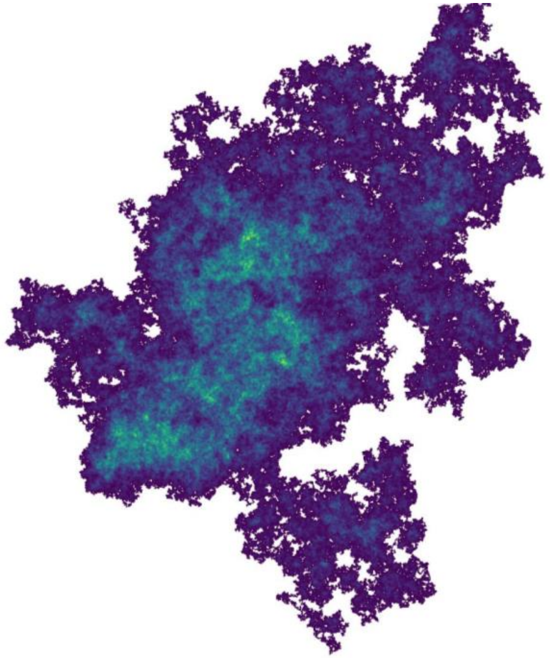
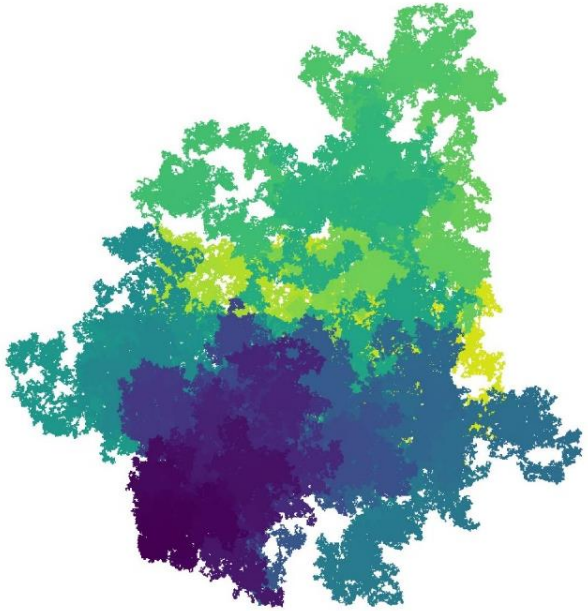
Prime numbers
&
Random Walks



Alberto Fraile O. Kinouchi, P. Dwivedi,
R. Martínez, T. E. Raptis and D. Fernández.

Sofia, October 20th, 2023



[1] A. Fraile, R. Martínez & D. Fernández. Jacob’s Ladder: Prime Numbers in 2D. Math. Comput. Appl. 25(1) (2020)

[2] A. Fraile, O. Kinouchi, P. Dwivedi, R. Martínez, T. E. Raptis, and D. Fernández. Prime numbers and random walks in a square grid. Phys. Rev. E 104, 054114 (2021)

[1] A. Fraile, R. Martínez & D. Fernández. Jacob’s Ladder: Prime Numbers in 2D. Math. Comput. Appl. 25(1) (2020)

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A065358 The Jacob's Ladder sequence: a(n) = Sum_{k=1..n} (-1)^pi(k), where pi = A000720. 10

0, 1, 0, 1, 2, 1, 0, 1, 2, 3, 4, 3, 2, 3, 4, 5, 6, 5, 4, 5, 6, 7, 8, 7, 6, 5, 4, 3, 2, 3, 4, 3, 2, 1, 0, -1, -2, -1, 0, 1, 2, 1, 0, 1, 2, 3, 4, 3, 2, 1, 0, -1, -2, -1, 0, 1, 2, 3, 4, 3, 2, 3, 4, 5, 6, 7, 8, 7, 6, 5, 4, 5, 6, 5, 4, 3, 2, 1, 0, 1, 2, 3, 4, 3, 2, 1, 0, -1, -2, -1, 0, 1, 2, 3, 4, 5, 6, 5, 4 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,5

COMMENTS Partial sums of A065357.

LINKS N. J. A. Sloane, Table of n, a(n) for n = 0..10000 (First 1000 terms from Harry J. Smith.)

Alberto Fraile, Roberto Martínez, and Daniel Fernández, Jacob's Ladder: Prime numbers in 2d, arXiv preprint arXiv:1801.01540 [math.HO], 2017. [They describe essentially this sequence except with offset 1 instead of 0 - N. J. A. Sloane,

Prime numbers; open problems

I Goldbach's Conjecture: Every even $n > 2$ is the sum of two primes.

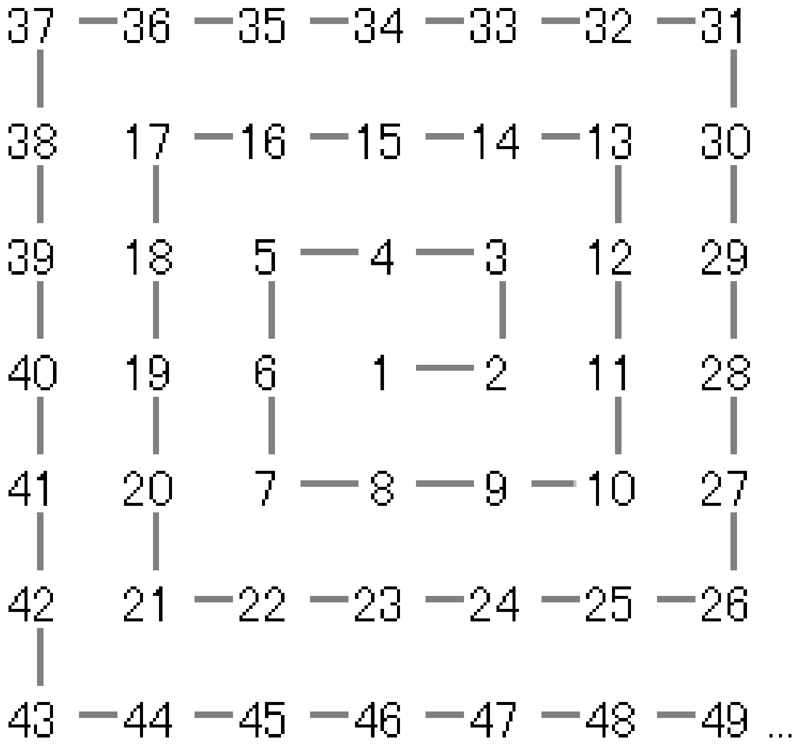
II Twin Prime Conjecture: There are infinitely many twin primes.

III Is there always a prime between n^2 and $(n+1)^2$?

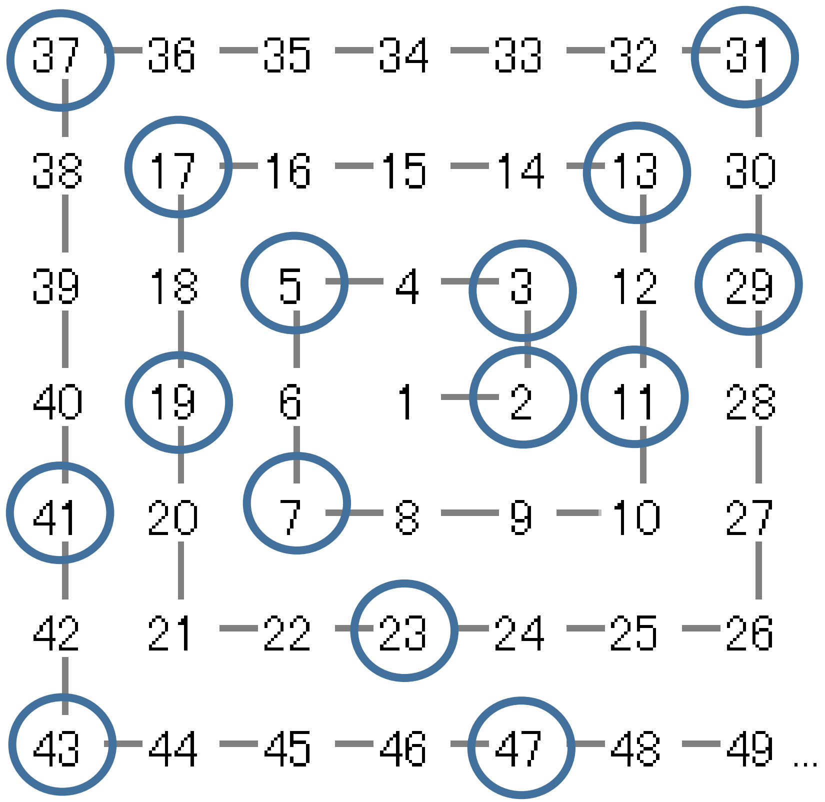
IV Riemann hypothesis

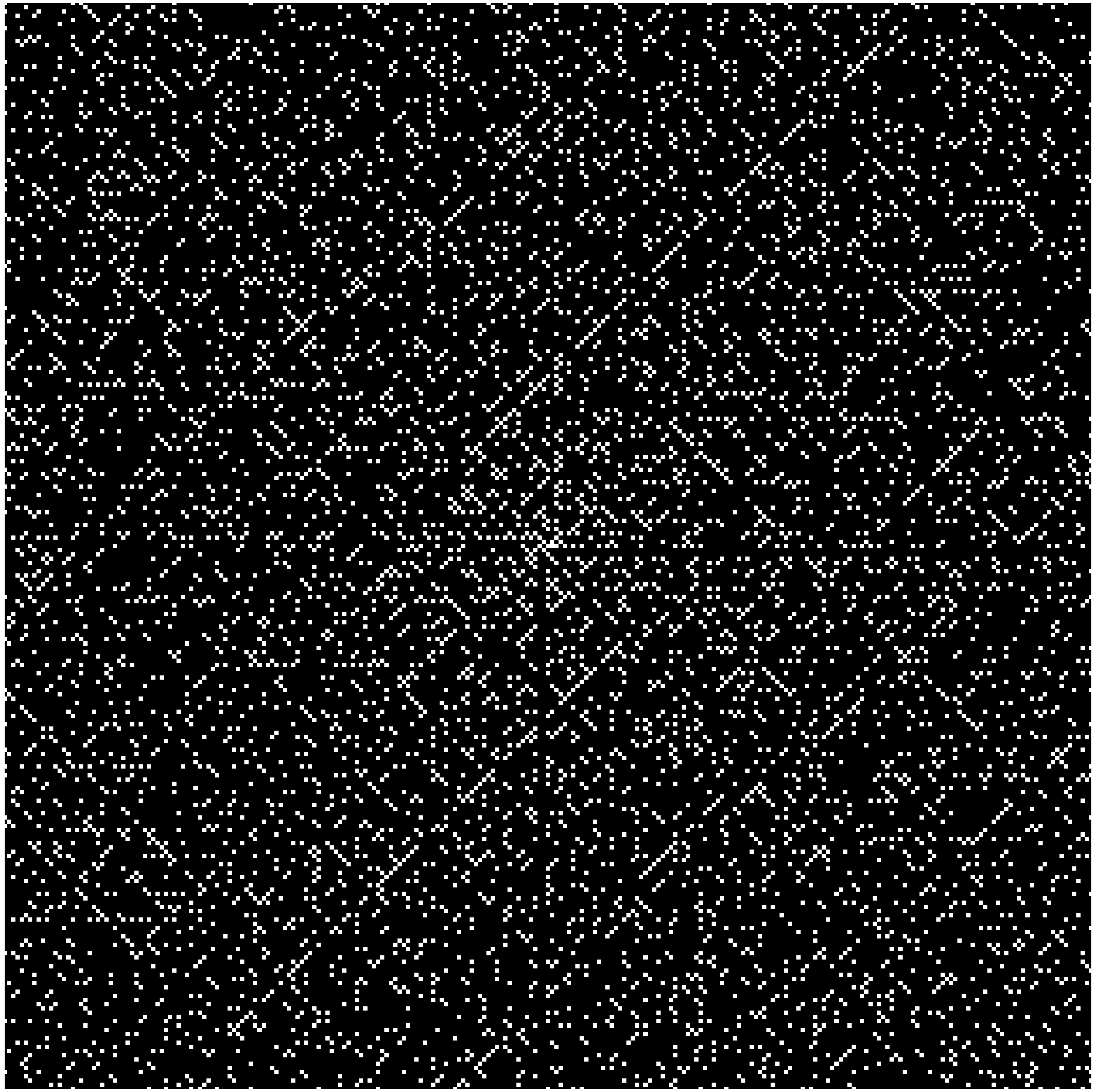
V …

Prime numbers in 2d.



