (Alfred Goldie) The following conditions are equivalent:

1. $R$ is a semiprime left Goldie ring;
2. The left classical ring of fractions $Q(R)$ exists and is semisimple. In this case $Q(R)$ is simple if and only if $R$ is prime.

The report is about group graded Goldie rings. In the theory of graded rings homogeneous elements and graded ideals (i.e., ideals containing with any element its homogeneous components) have a particular significance. Accordingly, concepts from classical theory of associative rings have graded analogues signed with prefix gr-. For example, the graded by a group $G$ ring $R = \bigoplus_{\sigma \in G} R_{\sigma}$ is called *gr-prime* if $IJ \neq 0$ for any nonzero graded ideals $I, J \subset R$. A left gr-Goldie ring is a ring without infinite ascending chains of graded left annihilators and without infinite direct sums of graded left ideals. The graded ring of fractions $Q^{gr}(R)$ is defined as $S^{-1}R$, where $S$ is the set of homogeneous regular
elements of $R$. A gr-simple ring is a ring without nontrivial graded ideals, and a gr-semisimple ring is a ring for which the graded module $R_R$ (or, which is equivalent, $R_R$) is decomposed into a direct sum of its gr-simple graded submodules. The graded analogue of implication (1) $\Rightarrow$ (2) of the Goldie theorem was proved to be false: in [1] there is a gr-semisimple ring which is a counterexample. It has been unknown whether there exists a counterexample for gr-prime Goldie rings, but the author has built such an example.

**Proposition.** There exists a gr-prime gr-Noetherian left but not gr-Artinian ring $R$ for which $S^{-1}R = R$, where $S$ is the set of homogeneous regular elements of $R$.

The graded analogue of the Goldie theorem is true with some extra restrictions: if $R$ is gr-semiprime, the theorem is true if group $G$ is periodic or if $R$ is a ring with finite support. If $R$ is gr-prime, it is true also if $G$ is abelian. The author has enhanced the last result by proving the following theorem.

**Theorem.** Let $R$ be a $G$-graded gr-prime left gr-Goldie ring, and let $G$ have a finite commutant. Then the classical graded left ring of fractions $S^{-1}R$ exists and is gr-semisimple.

The author is very grateful to his advisor A.L. Kanunnikov for the setting the problems, the useful discussions, and the care to his work.

**References**


Colength of $\ast$-polynomial identities of simple $\ast$-algebras

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**Keywords:** Algebras with polynomial identities, algebras with involution, colength.

**2010 Mathematics Subject Classification:** 16R10, 16R50, 16W10.

Let $(A, \ast)$ be an algebra with involution over a field $F$. Then $(A, \ast)$ is said to be $\ast$-simple if $A^2 \neq 0$ and it has no nontrivial $\ast$-invariant ideals. The list of finite dimensional $\ast$-simple algebras over an algebraically closed field $F$ consists of the matrix algebras $(M_n(F), t)$ with the transpose involution and $(M_n(F), s)$ with the symplectic involution (for even $n$), and of the algebras $(M_n(F) \oplus M_n(F)^{\text{op}}, \ast)$ with the exchange involution $(a,b)^\ast = (b,a)$, where $M_n(F)^{\text{op}}$ is the opposite algebra of $M_n(F)$ (with multiplication given by $a \circ b = ba$, where $ba$ is the product in $M_n(F)$).

Studying the $\ast$-polynomial identities of $\ast$-simple algebras over a field of characteristic 0, the first nontrivial cases are $(M_2(F), t), (M_2(F), s)$, and $(M_2(F) \oplus M_2(F)^{\text{op}}, \ast)$. Drensky and Giambruno [2] obtained the complete description of the cocharacters, codimensions and the Hilbert series of the polynomial identities of $(M_2(F), t)$ and $(M_2(F), s)$.

The subject of our study is the variety $\mathcal{M}$ of algebras with involution generated by the algebra $(M_2(F) \oplus M_2(F)^{\text{op}}, \ast)$. We show that the Hilbert series

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of the relatively free algebra in \( \mathfrak{M} \) freely generated by \( p \) symmetric and \( q \) skew-symmetric variables is

\[
H(F_{p,q}(\mathfrak{M}), T_p, U_q) = \frac{1}{\prod_{i=1}^{p} \prod_{j=1}^{q} (1 - t_i)(1 - u_j)} \times \\
\left( \prod_{i=1}^{p} \prod_{j=1}^{q} \frac{1}{(1 - t_i)(1 - u_j)} \sum_{a,b,c \geq 0} S_{(a+b,b)}(T_p)S_{(a+c,c)}(U_q) \right) \\
- \sum_{a+b > 0} S_{(a)}(T_p)S_{(b)}(U_q) - \sum_{a+b=3} S_{(1^a)}(T_p)S_{(1^b)}(U_q),
\]

where \( S_\lambda \) is the Schur function corresponding to the partition \( \lambda \). In the case of unitary algebras with ordinary PI-algebras the so called proper (or commutator) identities determine completely all polynomial identities. Similarly, for \(*\)-identities it is sufficient to know the \( Y\)-proper identities defined with the property that all symmetric variables \( y_i \) participate in commutators only. Let

\[
\xi_{i,j}(\mathfrak{M}) = \sum_{\lambda \vdash i} \sum_{\mu \vdash j} m_{\lambda,\mu}(\mathfrak{M}) \chi_\lambda \otimes \chi_\mu
\]

be the proper \( *\)-cocharacter sequence, where \( k \) and \( l \) are, respectively, the number of symmetric and skew-symmetric variables. Applying methods from [1] we have found the series of the \( Y\)-proper colengths

\[
l(t, u) = \sum_{i,j \geq 0} l_{i,j}(\mathfrak{M}) t^i u^j, \quad l_{i,j}(\mathfrak{M}) = \sum_{\lambda \vdash i} \sum_{\mu \vdash j} m_{\lambda,\mu}(\mathfrak{M}),
\]

which allows to express explicitly the \( Y\)-proper colengths \( l_{i,j}(\mathfrak{M}) \).

Joint work with V. Drensky and A. Giambruno.

References


On the multiplicity-free plethysms $p_2[s_\lambda]$

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Keywords: Schur functions, multiplicity-free, plethysm.

2010 Mathematics Subject Classification: 05E05, 20G05, 22E47.

Given two Schur functions $s_\mu(x)$ and $s_\lambda(x)$, where $x = (x_1, x_2, \ldots)$ is an infinite sequence of variables, $\mu$ and $\lambda$ are partitions of weight $m$ and $n$, respectively, the plethysm $s_\mu[s_\lambda(x)]$ is the symmetric function obtained by substituting the monomials of $s_\lambda(x)$ for the variables of $s_\mu(x)$. Littlewood [6] introduced this operation in the context of the representations of the general linear group and showed that for any partition $\mu$ of $m$,

$$s_\mu[s_\lambda(x)] = \sum_{\gamma \vdash mn} c_{\mu,\lambda}^\gamma s_\gamma(x)$$

where the sum runs over all partitions $\gamma$ of $mn$ and $c_{\mu,\lambda}^\gamma$ are non negative integers.

The problem of computing the coefficients $c_{\mu,\lambda}^\gamma$ is one of the fundamental open problems in the theory of symmetric functions and has proved to be very difficult. Essentially there are explicit formulas for $c_{\mu,\lambda}^\gamma$ in a few special cases.

Most algorithms for computing the expansions of $s_\mu[s_\lambda(x)]$ make use of the plethysm $p_k[s_\lambda(x)]$ of $s_\lambda(x)$ with the power symmetric function $p_k(x) = \sum_i x_i^k$ and involve multiplication of Schur functions ([1, 3, 4, 5]). We say that the plethysm $p_k[s_\lambda(x)]$ is multiplicity-free if every coefficient in the resulting Schur function expansion is $0, +1, -1$.

A well known example of multiplicity-free plethysm is $p_2[s_\lambda(x)]$, where $s_\lambda$ is the Schur function indexed by a rectangular partition ([3, 7]). Other examples of multiplicity-free plethysms are $p_2[s_\lambda(x)]$, where $\lambda$ is either a hook shape [1] or has two rows or two columns [3].

Here we will describe all the shapes $\lambda$ such that $p_2[s_\lambda]$ is multiplicity-free [2].
References


Formulae and bounds for the Brauer $p$-dimensions of Henselian valued fields

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Keywords: Brauer group, index-exponent pair, Brauer pair, Brauer/absolute Brauer $p$-dimension, Henselian field.


