A Reduction of Temporary Coalitions in Infinite Multiplayer Games

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Plan of Talk

Preliminaries: CGMs with incomplete information

$\Sigma \triangleq \{1, \ldots, N\}$ - a set players

$AP = \{p, q, \ldots\}$ - a set of atomic propositions

$$M \triangleq \langle W, W_I, \langle Act_i : i \in \Sigma \rangle, \langle \sim_i : i \in \Sigma \rangle, \langle P_i : i \in \Sigma \rangle, o, V \rangle$$

W - the set of states; $W_I \subseteq W$ - the initial states

Act_i - the actions of player $i \in \Sigma$

$\sim_i \subseteq W \times W$ - the indistinguishability relation of player i

$P_i : W \to \mathcal{P}(Act_i) \setminus \{\emptyset\}$ - i's protocol

$Act_\Gamma \triangleq \prod_{i \in \Gamma} Act_i$

$o : W \times Act_\Sigma \to W$ - the outcome function

$V \subseteq W \times AP$ is the valuation relation

$P_i(w') = P_i(w'')$ is required, if $w' \sim_i w''$.

Negotiations as Sequences of Conditional Promises

Negotiations are sequences of promises of the form:

> *$i \in \Sigma$ promises to choose an action from $B \subset Act_i$, if the players from $\Gamma \subseteq \Sigma \setminus \{i\}$ choose an action profile from $A \subset Act_\Gamma$.*

The promise is made known to the members of Γ.

Let $\hat{A} \mathrel{\hat{=}} \bigvee_{a \in A} \bigwedge_{j \in \Gamma} a_j$ for $A \subseteq Act_\Gamma$.
Then this promise can be written in epistemic logic:

$$\mathrm{promise}_{i,\Gamma}(A, B) \mathrel{\hat{=}} \mathrm{C}_{\{i\} \cup \Gamma}(\mathrm{K}_i \hat{A} \Rightarrow \mathrm{C}_\Gamma \hat{B}) \ .$$

Conditional Promises: Example

$A, B, C \in \Sigma$ can agree on move $\langle a, b, c \rangle$ using the sequence of promises:

$A: \; \mathrm{C}_{\{A,B,C\}}(\mathrm{K}_A(b \wedge c) \Rightarrow \mathrm{C}_{\{B,C\}} a)$

$B: \; \mathrm{C}_{\{A,B,C\}} \left(\mathrm{K}_B c \Rightarrow \mathrm{C}_{\{C,A\}} b\right)$

$C: \; \mathrm{C}_{\{A,B,C\}} c$

$\mathrm{C}_{\{A,B,C\}}(a \wedge b \wedge c)$ is a logical consequence of all the promises:

C's promise can be simplified to $\mathrm{C}_{\{A,B,C\}} c$.
Similarly B's promise simplifies to $\mathrm{C}_{\{A,B,C\}}(\mathrm{K}_B c \Rightarrow \mathrm{C}_{\{C,A\}} b)$.

C's promise and B's promise imply $C_{\{A,B,C\}}(b \wedge c)$.

Now $\mathrm{C}_{\{A,B,C\}}(a \wedge b \wedge c)$ follows from A's promise.

Honorability of Promises

A conjunction of promises s is honorable, if

(1) for every $a \in Act_\Sigma$, either $s \wedge C_\Sigma \bigwedge_{j \in \Sigma} a_j$ is consistent,

or $s \wedge C_\Sigma a_i$ is inconsistent for some particular $i \in \Sigma$.

and

(2) there exists an $a \in Act_\Sigma$ such that $s \wedge C_\Sigma \bigwedge_{i \in \Sigma} a_i$ is consistent.

Condition (2) states that what is promised in s is doable.

Condition (1) states that a player can independently assess whether its action is consistent with its promises.