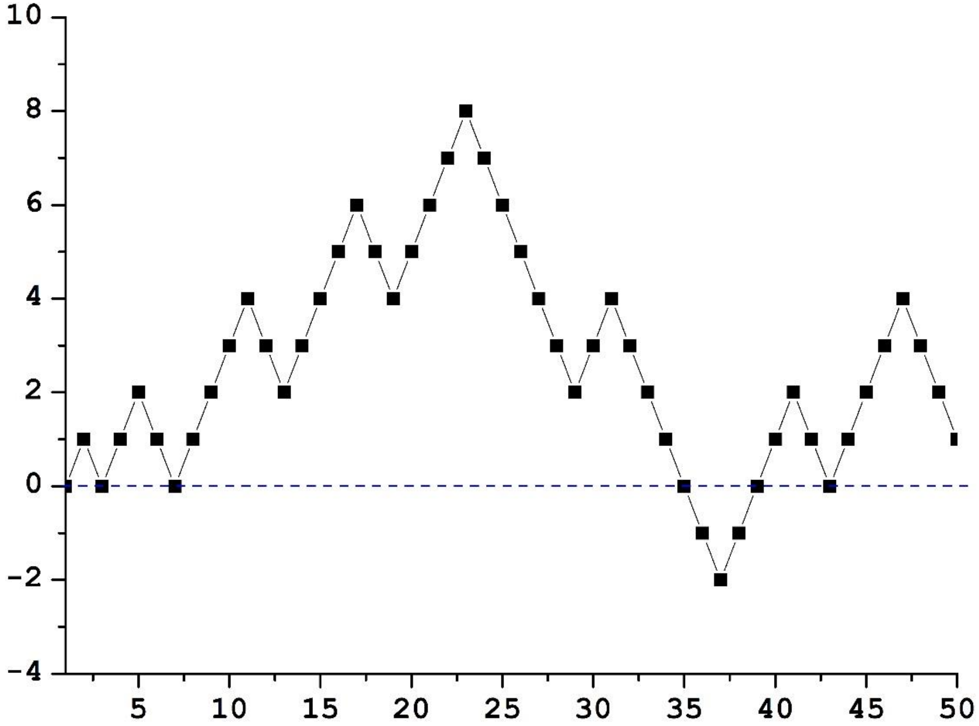
Prime numbers in 2d. Ulam spiral

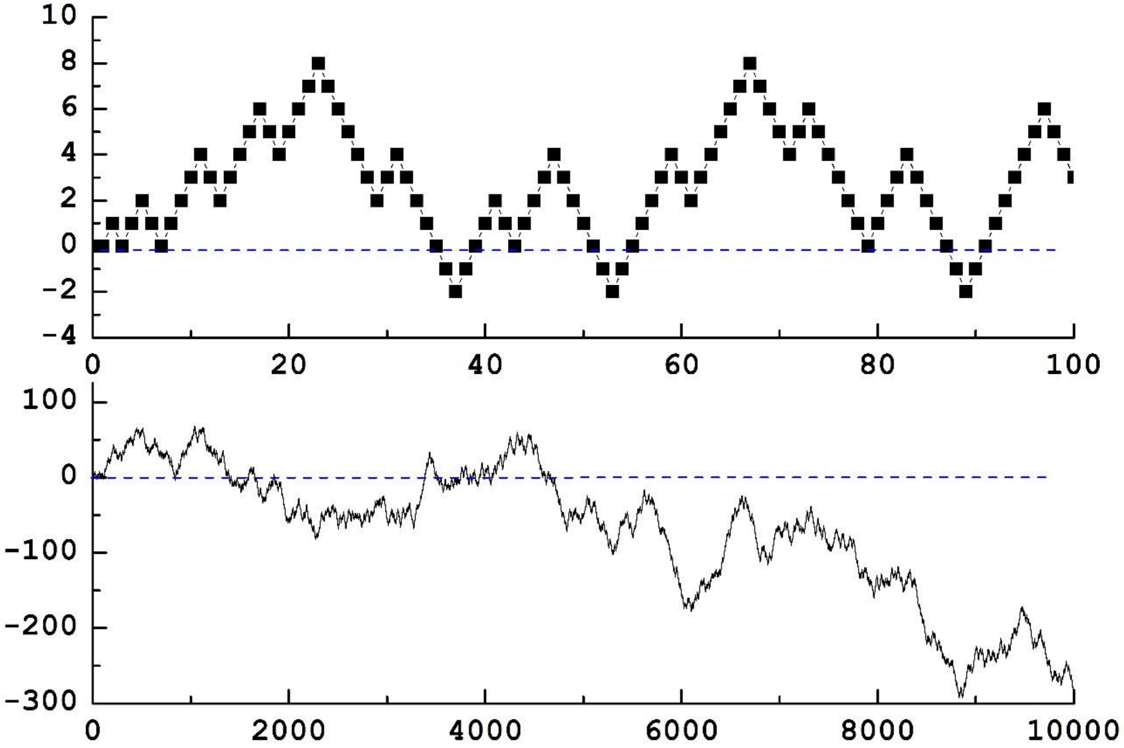




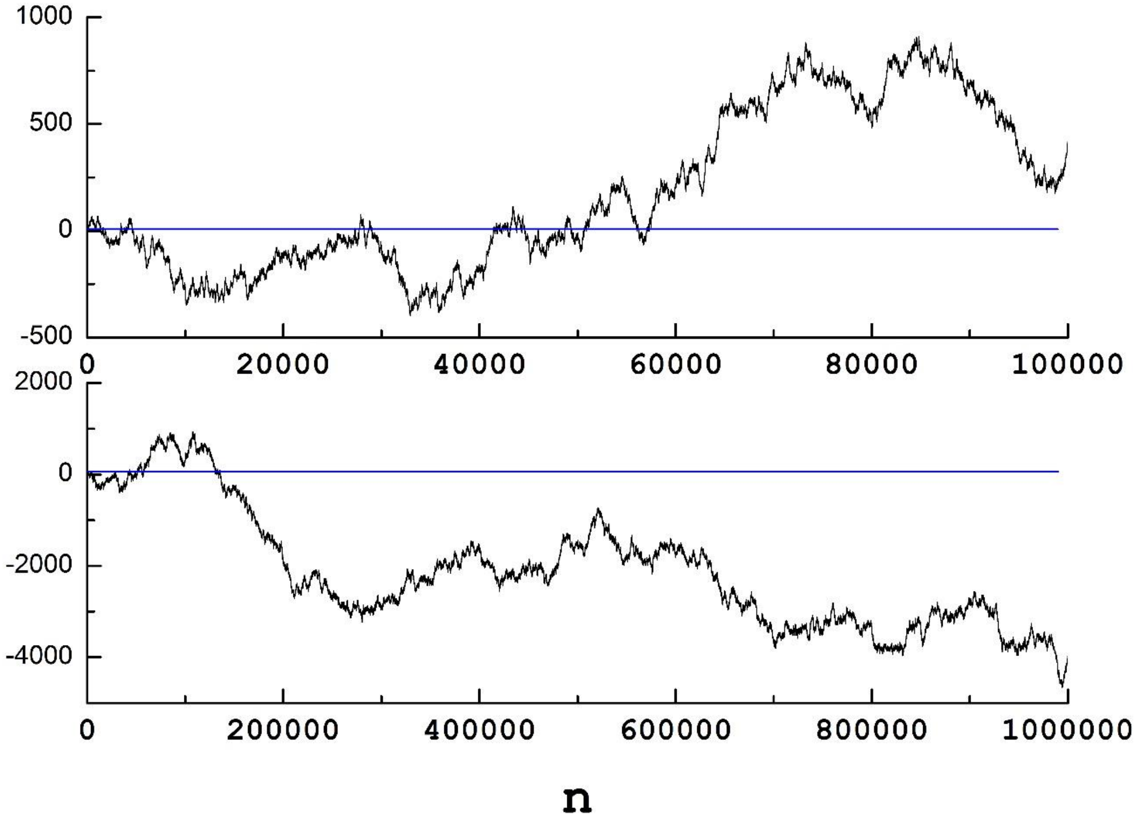
Stein, M. L.; Ulam, S. M.; Wells, M. B. (1964), "A Visual Display of Some Properties of the Distribution of Primes", American Mathematical Monthly, Mathematical Association of America, 71 (5): 516–520

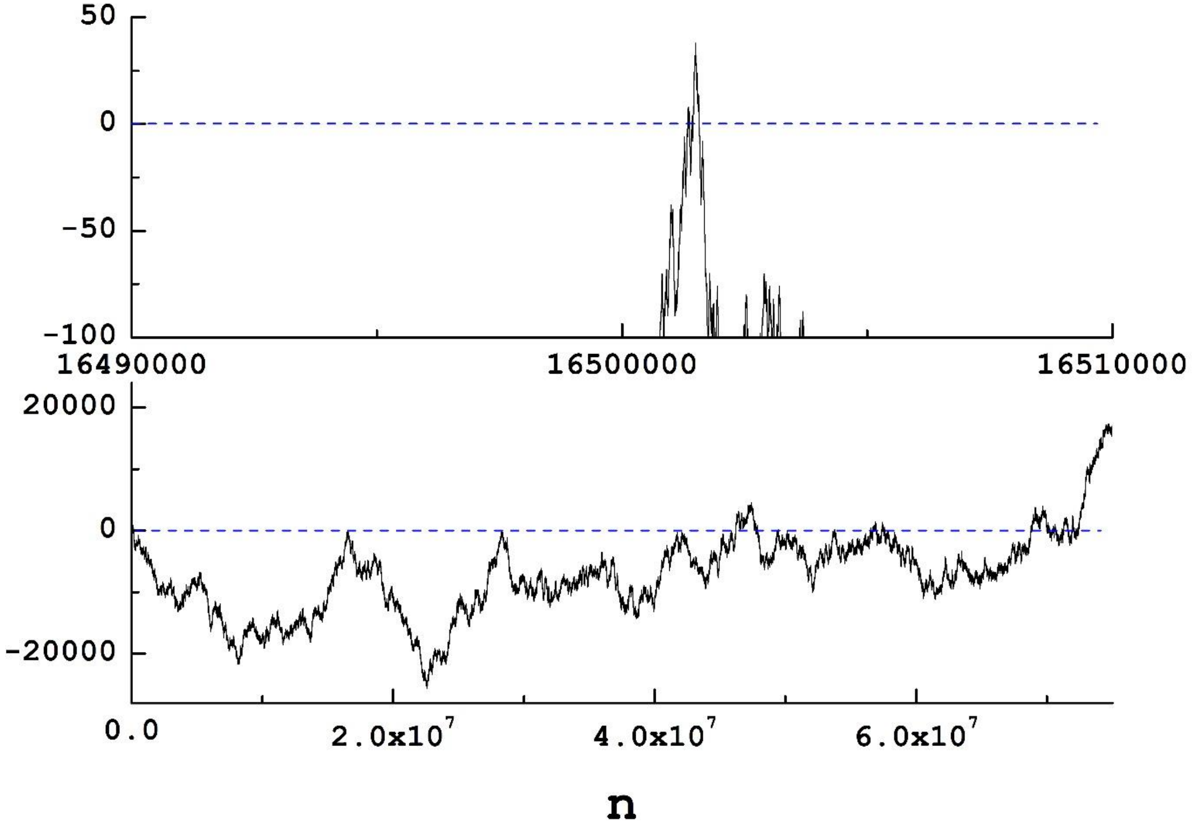
Prime numbers in 2d



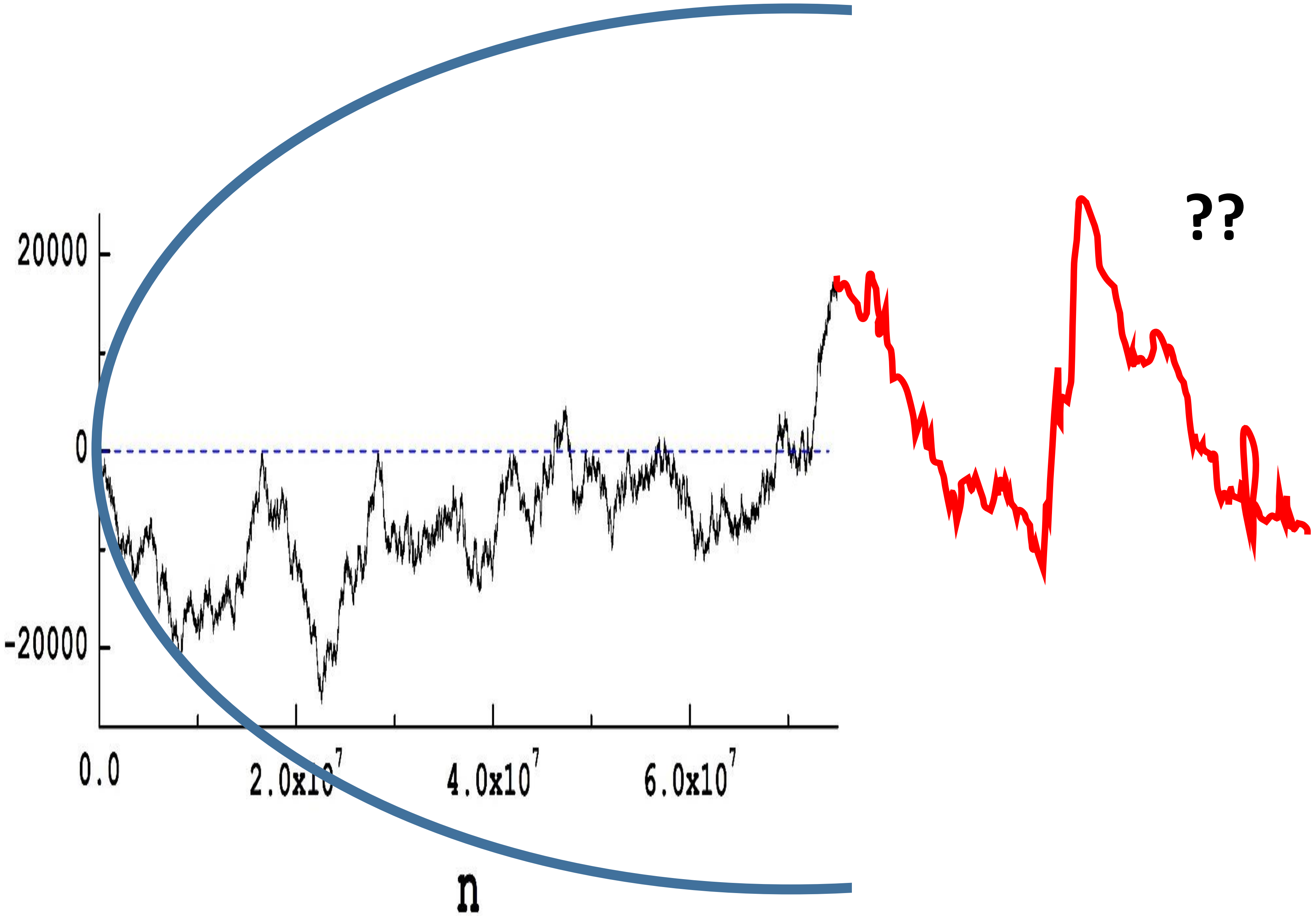


10
8
6
4
2
0
-2
-4
0
20
40
60
80
100
100
0
-100
-200
-300
0
2000
4000
6000
8000
10000


1000
500
0
-500
0
20000
40000
60000
80000
100000
2000
0
-2000
-4000
0
200000
400000
600000
800000
1000000
n



50
0
-50
-100
16490000
16500000
16510000
20000
0
-20000
0.0
2.0x10^7
4.0x10^7
6.0x10^7
n


20000
0
-20000
0.0
2.0x10^7
4.0x10^7
6.0x10^7
n
??

Mertens conjecture

Mertens function:

$$M(n) = \sum_{1 \le k \le n} \mu(k),$$

μ(k) is the Möbius function;

$\mu(n) = +1$ if n is a square-free positive integer with an even number of prime factors. (6, 10,..)
$\mu(n) = -1$ if n is a square-free positive integer with an odd number of prime factors. (2, 3, …)
$\mu(n) = 0$ if n has a squared prime factor. (4, 9, …)

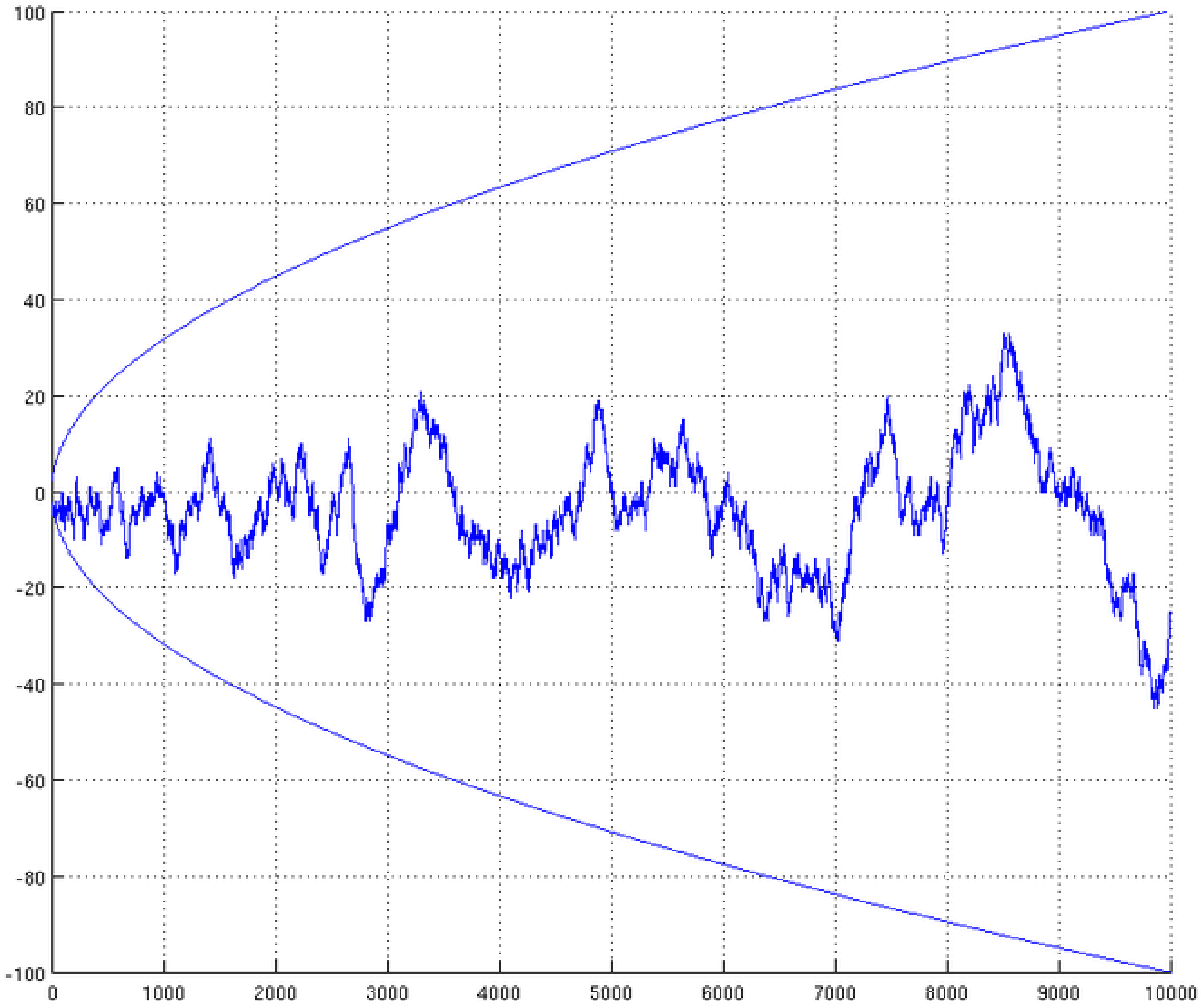
n	1	2	3	4	5	6	7	8	9	10
$\mu(n)$	1	−1	−1	0	−1	1	−1	0	0	1

Mertens conjecture

… for all n > 1,

$$|M(n)| < \sqrt{n}.$$

??