Let $E$ be a field, $E_{\text{sep}}$ a separable closure of $E$, $s(E)$ the class of associative finite-dimensional central simple $E$-algebras, $d(E)$ the subclass of division algebras $D \in s(E)$, $\mathbb{P}$ the set of prime numbers, and for each $A \in s(E)$, let $\text{ind}(A)$ be the Schur index of $A$ and $\text{exp}(A)$ the exponent of $A$ (the order of the equivalence class of $A$ in the Brauer group $\text{Br}(E)$). It is known that: $\text{Br}(E)$ is abelian; $(\text{ind}(A), \text{exp}(A))$ is a Brauer pair, i.e., $\text{exp}(A)$ divides $\text{ind}(A)$ and is divisible by $\text{rad}(\text{ind}(A))$; the Schur index function $\text{Ind}: \text{Br}(E) \to \mathbb{N}$ is multiplicative. Thus the study of the pairs $\text{ind}(A), \text{exp}(A)$, $A \in s(E)$, reduces to the case of $p$-primary $E$-pairs. It concentrates on the Brauer $p$-dimensions $\text{Br}_p(E): p \in \mathbb{P}$, and their supremum, the Brauer dimension $\text{Br}_d(E)$. We say that $\text{Br}_d(E) = n < \infty$, for a given $p$, if $n$ is the least integer $\geq 0$, for which $\text{ind}(A_p) \mid \exp(A_p)^n$ when $A_p \in s(E)$ and $\exp(A_p)$ is a power of $p$; if no such $n$ exists, $\text{Br}_d(E): = \infty$.

The absolute Brauer $p$-dimension $\text{abr}_d(E)$ is the supremum of $\text{Br}_d(R)$, taken over the finite extensions $R$ of $E$ in $E_{\text{sep}}$; the absolute Brauer dimension is

$$\text{abr}_d(E) = \sup\{\text{abr}_d(E): p \in \mathbb{P}\}.$$
We have $\text{Brd}_p(E) = \text{abrd}_p(E) = 1$, for all $p \in \mathbb{P}$, if $E$ is a global or local field (class field theory), or the function field of an algebraic surface over an algebraically closed field (de Jong, 2004; Lieblich, 2008). Also, $\text{abrd}_p(F_m) < p^{m-1}$, $p \in \mathbb{P}$, if $F_m$ is a field of $C_m$-type, in the sense of Lang, for some $m \in \mathbb{N}$ (Matzri, 2016).

This talk presents lower and upper bounds on $\text{Brd}_p(E)$ and $\text{abrd}_p(E)$, in case $E$ possesses a Henselian valuation $v$. They are expressed by invariants of the value group $v(E)$, including $\text{Brd}_p(E)$, the dimension of the group $v(E)/pv(E)$ as a vector space over the field $\mathbb{F}_p$ with $p$ elements, and the rank of $\mathcal{G}(\hat{E}(p)/E)$ as a pro-$p$-group $(\hat{E}(p)$ is the maximal $p$-extension of $\hat{E})$. When $p \neq \text{char}(\hat{E})$, these bounds depend on whether or not $\hat{E}$ contains a primitive $p$-th root of unity. This yields formulae for $\text{Brd}_p(E)$ and $\text{abrd}_p(E)$ in the following cases (see [3], [4]): $\hat{E}$ is a global field or the function field of an algebraic surface over an algebraically closed field; $\text{Brd}_p(\hat{E}) = 0$ or $\hat{E}$ is a local field; $\hat{E}$ is the function field of an algebraic curve over a perfect PAC-field; $\text{char}(E) = p$ and $(E, v)$ is maximally complete. In addition, a formula for $\text{Brd}_p(Y)$ is found, for any field $Y$ with a primitive $p$-th root of unity and a solvable Galois group $\mathcal{G}(Y(p)/E)$. The obtained results are used for describing the set of sequences $\text{abrd}_p(K), \text{Brd}_p(K) : p \in \mathbb{P}$, where $(K, v_K)$ runs across the class of Henselian fields with $\text{char}(\hat{K}) = 0$ (and in case $(K, v_K)$ are maximally complete, $\text{char}(K) = q > 0$ and $\hat{K}$ are perfect). As shown in [2], the description enables one to prove that, for each $(q, k) \in (\mathbb{P} \cup \{0\}) \times \mathbb{N} \cup (0, 0)$, there is a field $\Phi_{q,k}$, such that $\text{char}(\Phi_{q,k}) = q$, $\text{Brd}(\Phi_{q,k}) = k$, and finitely-generated transcendental extensions $F_{q,k}/\Phi_{q,k}$ satisfy the following:

(1) A Brauer pair $(n, m)$ is an index-exponent pair over $F_{q,k}$ in the following cases: $q = 0$ and $2 \nmid mn$; $q > 0$ and $\text{g.c.d.}\{mn, q - 1\} = 1$.

This solves Problem 4.4 of [1] (in the strongest presently known form).

References


On the maximal containments of lower central series ideals

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Keywords: Lower central series, polynomial identities, PBW grading, free associative algebra, \( GL_2 \)-representations.

2010 Mathematics Subject Classification: 16R10.

We begin a discussion about the maximal containments of lower central series ideals: ideals generated by products of two–sided ideals of the lower central series of the free associative algebra on \( n \) generators. We introduce two new ideas to the topic, the PBW grading and the pigeonhole principle, that help us give a complete classification of the containments for \( n = 2 \) and obtain partial results in the general case.

*The research of the author began at the Research Science Institute, Summer 2013, and was a part of the Undergraduate Research Opportunities Program at the Massachusetts Institute of Technology, Fall 2014 and Spring 2015.
On color analytic loops and their tangent spaces

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**Keywords:** Sabinin algebra, color algebra, color manifold, local integration

**2010 Mathematics Subject Classification:** 17D99, 20N05

A color algebra of color \((G, \chi)\), where \(G\) is an abelian group and \(\chi : G \times G \to \mathbb{C}\) is a bicharacter, is a \(G\)-graded algebra where the natural commutation on homogeneous elements is given by \(C : v \otimes w \mapsto \chi(|v|, |w|)(w \otimes v)\). Examples are, for instance, super-algebras for \((\mathbb{Z}_2, \chi(a, b) = (-1)^{ab})\) or \(G\)-graded algebras for \((G, \chi(a, b) = 1)\).

This commutation let us define varieties of color algebras for various familiar varieties as Lie algebras, associative algebras, Akivis algebras, Sabinin algebras, etc.

In this talk, I will present color manifolds and study the relation between color Sabinin algebras and color analytic loops.
Visualization in algebra

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Keywords: Applications of graph theory, algebras with polynomial identity, Burnside problem, nonassociative algebras.

2010 Mathematics Subject Classification: 00A08, 05C05, 05C25, 16P90, 16R10, 16R30, 17A30, 17A50, 20E08.

The purpose of the talk is to present a selection of algebraic results in a way understandable for the large mathematical and non-mathematical audience. The results discussed include also results obtained by some of the participants at the Conference.

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The Nowicki conjecture for free metabelian Lie algebras

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Keywords: Free metabelian Lie algebras, Weitzenböck derivations, algebras of constants.
2010 Mathematics Subject Classification: 13N15, 13A50, 17B01, 17B30, 17B40.

The classical theorem of Weitzenböck [7] states that the algebra of constants of a nonzero locally nilpotent linear derivation \( \delta \) of the \( m \)-generated polynomial algebra over a field \( K \) of characteristic 0 is finitely generated. Let \( m \) be even and let the Jordan normal form of \( \delta \) consist of only \( 2 \times 2 \) Jordan cells. We may assume that \( \delta \) acts on \( K[X,Y] = K[x_1, \ldots, x_n, y_1, \ldots, y_n], m = 2n, \) by

\[
\delta(y_i) = x_i, \delta(x_i) = 0, \quad i = 1, \ldots, n.
\]

Nowicki [6] conjectured that the algebra of constants \( K[X,Y]^\delta \) is generated by \( X \) and \( x_iy_j - x_jy_i \) for all \( 1 \leq i < j \leq n \). This conjecture was verified by several authors with proofs based on different ideas: The proof of Khoury in his Ph.D. thesis [3], see also [4], is very computational and uses essentially Gröbner basis techniques. Other unpublished proofs were given by Derksen and Panyushev who applied ideas of classical invariant theory. Drensky and Makar-Limanov [2] gave an elementary proof using easy arguments from undergraduate algebra and a simple induction only, without involving any invariant theory. Bedratyuk [5] reduced the Nowicki conjecture to the well known problem of classical invariant

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theory for the description of the algebra of joint covariants of $n$ linear binary forms. Finally, another simple proof was given by Kuroda [5].

