Invariant theory for reductive subgroups of reductive groups

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Abstract: Let $H \subset G$ be an embedding of connected complex reductive linear algebraic groups. A classical question with several important interpretations is: which irreducible G-modules contain nonzero H-invariant vectors? An approach based on the Geometric Invariant Theory of Hilbert-Mumford was developed in works of Heckman, Berenstein-Sjamaar, Belkale-Kumar and Ressayre, culminating in a description of the generalized Littlewood-Richadson cone - the convex hull of set of the highest weights of the G-modules containing H-invariants. The discrepancy between the convex hull and the actual set of weights presents the so-called saturation problem, famously solved by Knutson and Tao for diagonal embeddings of GL_n, and widely open in general. Ressayre's description of the cone demands extensive calculations even in relatively tame cases, which makes applications difficult. Further development of the structure theory seems desirable.

In this talk, based on joint works with Seppänen and Staneva, I will present some structural properties of generalized Littlewood-Richardson cones, allowing to partition the subgroups of a given G into types according to the properties of the cones. We derive a new numerical invariant of reductive groups, and use it to show that for "generic" subgroups the cone fills the entire Weyl chamber of G. This greatly reduces the difficulty of the saturation problem and allows it to be solved for some new cases, e.g. SL_2-subgroups of classical groups.