Disproof of the Mertens conjecture

By *A. M. Odlyzko* at Murray Hill and *H. J. J. te Riele* at Amsterdam

Odlyzko, A. M.; Riele, H. J. J. Disproof of the Mertens conjecture, Journal für die reine und angewandte Mathematik. (1985)

Conjectures

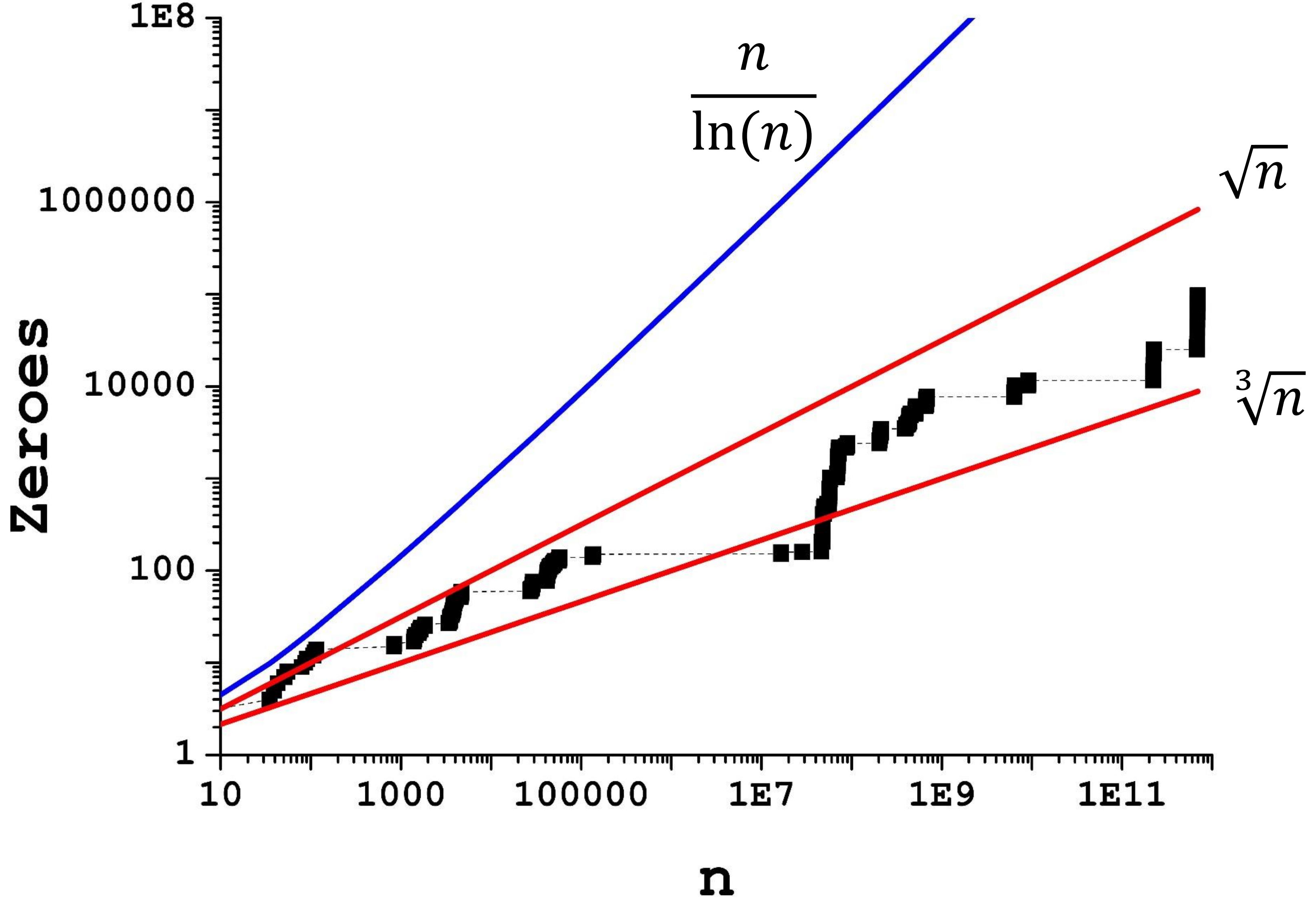
I. The number of cuts (zeroes) in the x axis tends to infinity. i.e, being Z(n) the number of zeroes in the Ladder

$$\lim_{n \to \infty} Z(n) = \infty$$

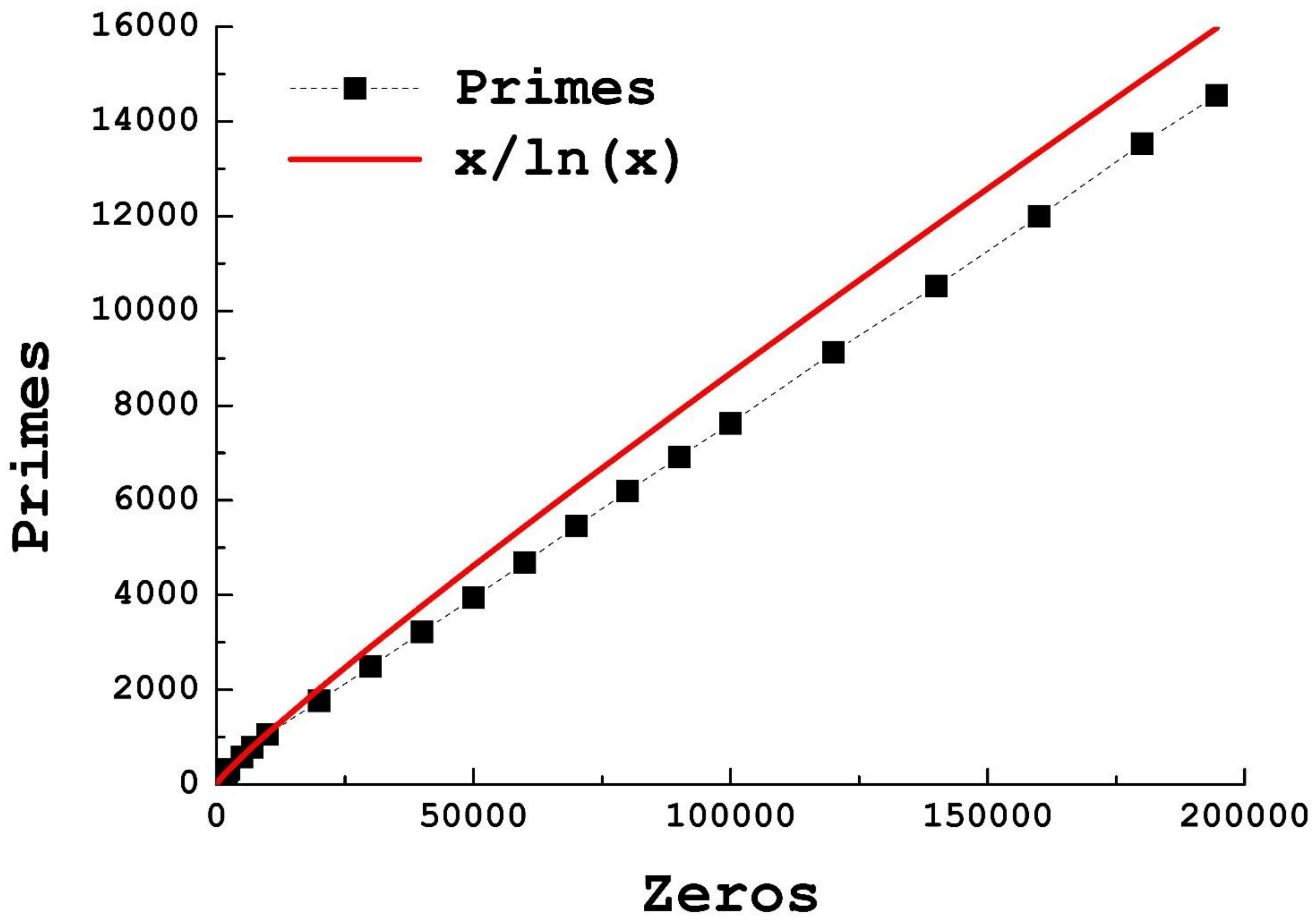
II. The slope, ε(n), of the Ladder is zero in the limit when n goes to infinity.