We study the Lie analogue of the problem assuming that the above derivation $\delta$ acts on the free metabelian Lie algebra $F(X,Y) = L(X,Y)/L''(X,Y)$, where $L(X,Y)$ is the free Lie algebra of rank $2n$. We have found a set of generators of algebra of constants $F(X,Y)^\delta$. Although $F(X,Y)^\delta$ is not finitely generated, our main result may be considered as an analogue of the Nowicki conjecture. The algebra $F(X,Y)^\delta$ is generated by the set $X$ and the constants $F'(X,Y)^\delta$ from the commutator ideal of $F(X,Y)$. We present an explicit finite set of generators of $F'(X,Y)^\delta$ as a $K[X,Y]^\delta$-module.

The results were obtained with Vesselin Drensky during the author’s visit to Institute of Mathematics and Informatics, Bulgarian Academy of Sciences.

References


On the analog of Amitsur’s conjecture for polynomial $H$-identities

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Keywords: Associative algebra, polynomial identity, Hopf algebra, $H$-module algebra, codimension.


Polynomial identities of an algebra $A$ are polynomials in non-commutative variables that vanish under all evaluations in $A$. Such polynomial identities form the ideal $\text{Id}(A)$ in the corresponding free algebra.

Study of polynomial identities is an important aspect of study of algebras themselves. It turns out that the asymptotic behaviour of numeric characteristics of polynomial identities of $A$, in particular, the codimensions $c_n(A)$, is tightly related to the structure of $A$ itself.

Let $P_n$ be the vector space of multilinear polynomials in $x_1, \ldots, x_n$. Then

$$c_n(A) := \dim \frac{P_n}{P_n \cap \text{Id}(A)}.$$

Conjecture 1 (S.A. Amitsur). Let $A$ be an associative PI-algebra (i.e., it possesses a non-trivial polynomial identity) over a field of characteristic 0. Then there exists

$$\text{PIexp}(A) := \lim_{n \to \infty} \sqrt[n]{c_n(A)} \in \mathbb{Z}_+.$$
The original Amitsur’s conjecture was proved in 1999 by A. Giambruno and M.V. Zaicev.

Algebras with an additional structure (e.g., grading, (semi)group, Hopf or Lie algebra action) appear in many areas of mathematics and physics. For such algebras, it is natural to consider polynomial identities involving this additional structure. Usually, to find such new identities is easier than to find the ordinary ones. Furthermore, these new identities completely determine the ordinary polynomial identities. Therefore the question arises whether the analog of Amitsur’s conjecture holds of polynomial identities involving the additional structure.

In the talk we shall discuss the generalizations together with positive and negative results on them. It seems that the analog of the conjecture in the following form is still true:

**Conjecture 2.** Let $A$ be a finite dimensional associative $H$-module PI-algebra for an arbitrary Hopf algebra $H$ over a field of characteristic 0. Then there exists

$$\text{PIexp}^H(A) := \lim_{n \to \infty} \sqrt[n]{c_n^H(A)} \in \mathbb{Z}_+,$$

where $c_n^H(A)$ is the $n$-th codimension of polynomial $H$-identities of $A$.

Conjecture 2 can be reduced to the existence for each $H$-simple component $B$ of $B/J^H(A)$ of an $H$-polynomial $f \notin \text{Id}^H(B)$ that has sufficiently many alternations. (Here $J^H(A)$ is the maximal nilpotent $H$-invariant ideal of $A$.) For example, Conjecture 2 is true in the case when the ordinary Jacobson radical $J(A)$ is an $H$-submodule. Also we will discuss how Conjecture 2 can be proved for Hopf algebras $H$ obtained from smaller Hopf algebras by Ore extensions.
Free subalgebras, graded algebras and nilpotent elements

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Keywords: Graded algebras, graded nil rings, graded nilpotent rings, free subalgebras, Jacobson radical, monomial algebras, Köthe conjecture, combinatorics on words, morphic words.

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The famous Köthe conjecture asserts that the sum of two nil left ideals is always nil. This still open problem has attracted many researchers and inspired a flurry of results toward a better understanding of its validity.

One of its most popular equivalent formulations is, that the polynomial ring $R[x]$ over a nil ring $R$ is equal to its own Jacobson radical.

The observation that $R[x]$ is naturally graded, and every homogeneous element is nilpotent (i.e., $R[x]$ is graded nil) motivated Bartholdi (2002) and Small and Zelmanov (in the case of uncountable fields, 2006) to ask whether a graded nil algebra is always Jacobson radical. As for the opposite direction, it is indeed the case that the Jacobson radical of a $\mathbb{Z}$-graded algebra is graded nil.

This was disproved by Smoktunowicz (2008), and should be mentioned together with another result by Smoktunowicz, disproving a conjecture of Makar-Limanov: Smoktunowicz constructed a nil ring $R$ such that after extension by several (in fact, 6) central variables it contains a free subalgebra. Such ring can exist only over countable base fields (by classical results of Amitsur).

In this talk we present a new construction, which provides over arbitrary base fields a monomial, graded nilpotent ring (a stronger property than graded
nil: every subring generated by homogeneous elements of the same degree is nilpotent) which contains a free subalgebra.

Our methods involve combinatorics of infinite words, and gluing together sequences of letters which arise from appropriate morphisms of free monoids. In particular, this resolves Small-Zelmanov’s question and could be thought of as a continuation to the uncountable base field case of Smoktunowicz’s counterexample to Makar-Limanov’s conjecture.

We finally show how, modifying a beautiful and not so well known example of Rowen (1989), one can produce graded algebras which are not Jacobson radicals over any field.

The talk is based on a recent joint work [1] with Jason P. Bell.

References

Invariants of symplectic and orthogonal groups acting on GL-modules

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Keywords: Invariant theory, classical groups, Hilbert series, Schur functions.

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Let $W$ be a polynomial $GL(n, \mathbb{C})$-module. Then $W$ can be written as a direct sum

$$W = \bigoplus_{\lambda} k(\lambda) V_{\lambda}^n,$$

where $\lambda = (\lambda_1, \ldots, \lambda_n) \in (\mathbb{Z}_{\geq 0})^n$ is a partition and $V_{\lambda}^n$ is the irreducible $GL(n, \mathbb{C})$-module with highest weight $\lambda$. We extend diagonally the action of $GL(n, \mathbb{C})$ on the symmetric algebra $\mathbb{C}[W]$ and study the algebras of invariants $\mathbb{C}[W]^G$, where $G$ is one of the classical groups $SL(n, \mathbb{C})$, $O(n, \mathbb{C})$, $SO(n, \mathbb{C})$, and $Sp(2k, \mathbb{C})$ (the latter in the case $n = 2k$). The Hilbert (or Poincaré) series of $\mathbb{C}[W]^G$ is the formal series

$$H(\mathbb{C}[W]^G, t) = \sum_{d\geq 0} \dim(\mathbb{C}[W]^G)_d t^d,$$

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$$(\mathbb{C}[W])_d = \sum_{\lambda} m(\lambda, d) V^n_\lambda,$$

then, starting with the decomposition of $W$ into a direct sum of irreducible components and using the theory of rational symmetric functions, they compute the multiplicity series

$$M(\mathbb{C}[W], z_1, \ldots, z_n, t) = \sum_{\lambda_i, d} m(\lambda, d) z_1^{\lambda_1} \cdots z_n^{\lambda_n} t^d.$$

Then $H(\mathbb{C}[W]^{SL(n, \mathbb{C})}, t)$ is obtained from the component of $M(\mathbb{C}[W], z_1, \ldots, z_n, t)$ for $\lambda_1 = \cdots = \lambda_n$. We further develop the method from [1] and show how to extend it to compute also the Hilbert series of the algebras of invariants $H(\mathbb{C}[W]^G, t)$ for $G = O(n, \mathbb{C}), SO(n, \mathbb{C})$, and $G = \text{Sp}(2k, \mathbb{C})$. We give explicit examples for computing $H(\mathbb{C}[W]^G, t)$, some of which were known and some of which were not known before. We discuss also further generalizations and applications of this method. This is a joint work with Vesselin Drensky.

References

Nondistributive rings

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Keywords: Nearrings, nondistributive rings, Ore localizations.
2010 Mathematics Subject Classification: 16Y30, 16S85, 16U20.