We let

$$s \star p \hat{=} \begin{cases} s \wedge p, & \text{if } s \wedge p \text{ is honorable;} \\ s, & \text{otherwise.} \end{cases}$$

Extending CGMs to Provide a Meaning for Promises

To enable the interpretation of $\mathsf{C}_{\{i\}\cup\Gamma}(\mathsf{K}_i\hat{A} \Rightarrow \mathsf{C}_\Gamma\hat{B})$ we

(1) include all actions in the vocabulary $AP^{\mathbf{1}} \triangleq AP \uplus \biguplus_{i\in\Sigma} Act_i$, and

(2) extend states to store latest actions.

$$M^{\mathbf{1}} \triangleq \langle W^{\mathbf{1}}, W_I^{\mathbf{1}}, \langle Act_i : i \in \Sigma\rangle, \langle \sim_i^{\mathbf{1}} : i \in \Sigma\rangle, \langle P_i : i \in \Sigma\rangle, o^{\mathbf{1}}, V^{\mathbf{1}}\rangle$$

$$W^{\mathbf{1}} \triangleq W \times (Act_\Sigma \uplus \{*\}), \quad W_I^{\mathbf{1}} \triangleq W_I \times \{*\}$$

$$\langle w', a'\rangle \sim_i^{\mathbf{1}} \langle w'', a''\rangle \triangleq w' \sim_i w'' \wedge a'_i = a''_i$$

$$o^{\mathbf{1}}(\langle w, b\rangle, a) \triangleq \langle o(w, a), a\rangle$$

$$V^{\mathbf{1}}(\langle w, b\rangle, p) \triangleq V(w, p), \text{ if } p \in AP$$

$$V^{\mathbf{1}}(\langle w, b\rangle, a) \triangleq b_i = a, \text{ if } a \in Act_i$$

Now $\mathsf{C}_{\{i\}\cup\Gamma}(\mathsf{K}_i\hat{A} \Rightarrow \mathsf{C}_\Gamma\hat{B})$ is fulfilled at $\langle w, a\rangle \in W^{\mathbf{1}}$, if

$$M^{\mathbf{1}}, \langle w, a\rangle \models \forall\bigcirc(\mathsf{C}_{\{i\}\cup\Gamma}(\mathsf{K}_i\hat{A} \Rightarrow \mathsf{C}_\Gamma\hat{B}))$$

CGMs with Negotiation: Negotiation Moves

To incorporate negotiation steps in CGMs, we

regard making promises as players' actions;

extend states to store promises.

We extend M^1 to

$$\check{M} \triangleq \langle \check{W}, \check{W}_I, \langle \check{Act}_i : i \in \Sigma \rangle, \langle \check{\sim}_i : i \in \Sigma \rangle, \langle \check{P}_i : i \in \Sigma \rangle, \check{o}, \check{V} \rangle$$

$$\mathcal{S}_i \triangleq \left\{ \text{promise}_{i,\Gamma}(A, B) : \Gamma \subseteq \Sigma \setminus \{i\}, A \subseteq Act_{\Gamma_k}, B \subseteq Act_i \right\}$$

$$\mathcal{S}^\wedge \triangleq \{ s_1 \wedge \ldots \wedge s_k : s_1, \ldots, s_k \in \mathcal{S}_i, i \in \Sigma \}$$

$$\check{Act}_i \triangleq Act_i \cup \mathcal{S}_i.$$

We end up with 2 kinds of moves. Move $a \in \check{Act}_\Sigma \triangleq \prod_{i \in \Sigma} Act_i \cup \mathcal{S}_i$ is

negotiation - if $a_i \in \mathcal{S}_i$ for some i;

implementation, if $a_i \in Act_i$ for all i.

CGMs with Negotiation: States and Outcome Function

In $\langle w, a, s\rangle \in \check{W} \mathrel{\hat{=}} W \times Act_\Sigma \times \mathcal{S}^\wedge$:

$w \in W$ is a state from the given game;

$a \in Act_\Sigma$ is the latest implementation move (as in M^1);

$s \in \mathcal{S}^\wedge$ stands for the promises accumulated since the last implementation move.

Implementation moves a update w and reset s:

$$\check{o}(\langle w, b, s\rangle, a) \mathrel{\hat{=}} \langle o(w, a), a, \mathrm{p}(o(w, a))\rangle$$

where $\mathrm{p}(v) \mathrel{\hat{=}} \mathrm{C}_\Sigma \bigwedge_{i\in\Sigma} (\bigvee P_i(v) \vee \bigvee \mathcal{S}_i)$ -

Negotiation moves update only s.