III. Envelope function?

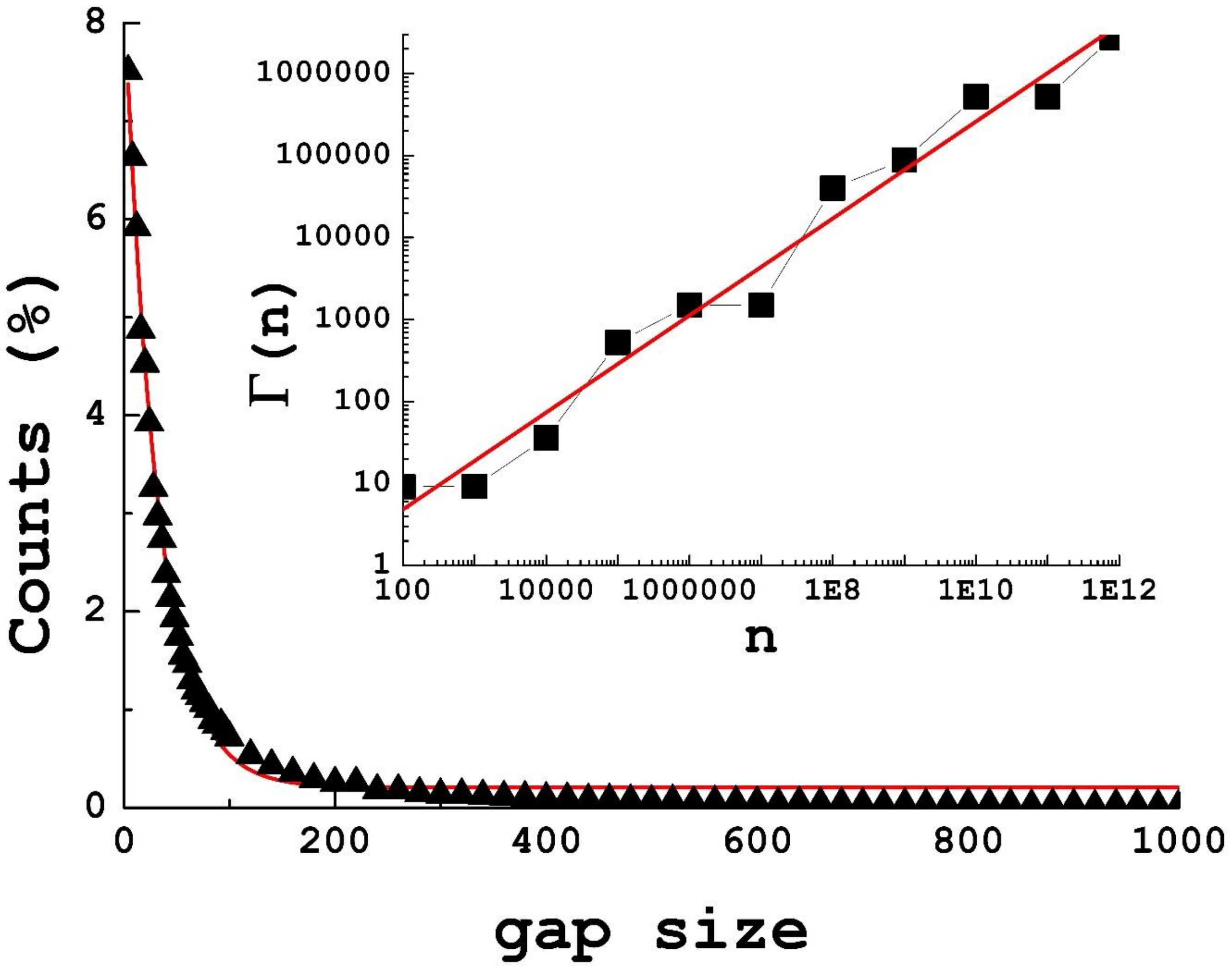
~200,000 zeroes in $8x10^{12}$



Results II. Prime numbers



Results III. Gaps



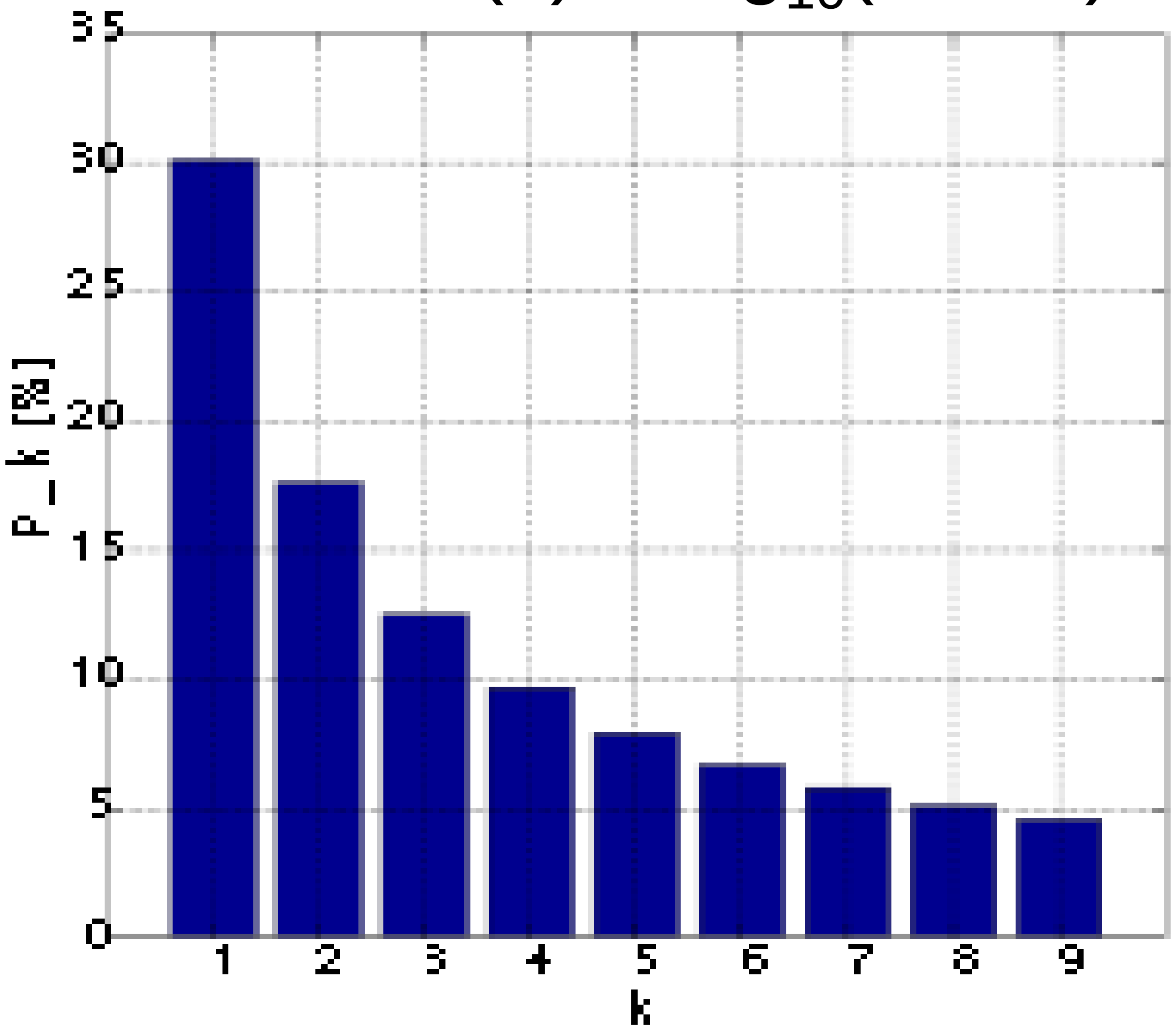
Benford Law

$$P(d) = \log_{10}(1+1/d)$$

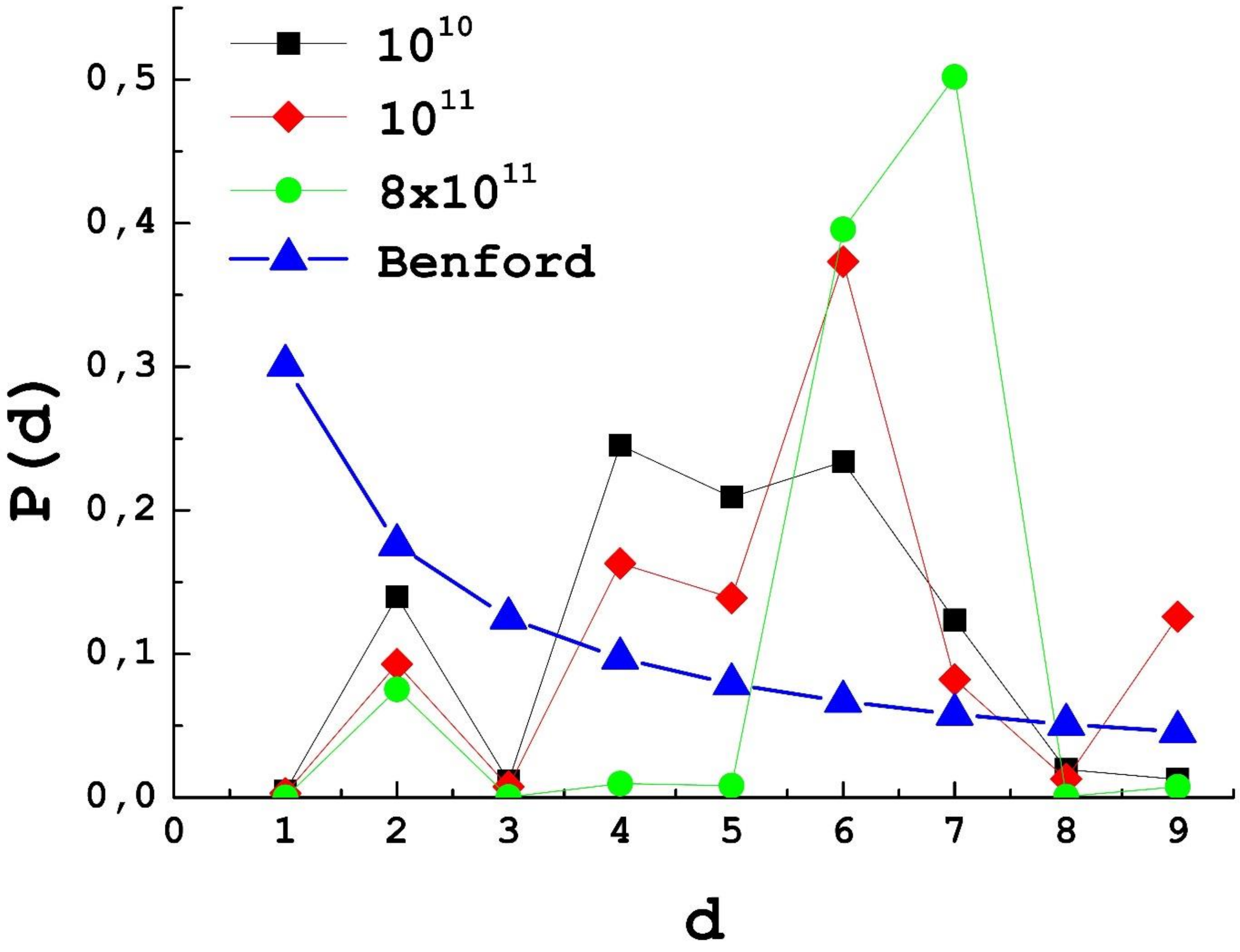
Examples

Fibonacci numbers
Factorials n!
Powers n^m
Binomial coefs $\binom{n}{m}$
Etc..

Length of rivers…



Benford Law



Determinants

Prime numbers sequence: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, …

$$D_1 = \begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix} = -1$$

$$D_2 = \begin{vmatrix} 11 & 13 \\ 17 & 19 \end{vmatrix} = -12$$

$$D_3 = \begin{vmatrix} 23 & 29 \\ 31 & 37 \end{vmatrix} = -48$$

…

Determinants

Jacob´s Ladder Zeroes sequence: 1, 3, 7, 35, 39, 43, 51, 55, 79, 87, 91, 107 …

$$D_1 = \begin{vmatrix} 1 & 3 \\ 7 & 35 \end{vmatrix} = 14$$

$$D_2 = \begin{vmatrix} 39 & 43 \\ 51 & 55 \end{vmatrix} = -48$$

$$D_3 = \begin{vmatrix} 79 & 87 \\ 91 & 107 \end{vmatrix} = 538$$

…

Determinants

Jacob´s Ladder Zeros sequence: 1, 3, 7, 35, 39, 43, 51…

Leading digit

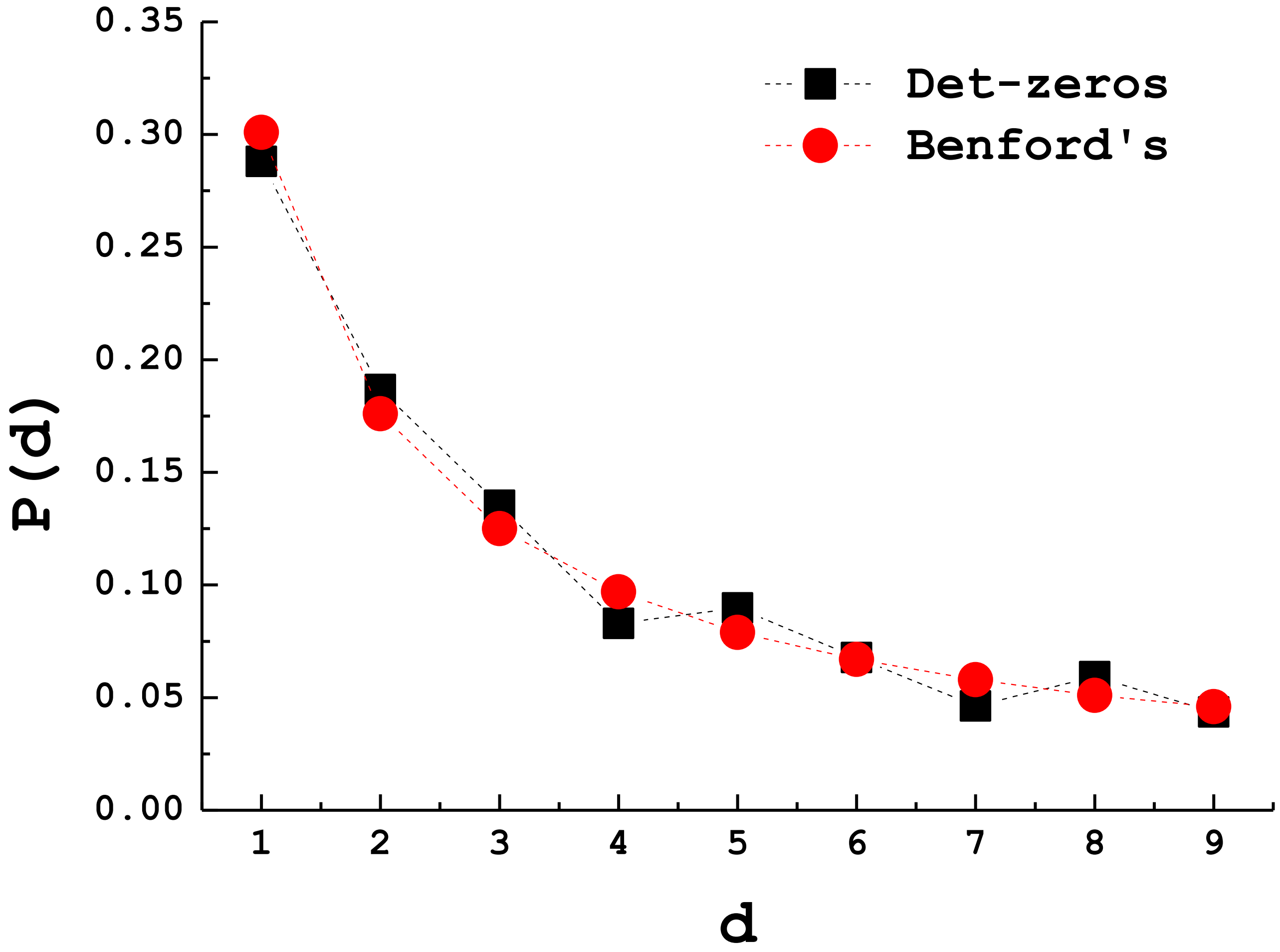
$$D_1 = \begin{vmatrix} 1 & 3 \\ 7 & 35 \end{vmatrix} = 14 \quad \rightarrow \quad 1$$

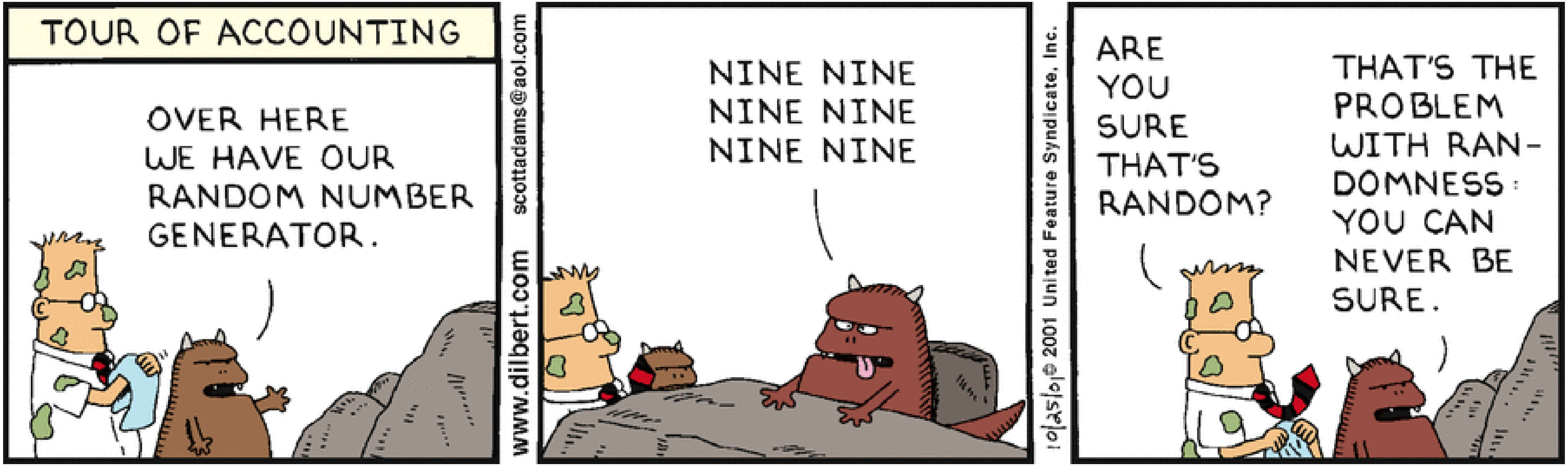
$$D_2 = \begin{vmatrix} 39 & 43 \\ 51 & 55 \end{vmatrix} = -48 \quad \rightarrow \quad 4$$

$$D_3 = \begin{vmatrix} 79 & 87 \\ 91 & 107 \end{vmatrix} = 538 \quad \rightarrow \quad 5$$

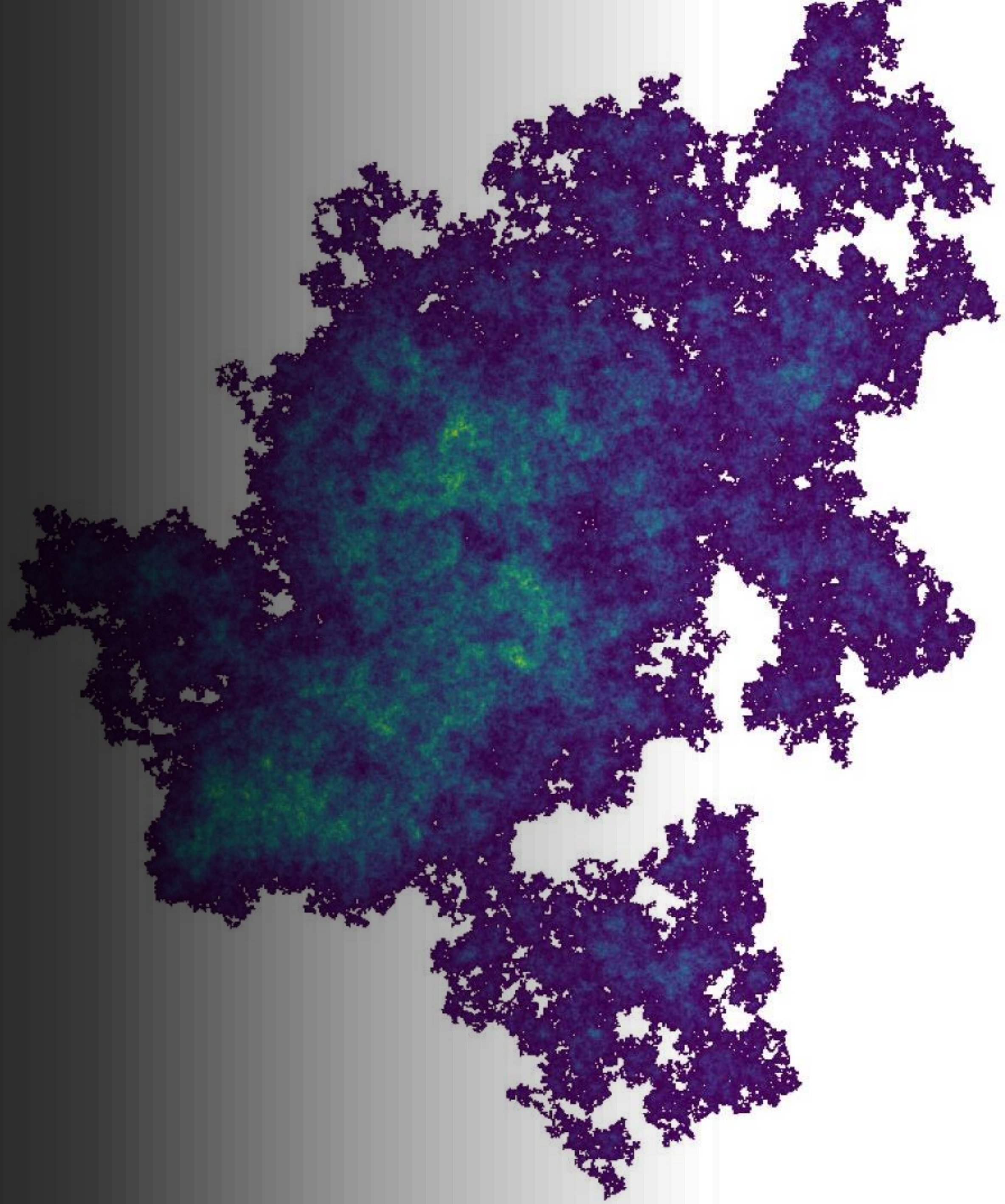
…

Determinants

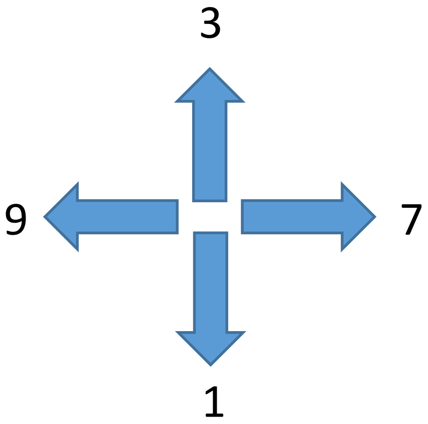



TOUR OF ACCOUNTING
OVER HERE WE HAVE OUR RANDOM NUMBER GENERATOR.
scottadams@aol.com
www.dilbert.com
NINE NINE NINE NINE NINE NINE
© 2001 United Feature Syndicate, Inc.
ARE YOU SURE THAT'S RANDOM?
THAT'S THE PROBLEM WITH RANDOMNESS: YOU CAN NEVER BE SURE.

