The Generators of the Semigroup of all Transformations Preserving a Crown

Jörg Koppitz

BAS

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- $\mathbf{F}_n =: ([n], F_n)$ fence (or zig-zag ordered set)

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- $I_3 = \left\langle \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \right\rangle$

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$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & - & 1 & 2 \end{pmatrix} \quad \gamma_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & - \end{pmatrix}$$

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$$\beta_2^{odd} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & - & - \end{pmatrix} \quad \beta_2^{even} = \begin{pmatrix} 1 & 2 & 3 \\ - & 1 & - \end{pmatrix}$$

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Merci