Let $p_1, \ldots, p_K$ be the promise actions in a. Then

$$\check{o}(\langle w, b, s\rangle, a) \mathrel{\hat{=}} \langle w, b, (\ldots(s \star p_1) \star \ldots \star p_{K-1}) \star p_K\rangle$$

CGMs with Negotiation

The rest of the components of

$$\check{M} \hat{=} \langle \check{W}, \check{W}_I, \langle \check{Act}_i : i \in \Sigma \rangle, \langle \check{\sim}_i : i \in \Sigma \rangle, \langle \check{P}_i : i \in \Sigma \rangle, \check{o}, \check{V} \rangle$$

are defined as expected:

Let $\mathrm{k}_i(s) \hat{=} \{s' \in \mathcal{S}^{\wedge} : \models s \Rightarrow \mathrm{K}_i s'\}$ - i's forecast on the next move.

Then

$$\check{W}_I \ \hat{=} \ \{\langle w, *, \mathrm{p}(w) \rangle : w \in W_I\}$$

$$\langle w', a', s' \rangle \check{\sim}_i \langle w'', a'', s'' \rangle \ \hat{=} \ \langle w', a' \rangle \ \sim_i^1 \ \langle w'', a'' \rangle \wedge \mathrm{k}_i(s') = \mathrm{k}_i(s'');$$

$$\check{P}_i(\langle w, a, s \rangle) \ \hat{=} \ P_i(w) \cup \mathcal{S}_i;$$

$$\check{V}(\langle w, a, s \rangle, x) \ \hat{=} \ V^1(\langle w, a \rangle, x), \ x \in AP \bigcup_{i \in \Sigma} Act_i.$$

Promises as Announcements on Kripke Models

Let $\langle w, a, s\rangle \models \eta$, if $\langle w, a, s\rangle$ is reached by an implementation move in $\check{M}$. Then the promises from s are kept at $\langle w, a, s\rangle$ in $\check{M}$, if

$$\check{M}, \langle w, a, s\rangle \models \forall(\neg\eta \,\mathsf{U}\, \eta \wedge s)$$

This relates the semantics of making promises to announcements.

Given an epistemic model $K \mathrel{\hat{=}} \langle W, \langle \sim_i : i \in \Sigma\rangle, V\rangle$ and $W' \subseteq W$, let

$$K|_{W'} \mathrel{\hat{=}} \langle W \cap W', \langle \sim_i |_{W' \times W'} : i \in \Sigma\rangle, V|_{W' \times AP}\rangle$$
$$[\![A]\!]_K \mathrel{\hat{=}} \{w \in W : M, w \models A\}$$

Then $K, w \models [A]B$ iff $K|_{[\![A]\!]_K}, w \models B$

Here $\forall(\neg\eta \,\mathsf{U}\, \eta \wedge \ldots)$ plays the role the announcement modality $[s]$. However, the transition relation $R(w, w') \mathrel{\hat{=}} (\exists a \in \prod_{i \in \Sigma} P_i(w))(w' = o(w, a))$ does not restrict players to honor their promises.

Hence promises from s are not guaranteed to be kept at $\langle w, a, s\rangle$ in $\check{M}$.

This can be achieved by appropriately revising players' protocols.

Honest Moves and Honest CGMs

Action $a_i \in Act_i$ is honest at $\langle w, a, s\rangle \in \check{W}$ in $\check{M}$, if $s \wedge a_i$ is consistent.

Promise $p \in \mathcal{S}_i$ is honest at $\langle w, a, s\rangle$, if $s \wedge p$ is honorable and $\not\models s \Rightarrow p$.

The condition $\not\models s \Rightarrow p$ guarantees that p has something 'substantial' in it.

By restricting protocols, a negotiation CGM can be transformed into a CGM where all moves are honest:

Consider $\bar{M} \hat{=} \langle \bar{W}, \bar{W}_I, \langle \bar{Act}_i : i \in \Sigma\rangle, \langle \bar{\sim}_i : i \in \Sigma\rangle, \langle \bar{P}_i : i \in \Sigma\rangle, \bar{o}, \bar{V}\rangle$

where

$$\bar{P}_i(\langle w, b, s\rangle) \hat{=} \{a \in P_i(w) : \not\models s \Rightarrow \mathsf{K}_i \neg a\} \cup \{p \in \mathcal{S}_i : \underbrace{\not\models s \Rightarrow \neg p}_{p \text{ is honorable}}, \underbrace{\not\models s \Rightarrow p}_{p \text{ is substantial}} \}.$$

The other components of $\bar{M}$ are as in $\check{M}$.

