A counterexample to the modular isomorphism problem

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Abstract

The modular isomorphism problem asks whether the isomorphism type of the modular group algebra of a p-group G over a field of characteristic p determines the isomorphism type of G. It was explicitly mentioned in a survey by Richard Brauer in 1963, and was the only classical version of the isomorphism problem for group rings which had resisted a solution, though it received considerable attention. Several partial positive solutions were obtained imposing very strong conditions on the group G, for instance the one of being metacyclic.

In a joint work with Leo Margolis and Ángel del Río, we solve the modular isomorphism problem in the negative by exhibiting a series of pairs of non-isomorphic 2-groups with isomorphic modular group algebras over every field of characteristic 2. These groups are two-generated with cyclic derived subgroup. We will also discuss the (lack of) posibility of obtaining, in a naive way, analogues to the counterexamples for $p > 2$ verifying this condition.