Referring to a graduate course in Abstract Algebra, by a ring we mean a set $R$ of no fewer than two elements, together with two binary operations called the addition and multiplication, in which (1) $R$ is an abelian group with respect to the addition, (2) $R$ is a semigroup with unit with respect to the multiplication, (3) $(r + s)t = rt + st$ and $r(s + t) = rs + rt$ for any $r, s, t \in R$. A nearring $N$ is a generalization of a ring, namely the addition needs not be abelian and only the right distributive law is required, additionally the left distributive law is replaced by $n0 = 0$ for every $n \in N$. The last postulate means that we require a nearring to be zerosymmetric. The talk is intended as a discussion on sets $N$ satisfying the nearring axioms except the right distributive law, which we replace by $0n$ for every $n \in N$. 
The first fundamental theorem for similarity groups and $G$-equivalence conditions of vectors given in $R^2$

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**Keywords:** First fundamental theorem, $G$-equivalence, homotethy, similarity groups.

**2010 Mathematics Subject Classification:** 13A50, 14L24, 03D50, 51L10.

Let $R[x_1, x_2, \ldots, x_n]^G$ be the ring of $G$-invariant real valued polynomials with $n$ vector variables and $R(x_1, x_2, \ldots, x_n)^G$ be the field of $G$-invariant real valued rational functions with $n$ vector variables. The generators of the field $R(x_1, x_2, \ldots, x_n)^G$ are called generator invariants of the vectors $\{x_1, x_2, \ldots, x_n\}$. Let $S(2)$ be the similarity transformations’ group in $R^2$ and $LS(2)$ be the linear similarity transformations’ group which is an important subgroup of $S(2)$.

In this study the first fundamental theorem which have great importance in the area of invariant theory, is stated for $G = LS(2)$ and $G = S(2)$.

Finally if two vector systems $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_n\}$ are given then the $G$-equivalence conditions of these systems are stated as generator invariants of the systems $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_n\}$.

**References**


Standard identities on matrix algebras with superinvolution

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Keywords: Polynomial identity, superinvolution, minimal degree.

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The celebrated theorem of Amitsur and Levitski (see [1]) states that the standard polynomial $St_{2n}$ is an identity of $M_n(F)$, the algebra of $n \times n$ matrices over a field $F$. Moreover, if $\text{char} F \neq 2$, $St_{2n}$ is, up to a scalar, the only identity of minimal degree of $M_n(F)$.

The general question whether the Amitsur-Levitski theorem could be extended to a special kind of matrices was consider by Kostant and then by Rowen. They proved some powerful results (see [3, 4, 5]) concerning symmetric and skew matrices with respect to the transpose or the symplectic involution.

I shall discuss analogous results in the setting of matrix algebras with superinvolution (see [2]).

References


Riemann Hypothesis Analogue
for locally finite modules over
the absolute Galois group of a finite field*

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Keywords: Rational ordinary generating functions, locally finite modules
over a profinite group, Riemann Hypothesis Analogue, Galois closure of a finite
covering of locally finite modules.


The present note translates Bombieri’s proof [5] of the Hasse-Weil Theorem for smooth
irreducible curves \(X/F_q \subset \mathbb{P}^n(F_q)\), defined over a finite field \(F_q\) in terms of
the locally finite action of the absolute Galois group \(\mathfrak{G} = \text{Gal}(F_q/F_q)\) of \(F_q\) on \(X\) and
the Galois closure of the Noether Normalization map \(X \dashrightarrow \mathbb{P}^1(F_q)\). It provides sufficient
conditions for an abstract locally finite \(\mathfrak{G}\)-module \(M\) to satisfy the Riemann Hypothesis Analogue with respect
to the projective line \(\mathbb{P}^1(F_q)\).

If \(\Phi_q : F_q \rightarrow F_q\), \(\Phi(a) = a^q\) is the Frobenius automorphism then \(\mathfrak{G} = \text{Gal}(F_q/F_q) = \langle \Phi_q \rangle \simeq (\mathbb{Z}, +)\) is
the profinite completion of \(\langle \Phi_q \rangle \simeq (\mathbb{Z}, +)\). A \(\mathfrak{G}\)-module \(M\) is locally finite if \(\forall \text{Orb}_{\mathfrak{G}}(x)\), \(x \in M\) is of finite degree 
\(\deg \text{Orb}_{\mathfrak{G}}(x) := |\text{Orb}_{\mathfrak{G}}(x)|\) and for \(\forall m \in \mathbb{N}\) there are at most finitely many \(\nu \in \text{Orb}_{\mathfrak{G}}(M)\) of
\(\deg \nu = m\). The formal power series \(\zeta_M(t) = \prod_{\nu \in \text{Orb}_{\mathfrak{G}}(M)} (1 - t^{\deg \nu})\) is called the
\(\zeta\)-function of \(M\). For \(\forall r \in \mathbb{N}\) we note that the set \(M^{\Phi_q^r} := \{x \in M \mid \Phi_q^r(x) = x\}\)

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67
of the $\mathbb{F}_{q^r}$-rational points is finite and $\zeta_M(t) = \exp \left( \sum_{r=1}^{\infty} \frac{\left| M^{\mathbb{F}_{q^r}} \right|}{t^r} \right)$. Recall that $\zeta_{\mathbb{P}^1(C)}(t) = \frac{1}{(1-t)(1qt)}$ for the projective line $\mathbb{P}^1(C)$.

**Definition 1.** A locally finite $\mathfrak{G} = \text{Gal}(\overline{\mathbb{F}}_q/\mathbb{F}_q)$-module $M$ satisfies the Riemann Hypothesis Analogue RHA/-$\mathbb{P}^1(\overline{\mathbb{F}}_q)$ with respect to $\mathbb{P}^1(\overline{\mathbb{F}}_q)$ if $P_M(t) = \frac{\zeta_M(t)}{\zeta_{\mathbb{P}^1(\overline{\mathbb{F}}_q)}(t)} = \prod_{i=1}^{d} (1-\alpha_it) \in \mathbb{Z}[t]$ is a polynomial of degree $P_M(t) = d \in \mathbb{N}$ and $\forall \alpha_i$ are of one and a same $|\alpha_i| = \sqrt[d]{|a_d|} \in \mathbb{R}_{>0}$, where $a_d = \text{LC}(P_M(t))$ is the leading coefficient of $P_M(t)$.

Note that $M$ is subject to RHA/-$\mathbb{P}^1(\overline{\mathbb{F}}_q)$ exactly when all zeroes $s_o \in \mathbb{C}$ of $\zeta_M(q^{-s})$, $s \in \mathbb{C}$ have $\text{Re}(s_o) = \log_q \sqrt[d]{|a_d|}$. By the Hasse-Weil Theorem, any curve $X/\mathbb{F}_q \subset \mathbb{P}^N(\overline{\mathbb{F}}_q)$ of genus $g \geq 1$ satisfies RHA/-$\mathbb{P}^1(\overline{\mathbb{F}}_q)$. This is analogous to the original Riemann Hypothesis $\text{Re}(z_o) = 1/2$ for the non-trivial zeroes $z_o \in \mathbb{C} \setminus (-2\mathbb{N})$ of Riemann’s $\zeta$-function $\zeta(z) := \sum_{n=1}^{\infty} \frac{1}{n^z}$, $z \in \mathbb{C}$, being equivalent to $\text{Re}(s_o) = 1/2$ for $\forall s_o \in \mathbb{C}$ with $\zeta_X(q^{-s_o}) = 0$.

A finite covering of locally finite $\mathfrak{G}$-modules is a $\mathfrak{G}$-equivariant map $\xi : M \to L$ with $|\xi^{-1}(x)| = \deg \xi \in \mathbb{N}$ for $\forall x \in L$. If $N$ is an infinite locally finite $\mathfrak{G}$-module and $G \leq Z(\mathfrak{G}) := \{ g \in \text{Sym}(N) \mid g\xi = \xi, \forall \alpha \in \mathfrak{G} \} \leq \text{Sym}(N)$ is a finite group with $G \cap \mathfrak{G} = \{ \text{Id}_N \}$ and $\text{Stab}_G(x) = \{ \text{Id}_N \}$ for $\forall x \in N$ then $\xi_G : N \to N/G$, $\xi_G(x) = \text{Orb}_G(x)$

is called a $G$-Galois covering. A finite covering $\xi : M \to L$ of locally finite $\mathfrak{G}$-modules has Galois closure $\xi_G : N \to L = N/G$ if $N$ is a locally finite $\mathfrak{G}_m = \text{Gal}(\overline{\mathbb{F}}_q/\mathbb{F}_{q^n})$-module and there are $G_1 \leq G \leq Z(\mathfrak{G}_m) \leq \text{Sym}(N)$ with $G \cap \mathfrak{G}_m = \{ \text{Id}_N \}$, $|G| < \infty$, $\text{Stab}_G(x) = \{ \text{Id}_N \}$ for $\forall x \in N$, such that $\xi_GG_1 = \xi_G$ as finite coverings of $\mathfrak{G}_m$-modules.

