

On the Degree of Dependence of Two Events as a Relative Invariant
of the Dihedral Group of order 8
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Abstract

The joint experiment $\mathfrak{J}_{(A,B)}$ of two binary trials $A \cup A^c$ and $B \cup B^c$ in a probability space can be produced not only by the ordered pair (A,B) but by a set consisting, in general, of 24 ordered pairs of events (named Yule's pairs). The probabilities $\xi_1, \xi_2, \xi_3, \xi_4$ of the four results of $\mathfrak{J}_{(A,B)}$ are linear functions in three variables $\alpha = \Pr(A)$, $\beta = \Pr(B)$, $\gamma = \Pr(A \cap B)$, and constitute a probability distribution. The symmetric group S_4 of degree four has an exact representation in the affine group $\text{Aff}(3, \mathbb{R})$, which is constructed by using the types of the form $[\alpha, \beta, \theta]$ of those 24 Yule's pairs. The corresponding action of S_4 permutes the components of the probability distribution $(\xi_1, \xi_2, \xi_3, \xi_4)$, and, in particular, its entropy function is S_4 -invariant. The function of degree of dependence of two events, defined via modifying the entropy function, turns out to be a relative invariant of the dihedral group of order 8.