Prime numbers and random walks in a square grid



Original PW. (Let´s call it A1)



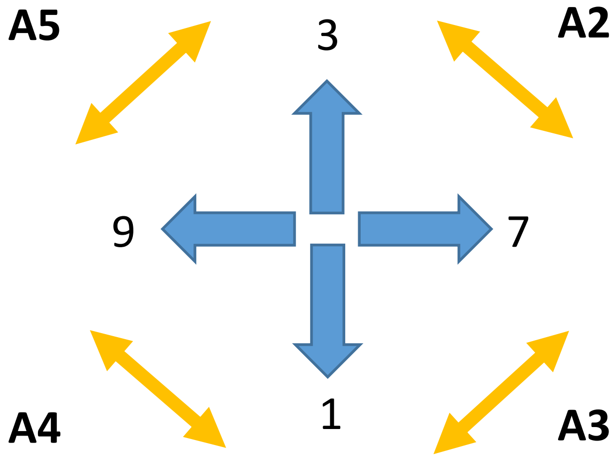
Posible “nonequivalent” algorithms?

Yellow arrow means changing 3 for 7 in algorithm A1 (leaving other numbers untouched)

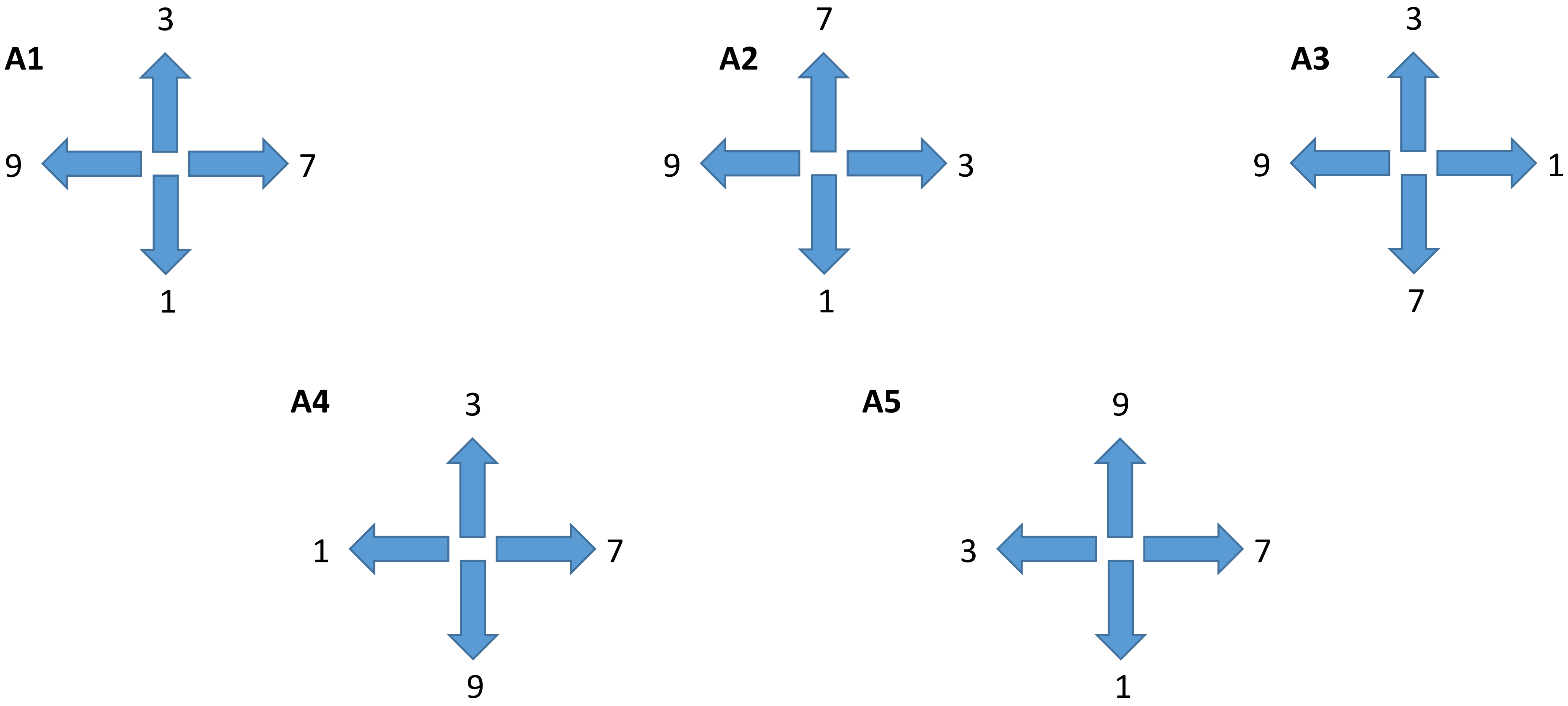
→ This creates Algorithm A2

Same for the rest.

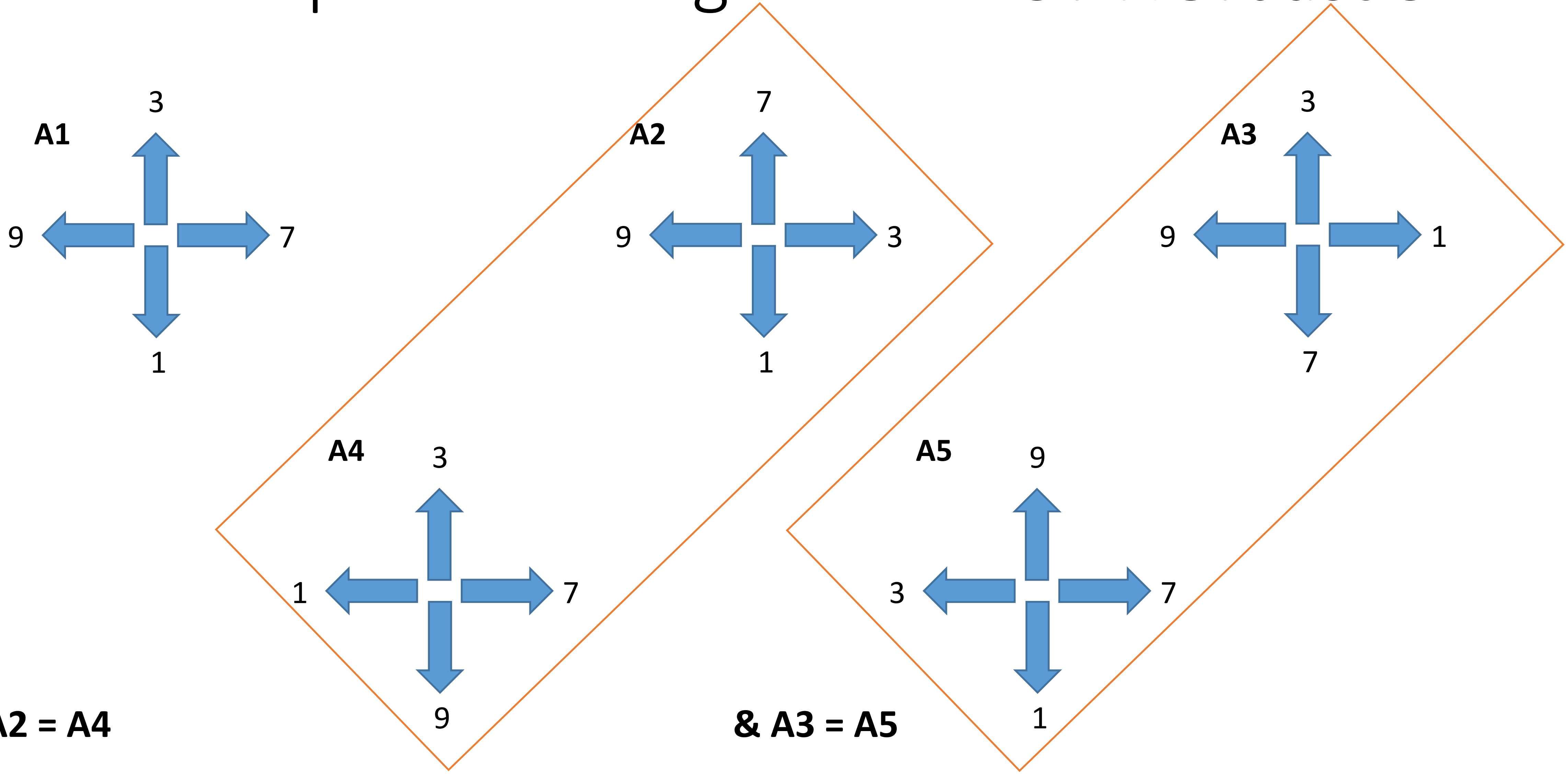
The rest are symmetries or rotations.



Posible “nonequivalent” algorithms: **5?**



Nonequivalent" algorithms: **5? NO. Just 3**



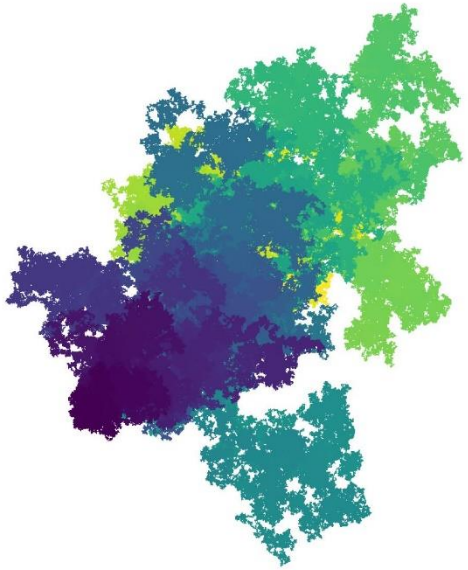
A2 = A4

& A3 = A5

[(246) Prime numbers and random walks in a square grid – YouTube]