For any smooth irreducible curve $X/\mathbb{F}_q \subset \mathbb{P}^n(\overline{\mathbb{F}}_q)$ there is a finite covering $\xi : X_o \to L_o$ of $\mathfrak{G} = (\overline{\mathbb{F}}_q)$-modules with $|X \setminus X_o| < \infty$, $|\mathbb{P}^1(\overline{\mathbb{F}}_q) \setminus L_o| < \infty$. Any such $\xi : X_o \to L_o$ has a finite Galois closure $\xi_G : Z_o \to L_o$ for some smooth quasi-projective curve $Z_o/\mathbb{F}_{q^m} \subset \mathbb{P}^s(\overline{\mathbb{F}}_q)$, defined over a finite extension $\mathbb{F}_{q^m} \supseteq \mathbb{F}_q$.

**Lemma-Definition 2.** If $\xi : M \to L$ is a finite covering of locally finite $\mathfrak{G}$-modules then the inertia map $e_\xi : M \to \mathbb{Q}_{>0}$,

$$e_\xi(x) := \frac{\deg \text{Orb}_{\mathfrak{G}}(x)}{\deg \text{Orb}_{\mathfrak{G}}(\xi(x))} = \frac{[\mathfrak{G} : \text{Stab}_{\mathfrak{G}}(x)]}{[\mathfrak{G} : \text{Stab}_{\mathfrak{G}}(\xi(x))]}$$
$$= \left[ \text{Stab}_G(\xi(x)) : \text{Stab}_G(x) \right] \text{ for } \forall x \in M$$

takes finitely many natural values $1 \leq e_\xi(x) \leq \deg \xi$, called the inertia indices of $\xi$.

**Definition 3.** (i) Let $\xi : M \to L_o \subseteq \mathbb{P}^1(\overline{\mathbb{F}}_q)$ be a finite covering of locally finite $\mathfrak{G} = (\overline{\Phi}_q)$-modules with $|\mathbb{P}^1(\overline{\mathbb{F}}_q) \setminus L_o| < \infty$, $M(1) := \{x \in M | \deg \text{Orb}_\mathfrak{G}(x) = \deg \text{Orb}_\mathfrak{G}(\xi(x)) \}$ be the submodule of the bijective restrictions of $\xi$ and $\lambda \in \mathbb{R}^>0$. We say that $\xi$ is of order $\lambda$ if there exist $C \in \mathbb{R}^>0$, $\nu \in \mathbb{N}$ with $|M(1)^{\Phi_{q^r}}| - \mathbb{P}^1(\overline{\mathbb{F}}_q)^{\Phi_{q^r}} \leq C |\mathbb{P}^1(\overline{\mathbb{F}}_q)^{\Phi_{q^r}}|^{\lambda}$ for $\forall r \in \mathbb{N}$ and the non-trivial inertia indices $e_\xi(M) \setminus \{1\}$ have $\min(e_\xi(M) \setminus \{1\}) \geq \lambda^{-1}$.

(ii) Let us suppose that $\xi_G : N \to L_o \subseteq \mathbb{P}^1(\overline{\mathbb{F}}_q)$ is a finite $G$-Galois covering of locally finite $\mathfrak{G} = (\overline{\Phi}_q)$-modules with $|\mathbb{P}^1(\overline{\mathbb{F}}_q) \setminus L_o| < \infty$ and $\lambda \in \mathbb{R}^>0$. Then $\xi_G$ is of $G$-order $\lambda$ when for any $g \in G$ there exist constants $C = C(g) \in \mathbb{R}^>0$, $\nu = \nu(g) \in \mathbb{N}$ with $|M(1)^{\Phi_{q^r}g}| - \mathbb{P}^1(\overline{\mathbb{F}}_q)^{\Phi_{q^r}} \leq C |\mathbb{P}^1(\overline{\mathbb{F}}_q)^{\Phi_{q^r}}|^{\lambda}$ for $\forall r \in \mathbb{N}$ and the non-trivial inertia indices $e_{\xi_G}(M) \setminus \{1\}$ have $\min(e_{\xi_G}(M) \setminus \{1\}) \geq \lambda^{-1}$.

If $X_o/\mathbb{F}_q \subseteq \mathbb{P}^n(\overline{\mathbb{F}}_q)$ is a smooth irreducible quasi-projective curve then any finite covering $\xi : X_o \to L_o \subseteq \mathbb{P}^1(\overline{\mathbb{F}}_q)$ of $\mathfrak{G} = (\overline{\Phi}_q)$-modules with $|\mathbb{P}^1(\overline{\mathbb{F}}_q) \setminus L_o| < \infty$ is of order $1/2$. Any finite $G$-Galois covering $\xi_G : X_o \to L_o \subseteq \mathbb{P}^1(\overline{\mathbb{F}}_q)$ of $\mathfrak{G} = (\overline{\Phi}_q)$-modules with $|\mathbb{P}^1(\overline{\mathbb{F}}_q) \setminus L_o| < \infty$ is of $G$-order $1/2$.

**Theorem 4.** Let $M$ be an infinite locally finite $\mathfrak{G} = \text{Gal}(\mathbb{F}_q/\mathbb{F}_q)$-module with polynomial $\zeta$-quotient $P_M(t) = \zeta M(t) \in \mathbb{Z}[t]$ of $\deg P_M(t) = d \in \mathbb{N}$ and $\text{LC}(P_M(t)) = a_d \in \mathbb{Z}\setminus\{0, \pm 1\}$. If there are $\mathfrak{G}$-submodules $M_o \subseteq M$, $L_o \subseteq \mathbb{P}^1(\overline{\mathbb{F}}_q)$ with $|M \setminus M_o| < \infty$, $|\mathbb{P}^1(\overline{\mathbb{F}}_q) \setminus L_o| < \infty$ and a finite covering $\xi : M_o \to L_o$ of $\mathfrak{G}$-modules of order $\lambda := \log_q \sqrt[\lambda]{|a_d|}$ with Galois closure $\xi_G : N_o \to L_o$ of $G$-order $\lambda$ then $M$ satisfies RHA/$\mathbb{P}^1(\overline{\mathbb{F}}_q)$.

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Gradings by abelian groups on Lie algebras

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The root space decomposition of a semisimple Lie algebra with respect to a Cartan subalgebra can be regarded as a grading by a free abelian group (the root lattice). In the 1960’s, V. Kac classified all gradings by cyclic groups on finite-dimensional simple Lie algebras over the complex numbers. Since then, there has been considerable interest in gradings on Lie algebras, especially the simple ones, by arbitrary abelian groups. Such gradings arise in the theory of symmetric spaces, infinite-dimensional Lie algebras, and colour Lie superalgebras. So-called fine gradings have recently been classified, by efforts of many authors, for all finite-dimensional simple Lie algebras over an algebraically closed field of characteristic 0. We will review some of the classification results and open problems in this area.
Varieties of algebras with superinvolution of polynomial growth

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Keywords: Polynomial identity, superinvolution, growth.


Let \( A \) be an associative algebra over a field \( F \) of characteristic zero and let \( c_n(A), n = 1, 2, \ldots \), be its sequence of codimensions. It is well known that the sequence of codimensions of a PI-algebra either grows exponentially or is polynomially bounded. A celebrated theorem of Kemer characterizes the algebras whose sequence of codimensions is polynomially bounded as follows. Let \( G \) be the infinite dimensional Grassmann algebra over \( F \) and let \( UT_2 \) be the algebra of \( 2 \times 2 \) upper triangular matrices. Then \( c_n(A), n = 1, 2, \ldots \), is polynomially bounded if and only if \( G, UT_2 \notin \text{var}(A) \), where \( \text{var}(A) \) denotes the variety of algebras generated by \( A \).