$\bar{M}$'s plays are exactly those of $\check{M}$'s plays in which promises are kept at all moves and negotiation is always terminates.

Temporal Projection: Translating Winning Conditions

To use $\bar{M}$, winning conditions $L \subseteq R_M^{\mathrm{inf}}$ must be translated to account of negotiation moves.

In $\bar{M}$, η can be defined by putting $V(\langle w, a, s\rangle, \eta) \leftrightarrow s \neq \mathrm{p}(w)$.

Given a play $\mathbf{w} \triangleq \langle w^0, *, \mathrm{p}(w^0)\rangle\langle w^1, a^1, s^1\rangle \ldots$, let $k_0 < k_1 < \ldots$ be the sequence of the positions k in $\mathbf{w}$ such that $\langle w^k, a^k, s^k\rangle \models \neg\eta$.

Since infinite negotiation is ruled out in $\bar{M}$, $k_0 < k_1 < \ldots$ is infinite.

Furthermore $\mathbf{w}|_{\neg\eta} \triangleq \mathbf{w}^{k_0}\mathbf{w}^{k_1} \ldots$ is a play in M, except for the additional $\bar{M}$ -specific components of the states involved.

$\mathbf{w}|_{\neg\eta}$ is obtained from $\mathbf{w}$ by hiding negotiation moves.

Hence the $\bar{M}$-equivalent $\bar{L}$ of a temporal condition L written for M must satisfy the equivalence:

$$\bar{M}, \mathbf{w} \models \bar{L} \text{ iff } M, \mathbf{w}|_{\neg\eta} \models L$$

where the states $\langle w, a, s\rangle$ from $\mathbf{w}|_{\neg\eta}$ are additionally stripped of their a- and s-components.

Temporal Projection

$$M, \mathbf{w} \models (A \sqcap B) \text{ iff } |\mathbf{w}|_A| = \omega \text{ and } M, \mathbf{w}|_A \models B.$$

where $\mathbf{w}|_A$ consists of $\mathbf{w}$'s states which satisfy A.

$(.\sqcap.)$ is known to be expressible in LTL, propositional ITL, etc.

E.g., if

$$B ::= \bot \mid p \mid B \Rightarrow B \mid \bigcirc B \mid (B \,\mathsf{U}\, B)$$

we have

$$(A \sqcap \bot) \Leftrightarrow \bot \qquad (A \sqcap p) \Leftrightarrow (\neg A \,\mathsf{U}\, A \wedge p)$$

$$(A \sqcap \bigcirc B) \Leftrightarrow (\neg p \,\mathsf{U}\, p \wedge \bigcirc (A \sqcap B))$$

$$(A \sqcap (B_1 \Rightarrow B_2)) \Leftrightarrow \Box\Diamond A \wedge ((A \sqcap B_1) \Rightarrow (A \sqcap B_2))$$

$$(A \sqcap (B_1 \,\mathsf{U}\, B_2)) \Leftrightarrow ((A \sqcap B_1) \,\mathsf{U}\, (A \sqcap B_2))$$

$(.\sqcap.)$ was first introduced in

Halpern, Manna and Moszkowski, ICALP 1983

and rediscovered in

Eisner, Fisman, Havlicek, McIsaac and Van Campenhout, ICALP 2003

Summary

A game on some given CGM M as the arena and $L_1, \dots, L_N$ as the relevant winning conditions, the analysis of this game with the possibility to have temporary coalitions in account can be reduced to the analysis of the game based on $\bar{M}$ and $\neg\eta\sqcap L_1, \dots, \neg\eta\sqcap L_N$.

Assuming temporary coalitions to be formed by the exchange of promises, this is achieved by subsequently extending M to

- enable specifying promises ($M^{\mathbf{1}}$),
- allow inserting negotiation moves ($\check{M}$), and
- rule out breaking promises and infinite negotiation ($\bar{M}$).

If the given M is finite, then so are $M^{\mathbf{1}}$, $\check{M}$ and $\bar{M}$.

Winning conditions written for M can be translated to apply to $\bar{M}$ too.

The End