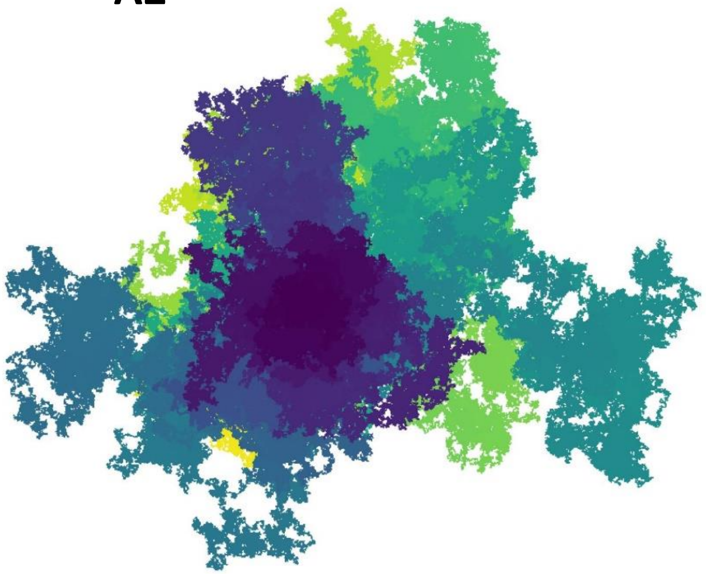
N=10E8

A1



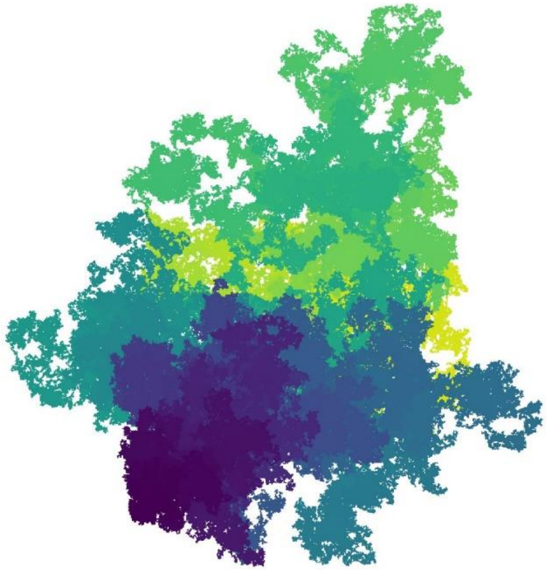
A = 245970

A2



A2 = 271026

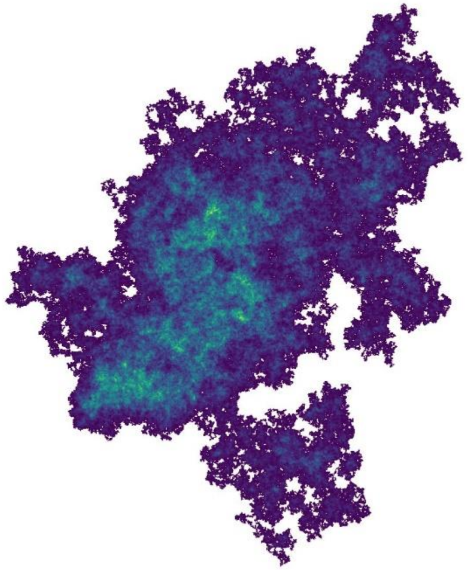
A3



A3 = 209327

N=10E8

A1

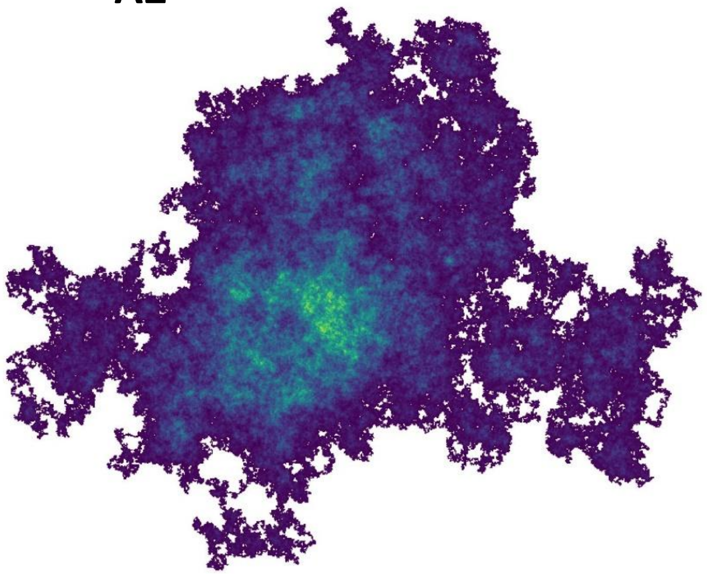


A = 245970

Z_M = 3126

V = 768,902,220

A2

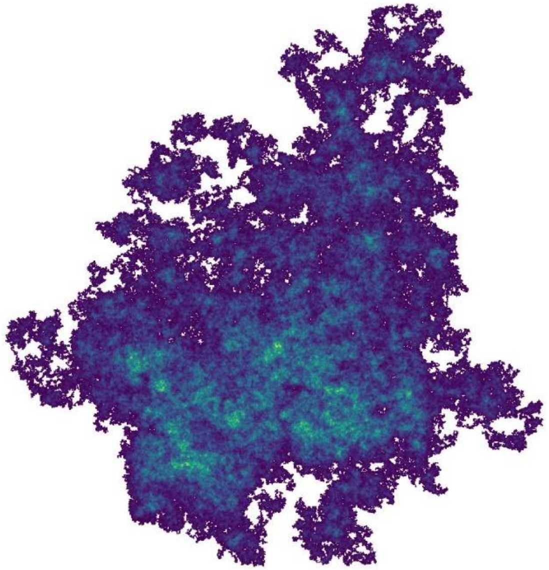


A2 = 271026

Z_M = 2646

V = 717,134,796

A3

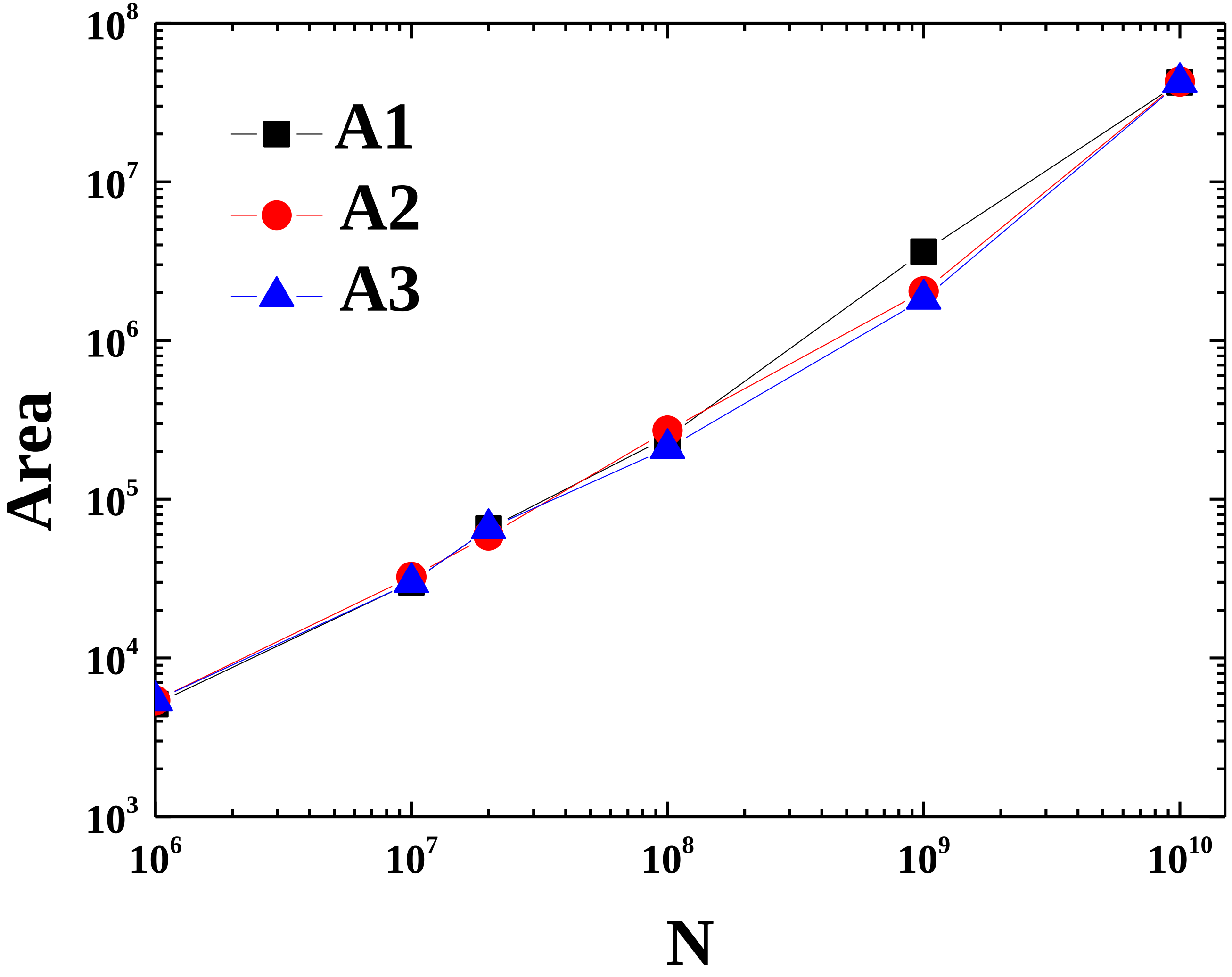


A3 = 209327

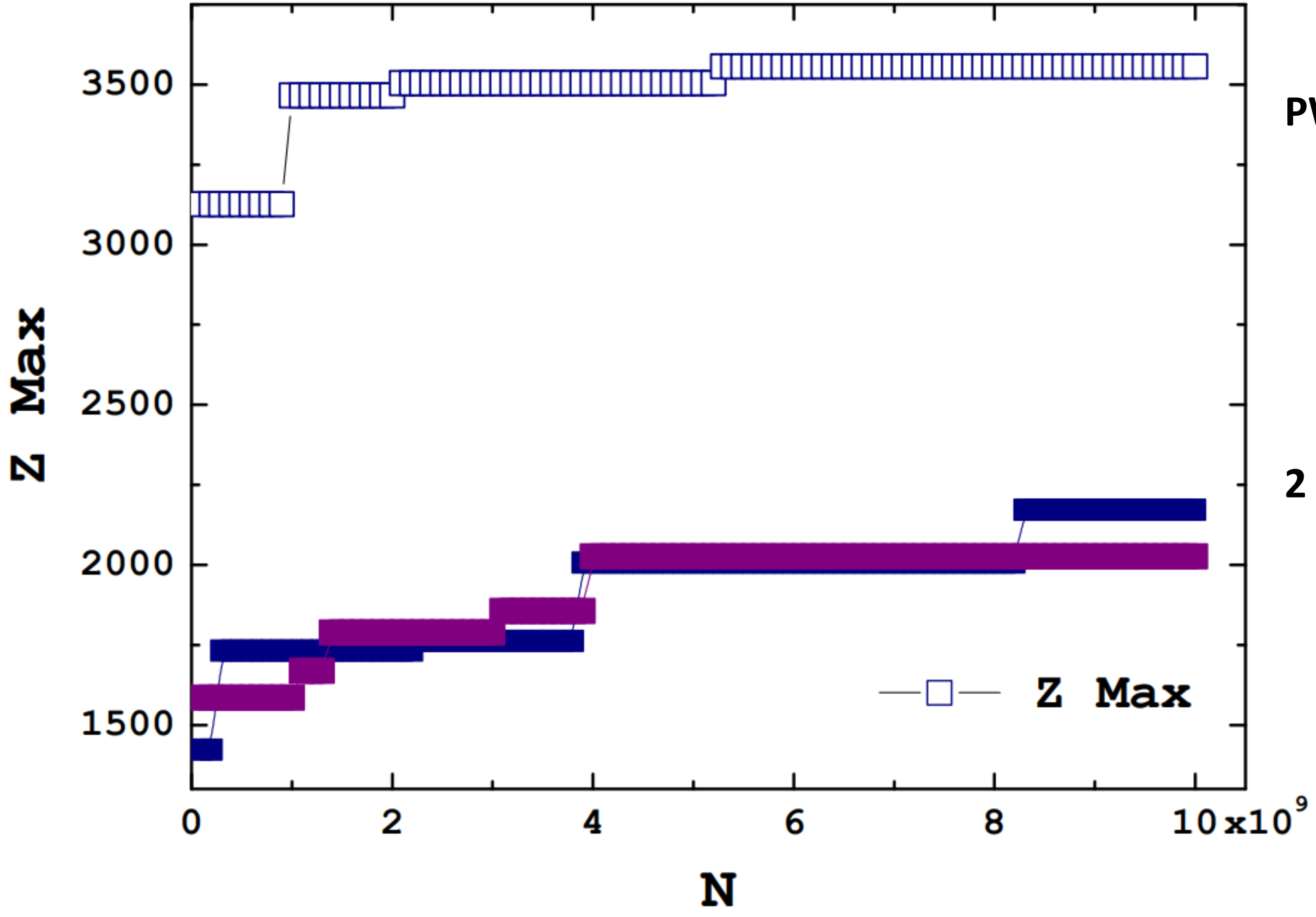
Z_M = 4088

V = 855,728,776

Area



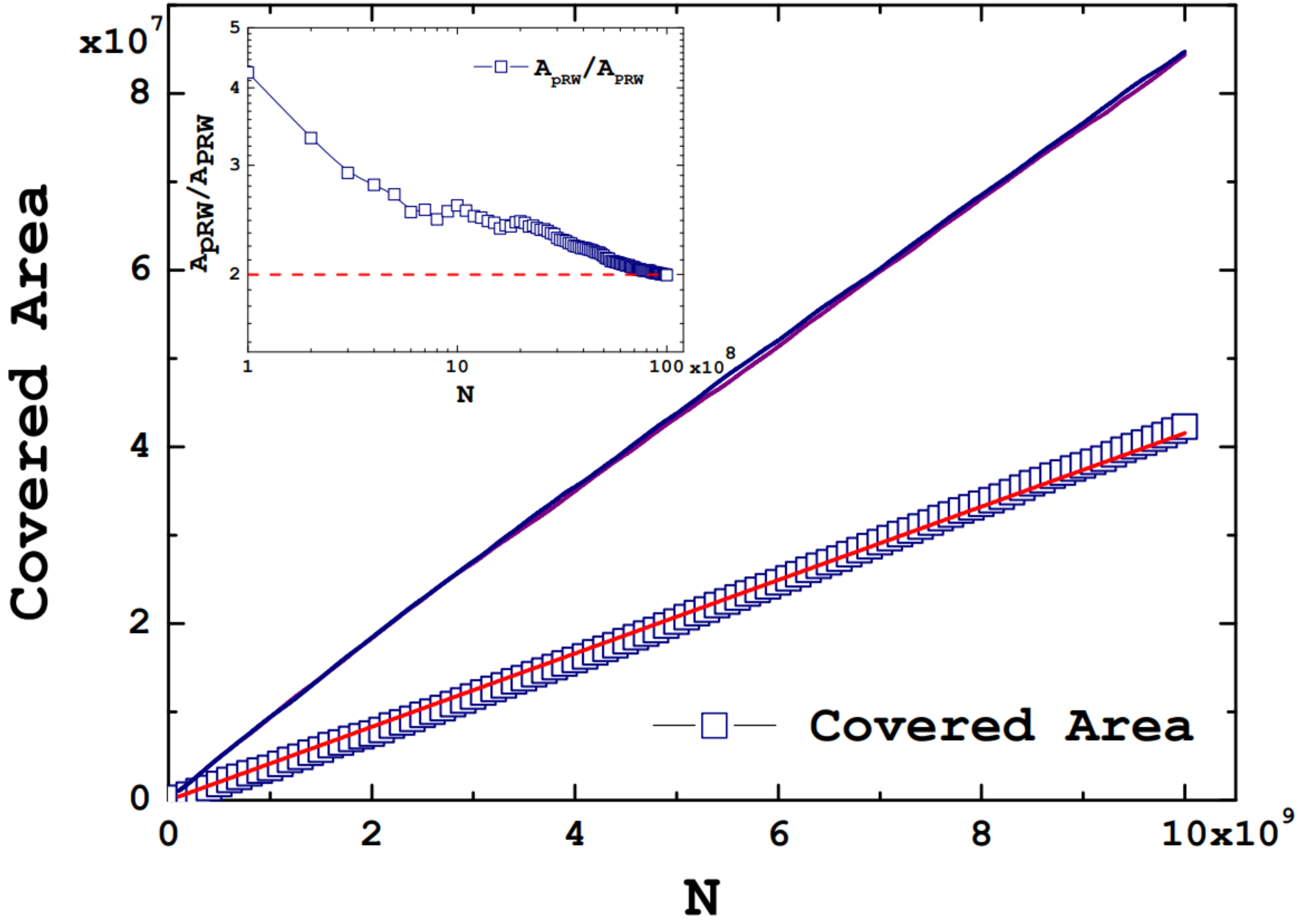
Z Max



PW Unlimited

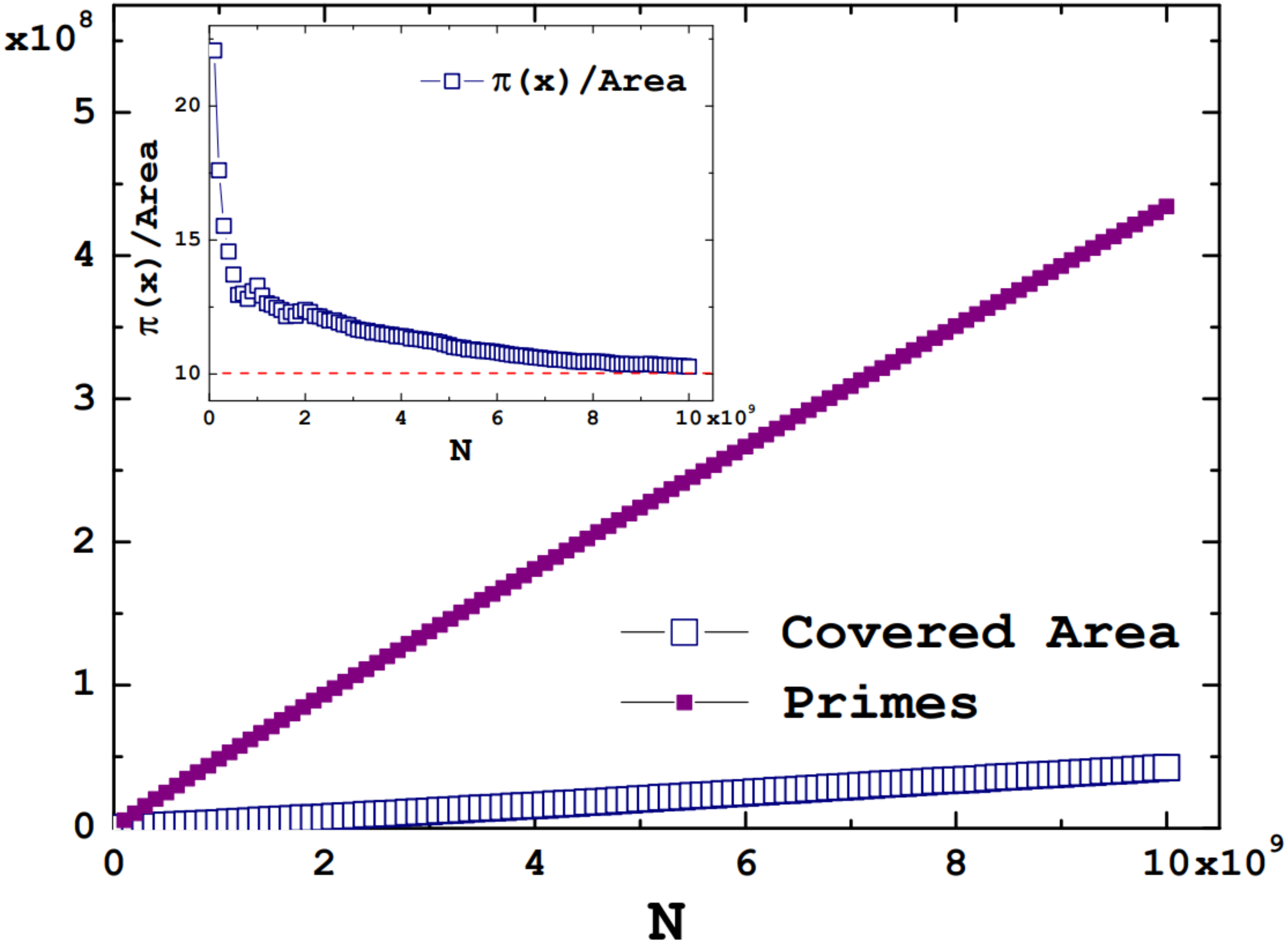
2 dif. pRWs

PW vs pseudoRW



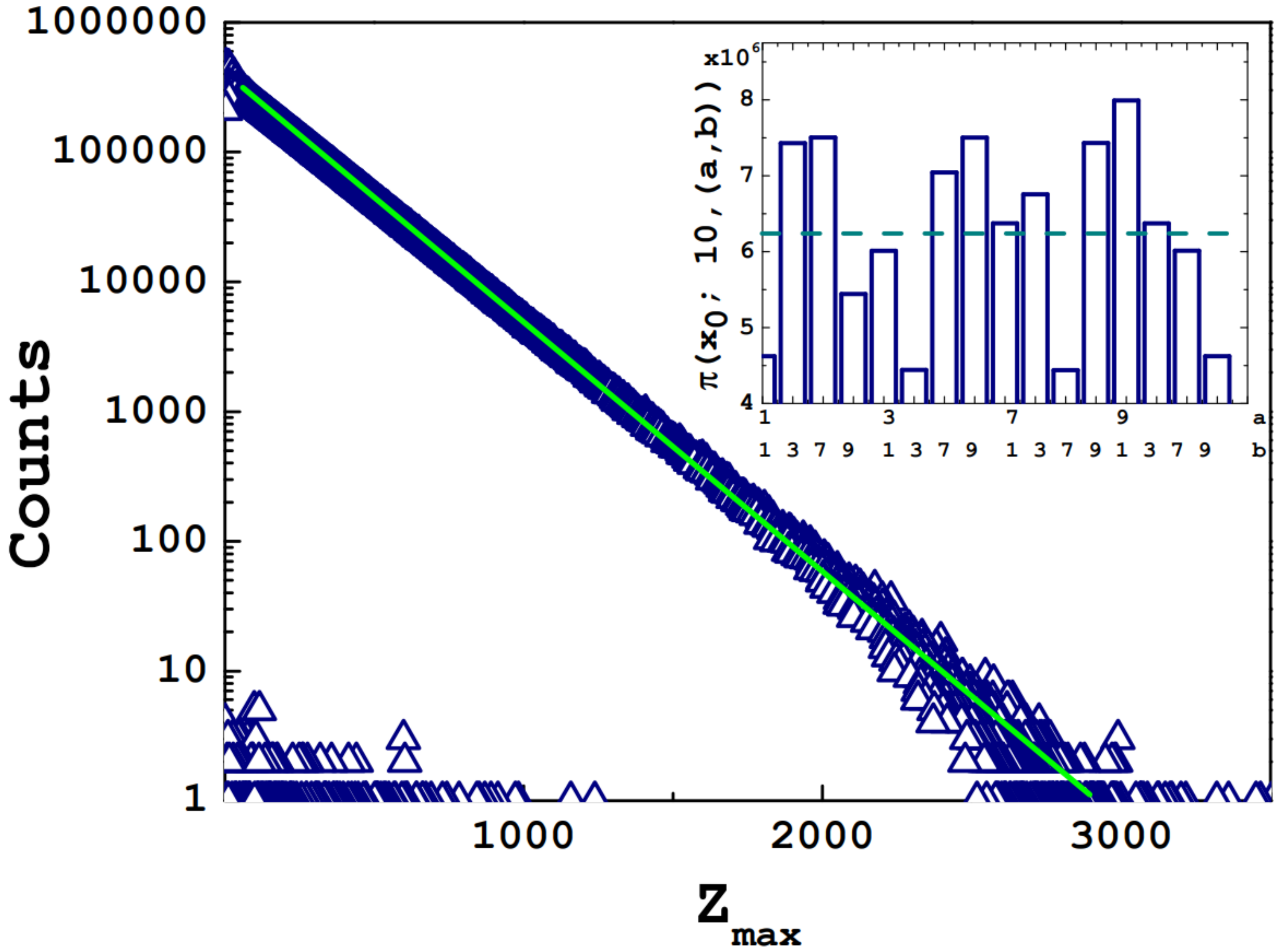
lim = 2.0 ?

Area *vs* Number of primes



lim = 10.0 ?

Why?



Chebyshev's bias ?

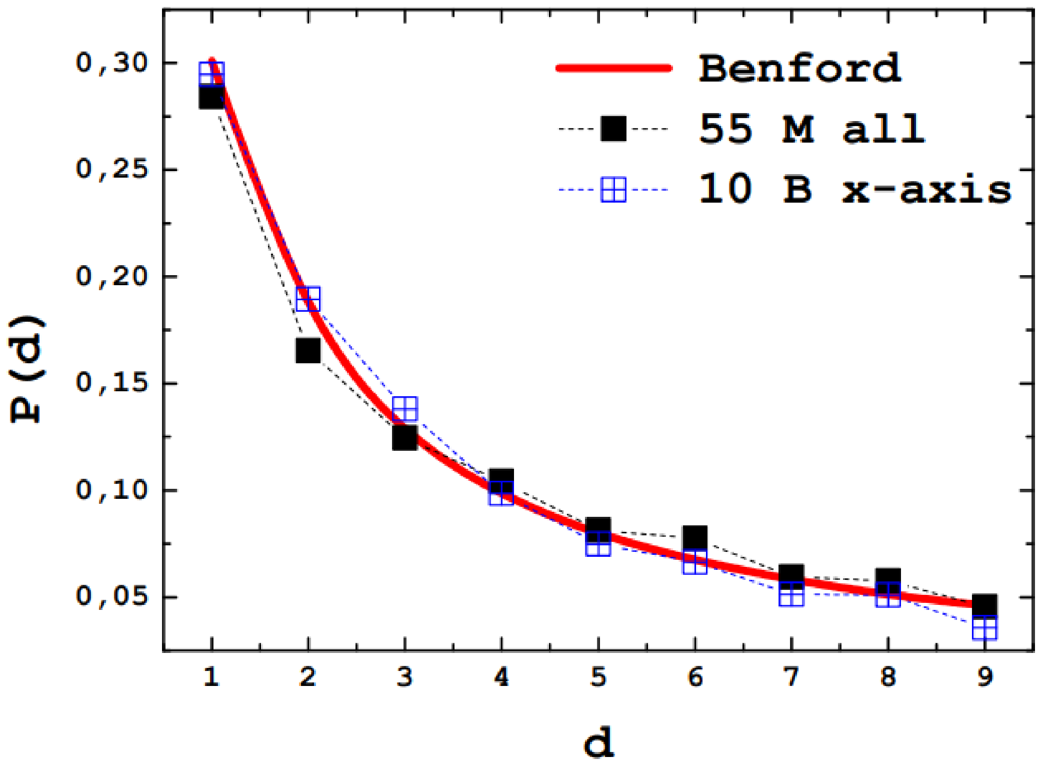