In the setting of algebras with superinvolution, in analogy with the ordinary case, one defines the sequence of \(*\)-codimensions of \( A \) and it turns out that if \( A \) satisfies an ordinary identity, then its sequence of \(*\)-codimensions is exponentially bounded. Moreover, if \( A \) is finite dimensional such sequence is polynomially bounded if and only if \( A \) generates a variety not containing a finite number of \(*\)-algebras: the group algebra of \( \mathbb{Z}_2 \) and a 4-dimensional subalgebra of the \( 4 \times 4 \) upper triangular matrices with suitable superinvolutions (see [1]).

In this note we focus our attention on such algebras which are the only finite dimensional \(*\)-algebras generating varieties of almost polynomial growth, i.e., varieties of exponential growth such that any proper subvariety has polynomial
growth and we classify the subvarieties of such varieties by giving a complete list of generating finite dimensional $\ast$-algebras. Along the way we classify all minimal varieties of polynomial growth and surprisingly we show that their number is finite for any given growth (see [2]).

References


Evaluations of non-commutative polynomials on finite dimensional algebras

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**Keywords:** Finite dimensional algebras, non-commutative polynomials, PI-algebras, Kaplansky conjecture.

**2010 Mathematics Subject Classification:** 16R30, 15A24, 17B60.

Let $p$ be a multilinear polynomial in several non-commuting variables with coefficients in an arbitrary field $K$. Kaplansky conjectured that for any $n$, the image of $p$ evaluated on the set $M_n(K)$ of $n$ by $n$ matrices is either zero, or the set of scalar matrices, or the set $sl_n(K)$ of matrices of trace 0, or all of $M_n(K)$. We will present results related to Kaplansky conjecture in cases $n = 2$ and $n = 3$. We will also present a classification of possible images of multilinear polynomials evaluated on quaternion algebra.
Polynomial growth and star-varieties

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2010 Mathematics Subject Classification: 16R10, 16R50.

Let \( V \) be a variety of associative algebras with involution over a field \( F \) of characteristic zero and let \( c_n(V), n = 1, 2, \ldots, \) be its \(*\)-codimension sequence. Such a sequence is polynomially bounded if and only if \( V \) does not contain the commutative algebra \( F \oplus F \), endowed with the exchange involution, and \( M \), a suitable 4-dimensional subalgebra of the algebra of \( 4 \times 4 \) upper triangular matrices (see [1]). Such algebras generate the only varieties of \(*\)-algebras of almost polynomial growth, i.e., varieties of exponential growth such that any proper subvariety is polynomially bounded.

In this talk I shall present the results of [2] in which all subvarieties of the \(*\)-varieties of almost polynomial growth were classified by giving a complete list of finite dimensional \(*\)-algebras generating them.

References


Maximal subalgebras of modular simple Lie (super)algebras of Cartan type

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Keywords: Modular simple Lie algebras, maximal subalgebras, algebras of Cartan type.

2010 Mathematics Subject Classification: 17B05, 17B20, 17B50.

In Lie Theory, there are a number of well-known problems where the maximal subsystems play a crucial role. In general, the study of the maximal subsystems of an algebraic system, such as finite groups, Lie groups, and Lie (super)algebras, is an essential part of the structural characterization of the system. One of the beautiful results of classical Lie theory is the classification of maximal subalgebras of simple Lie algebras over the field of complex numbers.

In this talk we will review key points of classification of graded maximal subalgebras of the simple restricted Cartan type Lie algebras, and then extend that approach to characterize all maximal graded subalgebras of the restricted Lie superalgebras of Cartan type.
Maximal commutativity of a subalgebra in quantized coordinate rings

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Keywords: Quantized coordinate rings, quantum groups, Poisson algebra, semiclassical limit.

2010 Mathematics Subject Classification: Primary: 16T20, 20G42. Secondary: 13A50, 81R50.

The quantum special linear group is an appropriate deformation of the coordinate ring of the special linear group. In another approach, it is the Hopf algebraic dual of the Drinfeld-Jimbo algebra of the special linear Lie algebra. In this algebra, that is conveniently given by generators and relations, the subalgebra of cocommutative elements is generated by sums of certain elements that are called quantum minors (In the classical commutative case, these generators are the coefficients of the characteristic polynomial).

We show that for generic deformation parameter this subalgebra coincides with the centralizer of the trace, and hence is a maximal commutative subalgebra. In addition, we discuss the analogue of this statement in the case of its semi-classical limit Poisson algebra.

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Extending Johnson’s theorem on regularity of algebras of quotients to Jordan algebras

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Keywords: Jordan algebras, algebras of quotients, von Neumann regular, nonsingular.

2010 Mathematics Subject Classification: Primary: 17C10. Secondary: 17C05.

Johnson’s theorem on associative rings of quotients asserts that an associative ring is right nonsingular if and only if its maximal ring of right quotients is von Neumann regular. We examine here the corresponding assertion for Jordan algebras involving the maximal ring of quotients as defined by the author, and the notion of strong regularity [1, 2]. We show that although the natural Jordan version of Johnson’s theorem does not hold in general, the result holds for PI Jordan algebras.

References


Nil Lie superalgebras of slow growth

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**Keywords:** Growth, self-similar algebras, nil-algebras, Lie superalgebra, restricted Lie algebras.

**2010 Mathematics Subject Classification:** 16P90, 16N40, 16S32, 17B50, 17B66, 17B70.

The Grigorchuk and Gupta-Sidki groups play fundamental role in modern group theory. They are natural examples of self-similar finitely generated periodic groups. The author constructed their analogue in case of restricted Lie algebras of characteristic 2 [2], Shestakov and Zelmanov extended this construction to an arbitrary characteristic [4]. There are more examples of (self-similar) finitely generated restricted Lie algebras with a nil $p$-mapping [3]. In characteristic zero, similar examples of Lie algebras do not exist by a result of Martinez and Zelmanov [1].

Now we construct analogues of the Grigorchuk and Gupta-Sidki groups in the world of Lie superalgebras of arbitrary characteristic and study their properties and properties of their associative hulls. In these examples, $\text{ad}(a)$ is nilpotent, $a$ being even or odd with respect to $\mathbb{Z}_2$-grading as Lie superalgebras. This property is an analogue of the periodicity of the Grigorchuk and Gupta-Sidki groups. In particular, we obtain an example of a nil finely-graded Lie superalgebra of slow polynomial growth, which shows that an extension of a theorem due to Martinez and Zelmanov [1] for the Lie superalgebras of characteristic zero is not valid.

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We also construct a family of 2-generated restricted Lie algebras of slow polynomial growth with a nil $p$-mapping, where the field of positive characteristic is arbitrary. We obtain a continuum subfamily of nil restricted Lie algebras having Gelfand-Kirillov dimension one but the growth is not linear.

References


A simple solution of the ADS-problem

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Keywords: Ideal, accessible subring; Kurosh chain.


A radical class (radical) is a non-empty class of rings, which is homomorphically closed, closed under extensions and has the inductive property (cf. [7]). The radical classes play an important role in the development of the theory of rings.

For every non-empty isomorphically closed class of rings $\mathcal{M}$ there is a smallest radical class $l(\mathcal{M})$ that contains $\mathcal{M}$. The radical $l(\mathcal{M})$ is called the lower radical determined by the class $\mathcal{M}$. The lower radical determined by a class $\mathcal{M}$ can be described by usage of Kurosh’s chain $\{\mathcal{M}_\alpha\}_{\alpha \geq 1}$. Define $\mathcal{M}_1$ be a class of all homomorphic images of all rings from $\mathcal{M}$ and for any ordinal number $\alpha > 1$:

$$\mathcal{M}_\alpha = \left\{ A : \begin{array}{l}
\text{every non-zero homomorphic image of } A \\
\text{contains a non-zero ideal belonging to } \mathcal{M}_\beta \\
\text{for some } \beta < \alpha
\end{array} \right\}.$$

Suliński, Anderson and Divinsky in [4] found characterizations of classes $\mathcal{M}_\alpha$ using accessible subrings and showed that always $l(\mathcal{M}) = \mathcal{M}_\omega$. They gave examples of Kurosh chains stabilizing exactly in first, second and third step, and put the question (named after the ADS-problem) of a homomorphically closed class $\mathcal{M}$ such that for $n \geq 4$, $l(\mathcal{M}) = \mathcal{M}_n \neq \mathcal{M}_{n-1}$.

Full, positive solution to the ADS-problem was given in 1982 by Beidar in [5].

