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A simple solution of the ADS-problem

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A radical class (radical) is a non-empty class of rings, which is homomorphically closed, closed under extensions and has the inductive property (cf. [7]). The radical classes play an important role in the development of the theory of rings.

For every non-empty isomorphically closed class of rings $\mathcal{M}$ there is a smallest radical class $l(\mathcal{M})$ that contains $\mathcal{M}$. The radical $l(\mathcal{M})$ is called **the lower radical determined by the class** $\mathcal{M}$. The lower radical determined by a class $\mathcal{M}$ can be described by usage of Kurosh's chain $\{\mathcal{M}_\alpha\}_{\alpha\geq 1}$. Define $\mathcal{M}_1$ be a class of all homomorphic images of all rings from $\mathcal{M}$ and for any ordinal number $\alpha > 1$:

$$\mathcal{M}_\alpha = \left\{ A \ : \ \begin{array}{c} \text{every non-zero homomorphic image of } A \\ \text{contains a non-zero ideal belonging to } \mathcal{M}_\beta \\ \text{for some } \beta < \alpha \end{array} \right\}.$$

Suliński, Anderson and Divinsky in [4] found characterizations of classes $\mathcal{M}_\alpha$ using accessible subrings and showed that always $l(\mathcal{M}) = \mathcal{M}_\omega$. They gave examples of Kurosh chains stabilizing exactly in first, second and third step, and put the question (named after the ADS-problem) of a homomorphically closed class $\mathcal{M}$ such that for $n \geq 4$, $l(\mathcal{M}) = \mathcal{M}_n \neq \mathcal{M}_{n-1}$.

Full, positive solution to the ADS-problem was given in 1982 by Beidar in [5].

The objective of this talk is to present construction which is a modification of Beidar's example. It will be a generalization and simplification of its outcome (cf. [6]).

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