FIG. 6. (color online) Leading digit histogram of the $z_{\max}$ values (PW up to $5.5 \cdot 10^7$). The proportion of each of the $z_{\max}$ values is shown in black squares. Blue squares are another example, here up to 10^9, but the (x, y) points considered are only those in the x axis. The expected values according to Benford's law are shown by the red curve.

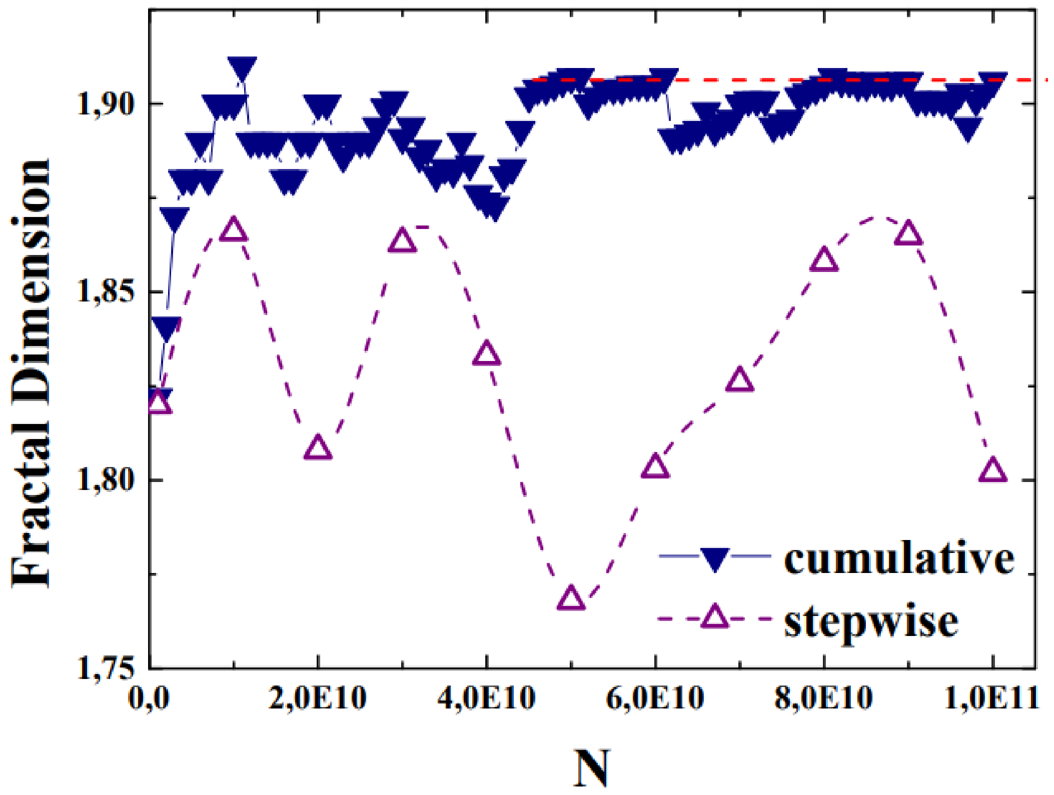


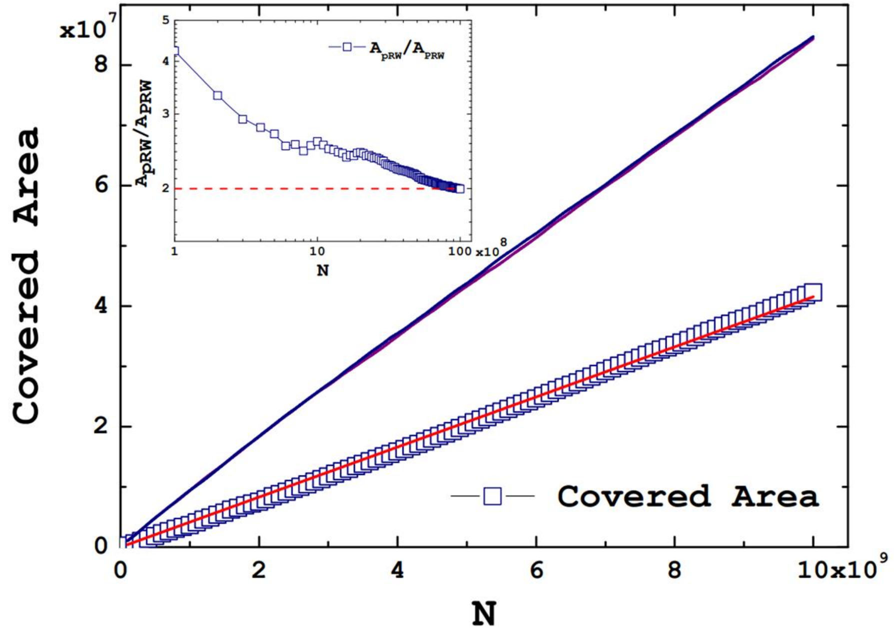
FIG. 7. Fractal dimension vs N calculated in a cumulative way (blue inverted triangles) and in steps of 10^8 (magenta empty triangles).

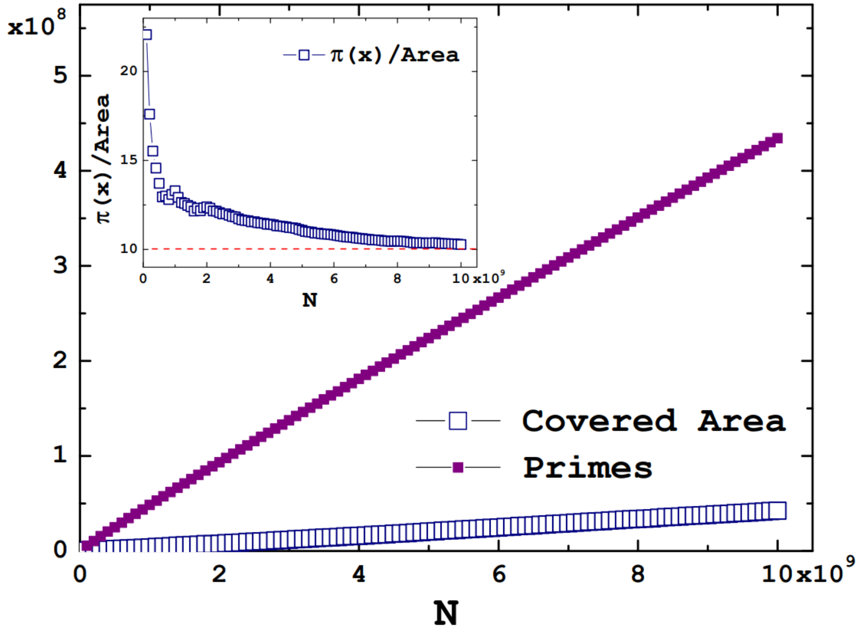
2 Big Open Questions

I $A_{pRW}/A_{PW} = 2.0$ when $N \rightarrow \infty$

II $\pi(N)/A_{PW} = 10.0$ when $N \rightarrow \infty$

III …





To think

“Simple” algorithm

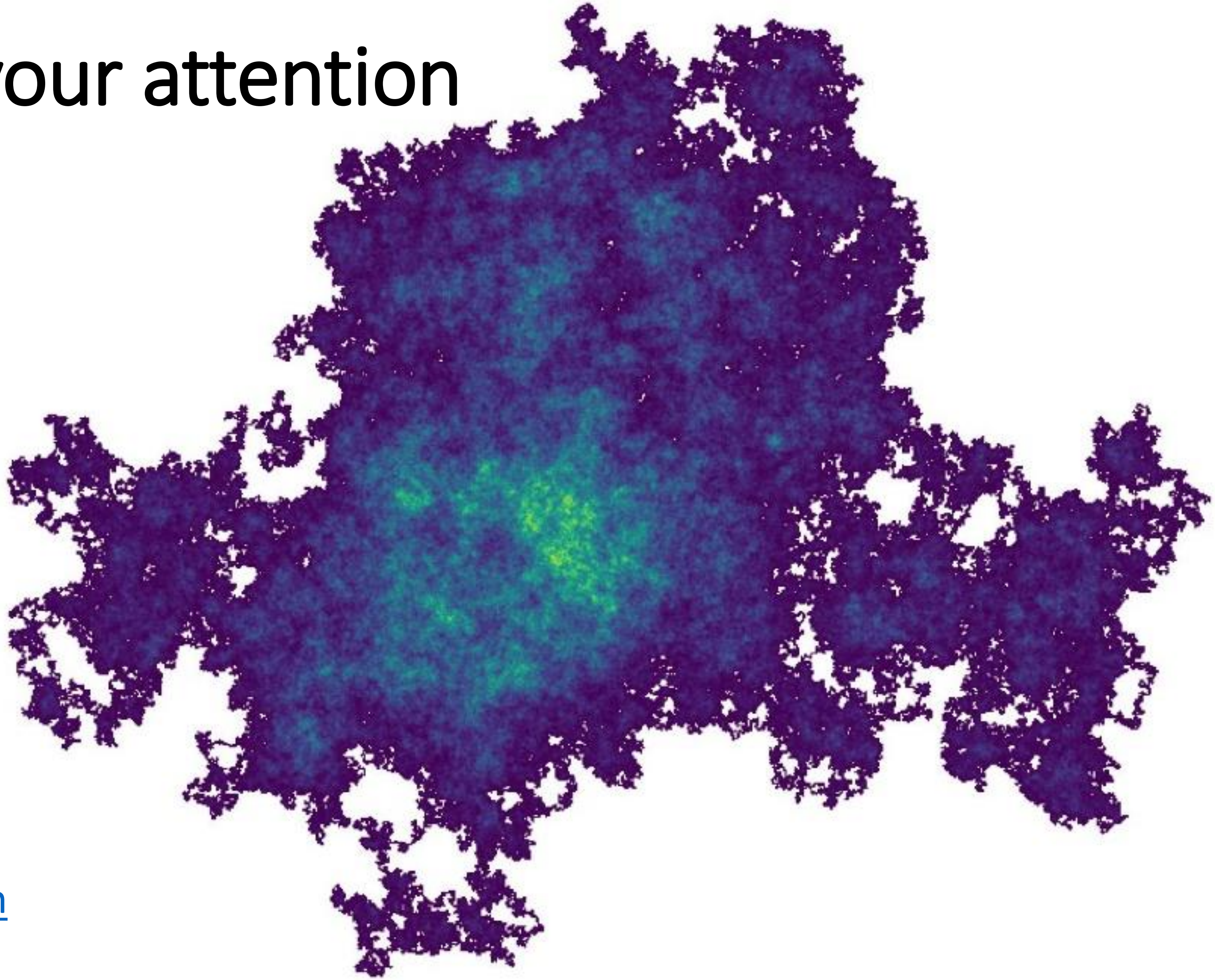
+

sequence of “random” numbers

=

“order” ??

