Modal definability of some classes of modal products

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(This work will be presented by Yana Rumenova)

Let $\mathcal{K}_{commute}$ be the class of structures for the language $\mathfrak{L}(R_1, R_2, \doteq)$ in which the two equivalence relations commute and let $\mathcal{K}_{partition}$ be the class of structures for the language $\mathfrak{L}(R, \doteq)$ of all partitions.
In this talk we will review the decidability of first-order theories of the following subclasses of $\mathcal{K}_{commute}$:

- Let for each $n \in \omega^+ \mathcal{K}^{R_1 \leq n}_{commute}$ be the class of all structures from $\mathcal{K}_{commute}$ such that for each matrix in the structure the rows have $\leq n$ number of cells (we shall introduce these notions later on);
- Let for each $n \in \omega^+ \mathcal{K}^{R_1 \leq n, R_2 < \omega}_{commute}$ be the class of all structures from $\mathcal{K}^{R_1 \leq n}_{commute}$ such that for each matrix in the structure the columns have a finite number of cells;
- Let for each $n, m \in \omega^+$ be the tighter subclass of $\mathcal{K}^{R_1 \leq n, R_2 < \omega}_{commute}$: $\mathcal{K}^{R_1 \leq n, R_2 \leq m}_{commute}$;
- Let $\mathcal{K}_{rectangle}$ be a subclass of $\mathcal{K}_{commute}$ such that the structures are modal products of structures from $\mathcal{K}_{partition}$;
- Let $\mathcal{K}_{square}$ be a subclass of $\mathcal{K}_{rectangle}$ such that the structures are modal products of a structure from $\mathcal{K}_{partition}$.

Because $\mathcal{K}_{commute}^{R_2 \leq n}$ is similar to $\mathcal{K}_{commute}^{R_1 \leq n}$ and $\mathcal{K}_{commute}^{R_2 \leq n, R_1 < w}$ is similar to $\mathcal{K}_{commute}^{R_1 \leq n, R_2 < \omega}$, we will only discuss FMP and the decidability problem of $\mathcal{K}_{commute}^{R_1 \leq n}$ and $\mathcal{K}_{commute}^{R_1 \leq n, R_2 < \omega}$. The same reasoning can be applied for obtaining the results for the other two classes.

We use methods from general/finite model theory like Ehrenfeucht-Fraïssé games and results on generalized products started by Mostowski and continued by Feferman and Vaught to demonstrate decidabilities of the first-order validity problems and the possession of the finite model property.

These classes almost fit the criteria of the definition of a stable class conjured by Balbiani and Tinchev, so we will call them pre-stable.
We will also show that the problem of deciding the validity of sentences in each of these subclasses of $\mathcal{K}_{commute}$ is reducible to the modal definability problem w.r.t. the subclass in question. Alas, this only gives us a lower bound of the complexity of the modal decidability problems w.r.t. each of these classes.