# Derivations of upper triangular matrix rings
# vs
# Derivations of upper triangular matrix semirings

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The motivation for this talk is the problem how to represent a derivation of a matrix ring and of an additively idempotent matrix semiring as a sum of well-known derivations.

The results of two of my articles, published in 2022, will be compared and we will draw conclusions about the advantages and disadvantages of these results.

We begin by considering the nature of derivations of triangular matrices over an additively idempotent semiring $R$ generated by left and right semicentral idempotents. Then we construct a semiring $\mathcal{D}$ of these derivations and find a basis of $\mathcal{D}$, considered as an $R$-semimodule. The main result of the first article states that an arbitrary derivation of $UTM_n(R)$ (the semiring of upper triangular matrices over an additively idempotent semiring $R$) is a linear combination of a derivations from the basis of $R$-semimodule $\mathcal{D}$.

When $R$ is an associative ring with identity and $UTM_n(R)$ is the ring of upper triangular $n \times n$ matrices over $R$ we propose a basis of an additive group $\mathcal{D}$ of derivations of $UTM_n(R)$ consisting of derivations $\delta_i$ such that $\delta_i(A) = [e_{ii}, A]$, where $A \in UTM_n(R)$ and $e_{ii}$ are diagonal matrix units for $i = 2, \ldots, n$. The main result states that if $D$ is an arbitrary derivation of the ring $UTM_n(R)$ and $A \in UTM_n(R)$, then there are matrices, such that the derivative $D(A)$ is a linear combination of the values of derivations $\delta_i \in \mathcal{D}$, $i = 2, \ldots, n$, of these matrices with coefficients the entries of the matrix $A$.