The objective of this talk is to present construction which is a modification of Beidar’s example. It will be a generalization and simplification of its outcome (cf. [6]).
References


A fourth order matrix superalgebra with involution and its PI-properties

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Keywords: Algebras with polynomial identity, Grassmann algebra, algebras with involution, superalgebras.

2010 Mathematics Subject Classification: 16R10, 16R50, 16W10, 16W55, 15A75.

The talk is a new research concerning PI-properties of the algebra \( A \) of the matrices over the Grassmann algebra \( E \) of type

\[
\begin{pmatrix}
y_1 & 0 & z_1 & 0 \\
0 & y_2 & 0 & z_2 \\
z_3 & 0 & y_3 & 0 \\
0 & z_4 & 0 & y_4
\end{pmatrix},
\]

for which \( \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} \) and \( \begin{pmatrix} y_3 & 0 \\ 0 & y_4 \end{pmatrix} \) have even entries, while the matrices \( \begin{pmatrix} z_1 & 0 \\ 0 & z_2 \end{pmatrix} \) and \( \begin{pmatrix} z_3 & 0 \\ 0 & z_4 \end{pmatrix} \) have odd entries.

In \( A \) we introduce an involution \( \psi \), namely

\[
\begin{pmatrix}
y_1 & 0 & z_1 & 0 \\
0 & y_2 & 0 & z_2 \\
z_3 & 0 & y_3 & 0 \\
0 & z_4 & 0 & y_4
\end{pmatrix}^{\psi} = \begin{pmatrix}
y_3 & 0 & z_1 & 0 \\
0 & y_4 & 0 & z_2 \\
-z_3 & 0 & y_1 & 0 \\
0 & -z_4 & 0 & y_2
\end{pmatrix}.
\]

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Some $\psi$-identities in only symmetric and in only skew-symmetric with respect to $\psi$ variables are shown.

A set of eight $\psi$-identities of degree 4 and 5 is given.

Low degrees’ $\psi$-identities for $\mathcal{A}$ in two variables are found as well, namely

\begin{align*}
[Y \circ Y, Z] &= 0, \quad [Y, Z, Z, Y] = 0, \\
[Z \circ Z, Y] &= 0, \quad [Y, Z, Z, Z] = 0, \\
[Z \circ Y, Z] &= 0, \quad [Y, Z]^2 = 0, \\
[Z \circ Y, Y] &= 0.
\end{align*}

We formulate a problem: Do there exist $\psi$-identities in $\mathcal{A}$ which are not consequences of the above ones?
On some Malcev algebras and their corresponding Moufang loops

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Keywords: Malcev algebra, Moufang loop, left automorphic loop.
2010 Mathematics Subject Classification: 20N05, 17D10.

We study the correspondence between some Malcev algebras and smooth Moufang loops. In particular, we will show that the every 3-generated subloop of a smooth Moufang loops which corresponds to Malcev algebra, defined by the following identities:

\[ [x, y] = -y, x \], \quad J(x, y, [z, x]) = [J(x, y, z), x] = 0

is left automorphic.
Cores in Yetter-Drinfel’d Hopf algebras

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Keywords: cocommutative cosemisimple Hopf algebras, Yetter-Drinfel’d Hopf algebras.

2010 Mathematics Subject Classification: 16T05.

By the structure theorem for cocommutative cosemisimple Yetter-Drinfel’d Hopf algebras over groups of prime order $p$, every such algebra, if it is not trivial, contains a $p$-dimensional Yetter-Drinfel’d Hopf subalgebra, called the core, with the property that the quotient by the core is trivial and therefore an ordinary group algebra, at least under suitable assumptions on the base field. In this case, it is not difficult to see that the action and the coaction of the group of prime order must be trivial. Therefore, every nontrivial such Yetter-Drinfel’d Hopf algebra is an extension of two trivial ones.

This result raises the question whether action and coaction on the core are also trivial over more general finite abelian groups. This is however, not the case: We describe an example of a core with a nontrivial action and a nontrivial coaction of an elementary abelian group of order 4. However, the core is still trivial as a Yetter-Drinfel’d Hopf algebra; i.e., it is an ordinary Hopf algebra. We conjecture that this is always the case, and describe some partial results in this direction. The talk is based on joint work with Yevgenia Kashina.
On sets of pp-generators of finite groups

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Keywords: Finite group, generating set, independent set, soluble group.

2010 Mathematics Subject Classification: Primary: 20D10. Secondary: 20D60, 20F05.

Let $G$ be a finite group. As in [1], we say that the group $G$ has property $B_{pp}$ if all its minimal pp-generating sets have the same size. By a pp-generating set we mean a generating set which elements have prime power order. Groups with property $B_{pp}$ are called $B_{pp}$-groups for short. A group $G$ is said to have the pp-basis property if all its subgroups are $B_{pp}$-groups. We give the classification of finite groups with the pp-basis property as products of $p$-groups and precisely described $\{p, q\}$-groups.

The talk is based on a common work with J. Krempa (University of Warsaw) [2].

References


Binary symmetry groups of polytopes
as quotients of braid groups

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**Keywords:** Braid groups, Coxeter groups.

**2010 Mathematics Subject Classification:** 20F36, 20F05.

The symmetry group of a regular $n$-polytope is a finite Coxeter group of rank $n$. Let $g \subset SO(n)$ be the normal subgroup of orientation-preserving symmetries of the polytope. A natural object to study is the group $G$ of homotopy classes of paths $R : [0, 1] \to SO(n)$ with $R(0) = Id$ and $R(1) \in g$. This group is a finite analog of the topological universal covering space of $SO(n)$. As a group we expect it to be a nontrivial central extension by $\mathbb{Z}_2$ of $g$. $G$ can be generated by a finite set of simple “generating paths” $R_i$. Words giving (explicitly) contractible closed paths must be set to the identity and this provides a set of relations among the generators. After simplifications it turns out that in a number of cases the generators $R_i$ satisfy Artin’s relations for the braid group plus one additional relation. Thus, for example the braid group $B_n$ factorized by the normal closure of the element $R_1^2R_2R_1R_2^{-2}$ is the so-called binary hyperoctahedral group – the group of rotational symmetries of the hyperoctahedron (also called cross-polytope) in dimension $n$. This provides a uniform way to present all double covers of the orientation-preserving subgroups of the Coxeter $B$-series.

The binary icosahedral group is obtained as $B_3/N$ with $N$ being the normal closure of $R_1R_2^2R_1R_2^{-3}$. (The full icosahedral group is the exceptional Coxeter group $H_3$). The binary tetrahedral group can be obtained as $B_3/N$ with $N$ the normal closure of $R_1R_2^2R_1R_2^{-1}$. The double covers of the Coxeter groups of the
A-series (which are the symmetric groups $S_n$) are obtained by imposing on $B_n$ the additional relation $R_1R_2R_1 = Id$.

Some finite factors of braid groups appear in classical works by Coxeter and Moser ([1, 2]). In particular the presentation for the binary octahedral group ($n = 3$) appears in [1]. In [2] the author studies the groups obtained by imposing the additional condition $R_1^n = Id$, $n > 2$. However it seems that the connection between braid groups and Coxeter groups has not been explored thoroughly and our work is a step in that direction.

As a by-product, constructing explicitly the binary hyperoctahedral group $G$ and studying its subgroup of homotopy classes of closed paths we have been able to provide a (new) proof (using just vector analysis and algebra) of the well-known fact that $\pi_1(SO(n)) = \mathbb{Z}_2$ for all $n \geq 3$ [3].

References


An estimate of exponential sums over primes

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Keywords: Exponential sums, primes in arithmetic progressions, almost primes.

2010 Mathematics Subject Classification: 11A41, 11N05, 11N80.

We consider the exponential sum:

\[ W(x, D) = \sum_{d \sim D} \xi(d) \sum_{n \sim x, n \equiv a(d)} \Lambda(n)e(\alpha n). \]  \(1\)

Here \(d \sim D\) means \(D/2 \leq d < D\),

\[ x \geq 1, \quad z = z(x) \geq 2, \quad D \leq z^\eta, \quad \eta \in (2, 3] \quad \text{is fixed}, \quad P(z) = \prod_{p \leq z} p, \]

\(\xi(d)\) is a real function such that

\[ \xi(d) \ll 1, \quad \text{if} \quad d \mid P(z) \quad \text{and} \quad d \leq D; \]
\[ \xi(d) = 0, \quad \text{otherwise}, \quad \]  \(2\)

\(\Lambda(n)\) is the von Mangoldt function and \(\alpha \in \mathbb{R}\) has a rational approximation \(a/q\) satisfying

\[ \left| \alpha - \frac{a}{q} \right| < \frac{1}{q^2}, \quad \text{where} \quad (a, q) = 1, \quad q \geq 1. \]  \(3\)

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The necessity of estimation of such sums arises in some applications of the sieve method.