Thank you for your attention



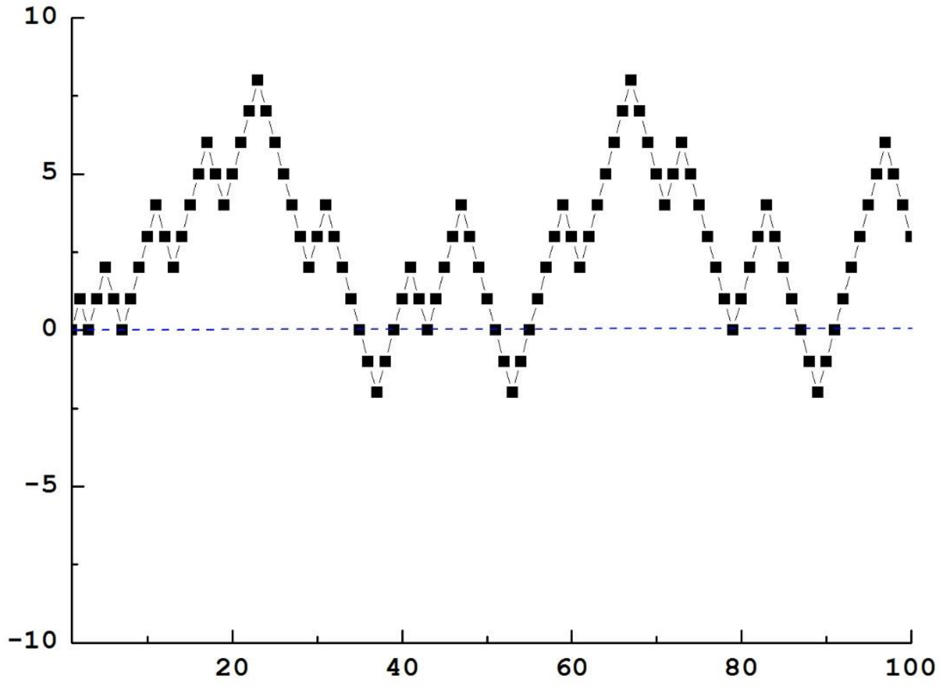
Alberto Fraile:

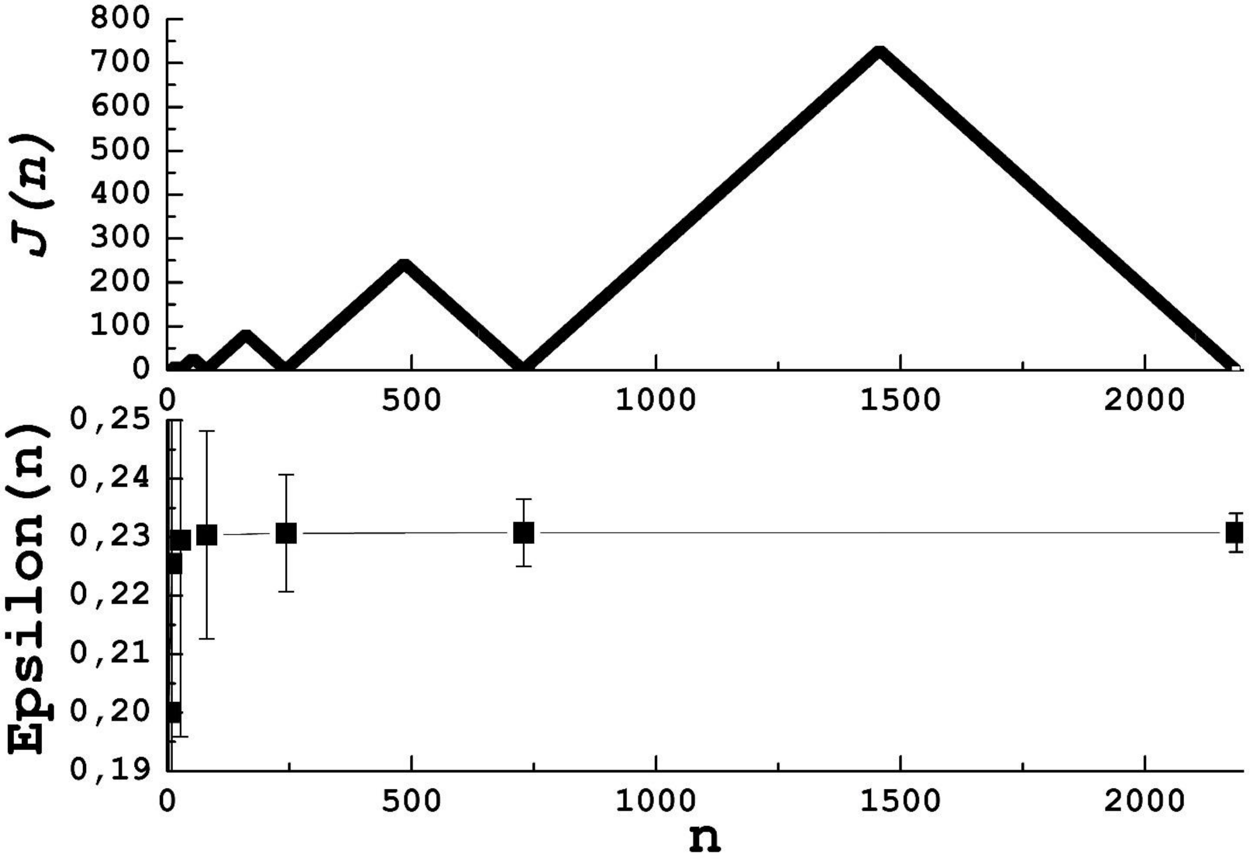
albertofrailegarcia@gmail.com

Conclusions

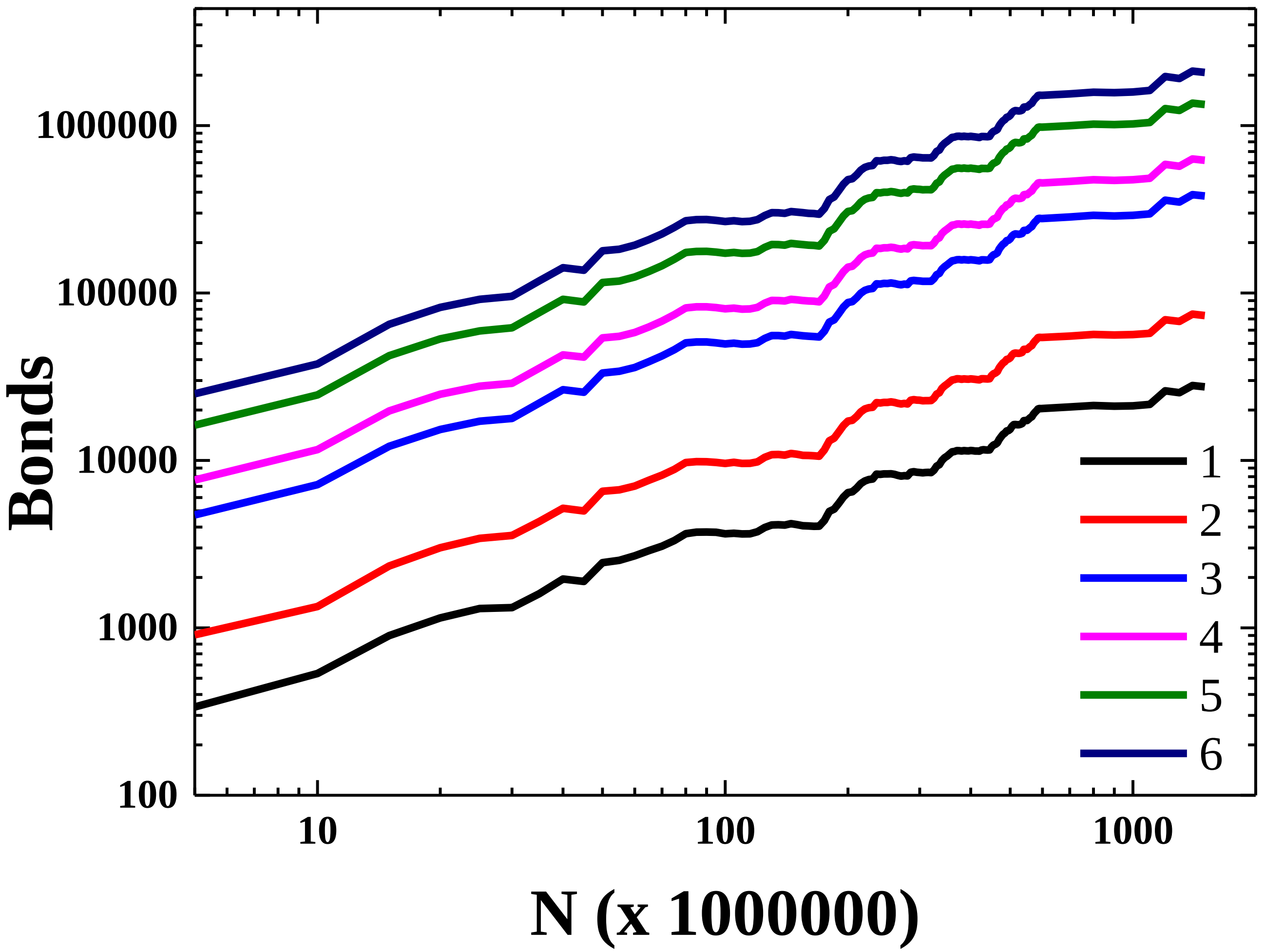
Interval	Zeroes	Primes	$n/\log n$	Diff (%)	Average gap Γ
$[1, 10^2]$	10	5	4.342	13.16	9.2
$[1, 10^3]$	16	6	5.770	3.82	9.25
$[1, 10^4]$	59	21	14.469	31.09	36.20
$[1, 10^5]$	139	36	28.169	21.75	526.57
$[1, 10^6]$	151	37	30.096	18.65	1503.97
$[1, 10^7]$	151	37	30.096	18.65	1503.97
$[1, 10^8]$	2,415	313	310.034	0.947	40170.11
$[1, 10^9]$	7,730	846	863.41	-2.058	887722.55
$[1, 10^{10}]$	11,631	1,161	1,242.438	-7.014	523588.07
$[1, 10^{11}]$	11,631	1,161	1,242.438	-7.014	523588.07
$[1, 8 \cdot 10^{11}]$	194,530	14,556	15,973	-9.734	2750072.04

Conjecture II



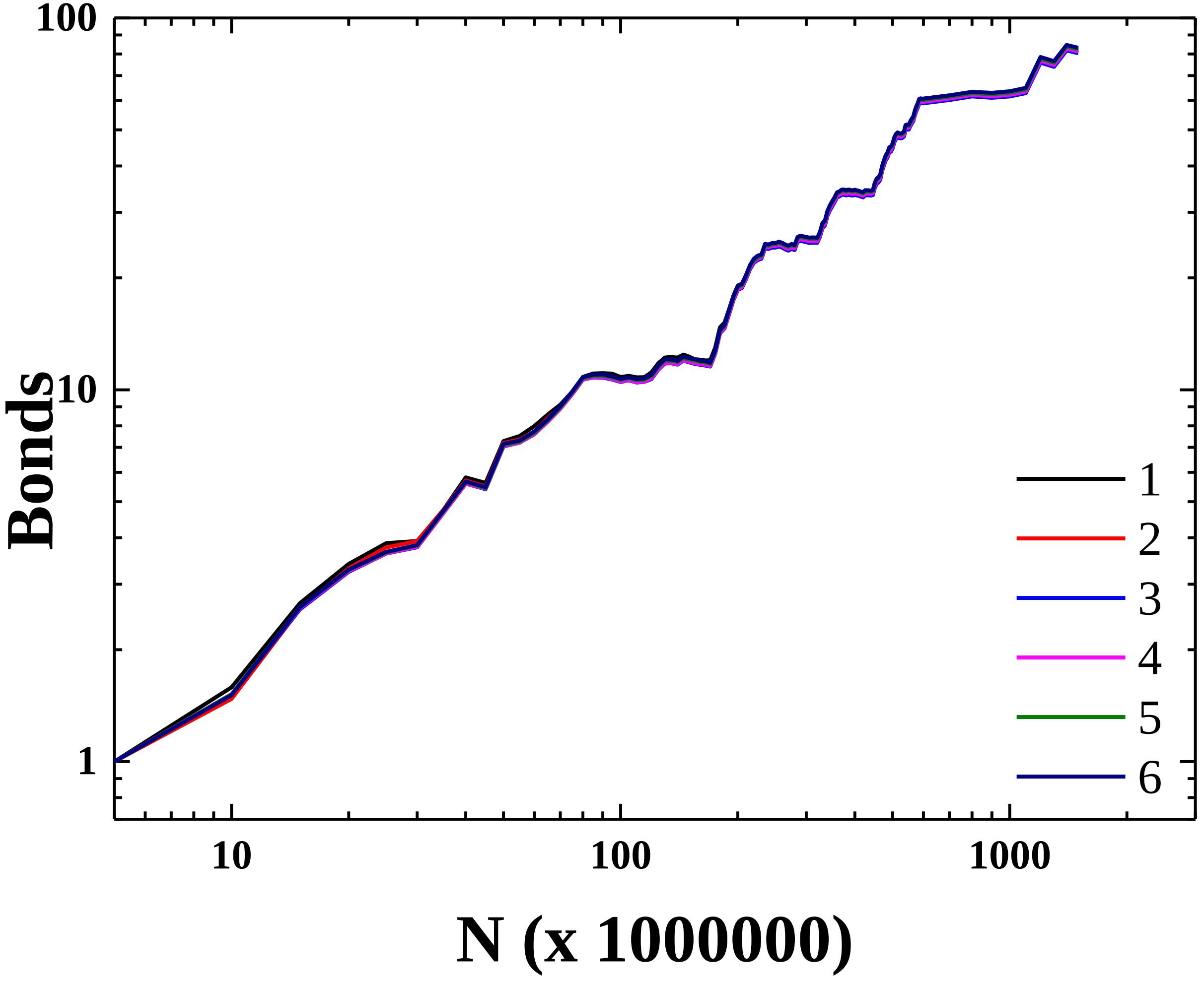


Bonds



Bonds
1000000
100000
10000
1000
100
10
100
1000
N (x 1000000)
1
2
3
4
5
6

Bonds



References:

[1] A. Fraile, R. Martínez & D. Fernández. Jacob's Ladder: Prime Numbers in 2D. Math. Comput. Appl. 2020, 25(1)

[2] A. Fraile, O. Kinouchi, P. Dwivedi, R. Martínez, T. E. Raptis, and D. Fernández. Prime numbers and random walks in a square grid. Phys. Rev. E 104, 054114 (2021)

[3] T. Raptis, A. Fraile. Persistent order in Schramm-Loewner Evolution driven by the primes last digit sequence