If we only use that $\xi(d) \ll 1$ then for $D \leq \frac{x^{1/3}}{(\log x)^A}$ with $A$-arbitrary large and fixed, Tolev [8, Lemma 1] and Dimitrov, Todorova [1, Lemma 6.4] proved that:

$$ W(x, D) \ll \left( \frac{x}{q^{1/4}} + \frac{x}{(\log x)^{A/2}} + x^{3/4}q^{1/4} \right) \log^{37} x. \quad (4) $$

In the case when the function $\xi(d)$ is well factorable K. Matomäki (see [6]) received result:

$$ W(x, D) \ll (\log x)^C x^{3/4 + \eta} \left( \frac{x}{q} + q + D^2 + x^{7/9+4\eta} + \min \left\{ D^{4+20\eta}, \frac{x}{D} \right\} \right)^{1/4-\eta}. $$

Here $\eta > 0$, $C = C(\eta) > 0$.

Using the conditions (2) we can improve the result of Tolev and Dimitrov, Todorova.

**Theorem 1.** Suppose $\alpha \in \mathbb{R}$ with a rational approximation $\frac{a}{q}$ satisfying (3), $\xi(d)$ are real numbers satisfying (2), $x$ is sufficiently large positive real number,

$$ z = \frac{x^{2/15}}{(\log x)^A}, \quad (\log x)^A < q < \frac{x}{(\log x)^A}, \quad A > 148 $$

and $D \leq z^3$. Then

$$ W(x, D) \ll \left( \frac{x}{q^{1/4}} + \frac{x}{(\log x)^{A/4}} + x^{3/4}q^{1/4} \right) (\log x)^{37}. \quad (5) $$

Applying the above theorem we improve a previous result of Dimitrov and Todorova [1] and prove that

**Theorem 2.** Let $B$ be an arbitrary large and fixed and

$\lambda_i \in \mathbb{R}$, $\lambda_i \neq 0$, $i = 1, 2, 3$;

$\lambda_1, \lambda_2, \lambda_3$ not all of the same sign;

$\lambda_1/\lambda_2 \in \mathbb{R} \setminus \mathbb{Q}$;

$\eta \in \mathbb{R}$. 

90
Then there are infinitely many ordered triples of primes $p_1, p_2, p_3$ with

$$|\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 + \eta| < [\log(\max p_j)]^{-B}$$

and

$$p_1 + 2 = P'_7, \quad p_2 + 2 = P''_7, \quad p_3 + 2 = P'''_7.$$ 

References


Properly discontinuous group actions on affine homogeneous spaces

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Keywords: Algebraic groups, discrete subgroups of Lie groups, discontinuous actions on homogeneous spaces.

2010 Mathematics Subject Classification: 57S30.

Let $G$ be a real algebraic group, $H \leq G$ an algebraic subgroup containing a maximal reductive subgroup of $G$, and $\Gamma$ a subgroup of $G$ acting on $G/H$ by left translations. We conjecture that $\Gamma$ is virtually solvable provided its action on $G/H$ is properly discontinuous and $\Gamma \backslash G/H$ is compact and we confirm this conjecture when $G$ does not contain simple algebraic subgroups of rank $\geq 2$. If the action of $\Gamma$ on $G/H$ (which is isomorphic to an affine linear space $\mathbb{A}^n$) is linear our conjecture coincides with the Auslander conjecture. We prove the Auslander conjecture for $n \leq 5$. 
Lie algebras of maximal class

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Keywords: Filiform Lie algebras, Lie algebras of maximal class, \(\mathbb{N}\)-gradings, nilpotent Lie algebras.

2010 Mathematics Subject Classification: 17B05, 17B30, 17B70.

In this talk we first survey finite-dimensional filiform Lie algebras, i.e., nilpotent Lie algebras with the maximal nilindex \(n-1\) for a given dimension \(n\). Filiform Lie algebras form a generic set of points of the affine variety consisting of all \(n\)-dimensional nilpotent Lie algebras. We then discuss a generalization of a filiform Lie algebra to the infinite-dimensional case, i.e., a Lie algebra of maximal class \(L\) over an algebraically closed field of zero characteristic. We additionally assume that \(L\) is equipped with \(\mathbb{N}\)-grading: \(L = \bigoplus_{i=1}^{\infty} L_i\), where \([L_i, L_j] \subseteq L_{i+j}, i, j \in \mathbb{N}\). If \(L\) is as above, then either \(L = \langle L_1 \rangle\) or \(L = \langle L_1, L_q \rangle\), where \(q \geq 2\) (for convenience, we assume that \(L_1 \neq \{0\}\)).

As was shown by A. Fialowski in [6], there are only three isomorphism types of \(\mathbb{N}\)-graded Lie algebras \(L = \bigoplus_{i=1}^{\infty} L_i\) of maximal class generated by \(L_1\) and \(L_2\), \(L = \langle L_1, L_2 \rangle\) (see also [10, 13]). In [15] M. Vergne described the structure of these algebras with the property \(L = \langle L_1 \rangle\). If \(L = \langle L_1, L_q \rangle\) where \(q > 2\), then we proved in [1] that under some technical condition, there can only be one isomorphism type of such algebras. For \(q = 3\) we fully classified them. Our results coincide with computer calculations of Vaughan-Lee who investigated the cases \(q = 3, 4\). In [13] A. Shalev and E. Zelmanov showed that if \(L\) is a nonsolvable \(\mathbb{N}\)-graded Lie algebra of maximal class, and \(L = \langle L_1, L_2 \rangle\), then \(L\) is isomorphic to the positive part of the Witt algebra. If \(L\) is an \(\mathbb{N}\)-graded Lie algebra of maximal class with polynomial multiplication (this implies that \(L\) is nonsolvable), then \(L\) must be isomorphic to the positive part of the Witt algebra or to a certain subalgebra of it (see [14]).
References


Exponential growth of the codimensions of PI-algebras with involution

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Keywords: Polynomial identity, involution, superinvolution, codimensions.

2010 Mathematics Subject Classification: Primary: 16R10, 16R50. Secondary: 16P90.

In this lecture we are concerned with algebras with involution and in particular we are interested in the growth of the identities of such algebras.

If $A$ is any PI-algebra with involution $\ast$ and $c_n^*(A)$, $n = 1, 2, \ldots$, is the corresponding sequence of $\ast$-codimensions, we are able to provide an exact estimate of the exponential rate of growth of $c_n^*(A)$, \[ \exp^*(A) = \lim_{n \to \infty} \sqrt[n]{c_n^*(A)}. \]

It turns out that the $\ast$-exponent, $\exp^*(A)$, is an integer and, as in the ordinary case (no involution), we also give an explicit way of computing it.
The triadjoints of \( d \)-bimorphisms

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**Keywords:** Arens adjoint, vector lattice, order bidual, \( d \)-algebra, \( d \)-bimorphism.

**2010 Mathematics Subject Classification:** 46A40, 47B65, 06F25.

In this paper we introduce a new concept of \( d \)-bimorphism on vector lattices and prove that, for vector lattices \( A \) and \( B \), the Arens triadjoint \( T^{**} : (A')_n' \times (A')_n' \rightarrow (B')_n' \) of a \( d \)-bimorphism \( T : A \times A \rightarrow B \) is a \( d \)-bimorphism. This generalizes the concept of \( d \)-algebra and some results on the order bidual of \( d \)-algebras.

**References**


Pauli grading on simple Lie superalgebras and graded codimension growth

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Keywords: Lie superalgebra, group grading, graded identities, codimension growth.

2010 Mathematics Subject Classification: 17B01, 17B02.

Group gradings play an important role in the study of algebras of distinct type. One of the most interesting questions is the description of gradings on simple algebras. The classification of all finite gradings on finite dimensional simple complex associative and Lie algebras is well known. The same problem for finite dimensional associative real simple algebras was also recently solved.

We discuss finite group gradings on simple Lie superalgebras. For some simple superalgebras we construct a grading by an elementary abelian 2-group based on $2 \times 2$ Pauli matrices. Then we apply this result for studying graded polynomial identities. In particular, we compute the precise value of the corresponding graded PI-exponent.

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