

# Отчетна научна сесия на секция „Алгебра и логика“

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## Резюмета на докладите

Веселин Дренски

New examples of fundamental algebras

(Joint project with Luisa Carini, University of Messina, Italy)

Fundamental algebras are the building blocks used to generate any variety of finite basic rank of associative algebras over a field of characteristic 0. Our first result describes the fundamental algebras which are tensor products of any number of finite dimensional Grassmann algebras. Then we show that the triangular product of two fundamental algebras is again fundamental. The proofs are based on the recent description of fundamental algebras in the language of cocharacter sequences due to Giambruno, Polcino Milies and Zaicev combined with other techniques from the theory of PI-algebras.

Васил Кънев

Модулярни пространства на Хурвиц параметризиращи покрития с фиксирана монодромна група

Hurwitz modular varieties parametrizing coverings with fixed monodromy group

Given a projective curve  $Y$ , a transitive subgroup  $G$  of the symmetric group  $S_d$  and a natural number  $n$  the talk is devoted to smooth families of coverings of  $Y$  of degree  $d$  branched in  $n$  points whose monodromy group is  $G$ . These families form a category whose morphisms correspond to the pullback by morphisms of the bases of the families. Under certain restrictions on the group  $G$  we construct a universal family in this category. We discuss how to change the category, so that the universal family exists without any restrictions on  $G$ .

Нека е дадена проективна крива  $Y$ , транзитивна подгрупа  $G$  на симетричната група  $S_d$  и естествено число  $n$ . Темата на доклада са гладките семейства от покрития на  $Y$  от степен  $d$  разклонени в  $n$  точки, чиято монодромна група е  $G$ . Тези семейства образуват категория, в която морфизмите съответстват на прообразите на семействата определени от морфизмите на базите на семействата. При определени ограничения върху групата  $G$  се конструира универсално семейство. Предлага се една модификация на категорията, така че универсално семейство да съществува без ограничения върху групата  $G$ .

Иван Чипчаков

Поleta с размерност единица, нормални над локално или глобално поле

Fields of dimension one, normal over a global or local field

Let  $E$  be a normal extension of a global or local field  $K$ . We show that if  $K$  is a local field,  $v$  is its (natural) discrete valuation, and  $q$  is the characteristic of the residue field of  $(K, v)$ , then  $E$  is a field of dimension  $\dim(E) \leq 1$  if and only if the following conditions hold: for each prime number  $p \neq q$ ,  $E$  contains as a subfield an unramified  $\mathbb{Z}_p$ -extension  $K_p$  of  $K$ ; the restriction  $p \neq q$  is dropped in case the value group  $v'(E)$  is  $q$ -in divisible, where  $v'$  is the unique, up-to equivalence, valuation of  $E$  extending  $v$ . When  $K$  is a global field and  $E/K$  is abelian and tamely ramified, nontrivial Krull valuations of  $E$  are discrete; also,  $\dim(E) \leq 1$  if and only if  $E$  is a nonreal field and the residue fields of these valuations are algebraically closed. Under the hypothesis that  $K$  is a global field,  $E/K$  is abelian and  $\dim(E) \leq 1$ , this is used for proving the existence, for each  $n \in \mathbb{N}$ , of  $n$ -variate homogeneous polynomials of degree  $n$  with coefficients in  $E$ , which violate the local-to global principle over  $E$ .

Нека  $E$  е нормално разширение на локално или глобално поле  $K$ . Първият резултат на доклада показва, че ако  $K$  е локално поле,  $v$  е (естественото) дискретно нормиране на  $K$  и  $q$  е характеристиката на полето от остатъци на  $(K, v)$ , то  $E$  е поле с размерност  $\dim(E) \leq 1$ , тогава и само тогава, когато са изпълнени следните условия: за всяко просто число  $p \neq q$ ,  $E$  притежава подполе  $K_p$ , което е неразклонено  $\mathbb{Z}_p$ -разширение на  $K$ ; ограничението  $p \neq q$  отпада в случай, че групата от стойности  $v'(E)$  е  $q$ -неделима, където  $v'$  е единственото, с точност до еквивалентност, нормиране на  $E$ , продължаващо  $v$ . Когато  $K$  е глобално поле, а  $E/K$  е абелево и слабо разклонено, нетривиалните нормирания на Крул на  $E$  са дискретни; освен това,  $\dim(E) \leq 1$ , точно тогава, когато  $E$  не може да бъде наредено и полетата от остатъци на споменатите нормирания са алгебрически затворени. При предположение, че  $K$  е глобално поле,  $E/K$  е абелево и  $\dim(E) \leq 1$ , това се използва за доказателство на съществуването, за всяко  $n \in \mathbb{N}$ , на хомогенни полиноми от степен  $n$  и  $n$  променливи с коефициенти в  $E$ , които нарушават локално-глобалния принцип над  $E$ .

Татяна Гатева-Иванова

Associative algebras and Lie algebras defined by Lyndon words

Assume that  $X = \{x_1, \dots, x_g\}$  is a finite alphabet and  $\mathbf{k}$  is a field. We study the class  $\mathcal{C}(X;W)$  of associative graded  $\mathbf{k}$ -algebras  $A$  generated by  $X$  and with a fixed obstructions set  $W$  consisting of Lyndon words in the alphabet  $X$ . Important examples are the monomial algebras  $A = \mathbf{k}\langle X \rangle / (W)$ , where  $W$  is an antichain of Lyndon words of arbitrary cardinality and the enveloping algebra  $U\mathfrak{g}$  of any  $X$ -generated Lie  $\mathbf{k}$ -algebra  $\mathfrak{g} = \text{Lie}(X) = ([W])$ , whenever the set of standard bracketings  $[W] = \{[w] \mid w \in W\}$  is a Gröbner-Shirshov Lie basis. We prove that all algebras  $A$  in  $\mathcal{C}(X;W)$  share the same Poincare-Birkhoff-Witt type  $\mathbf{k}$ -basis built out of the so called *Lyndon atoms*  $N$  (determined uniquely by  $W$ ) but, in general,  $N$  may be infinite. Moreover,  $A$  has polynomial growth if and only if the set of Lyndon atoms  $N$  is finite. In this case  $A$  has a  $\mathbf{k}$ -basis  $\Pi = \{I_1^{\alpha_1} I_2^{\alpha_2} \dots I_d^{\alpha_d} \mid \alpha_i \geq 0, 1 \leq i \leq d\}$ , where  $N = \{I_1, \dots, I_d\}$ . Surprisingly, in the case when  $A$  has polynomial growth its global dimension does not depend on the shape of its defining relations but only on the set of obstructions  $W$ : We prove that if  $A$  has polynomial growth of degree  $d$  then  $A$  has global dimension  $d$  and is standard finitely presented, with  $d-1 \leq |W| \leq d(d-1)/2$ . We study when the set of standard  $[W] = \{[w] \mid w \in W\}$  is a Gröbner-Shirshov Lie basis. We use our general results to classify the

Artin-Schelter regular algebras  $A$  generated by two elements, with defining relations  $[W]$  and global dimension  $\leq 7$ .

### References

- [1] Tatiana Gateva-Ivanova, *Algebras defined by Lyndon words and Artin-Schelter regularity*, To appear in The Transactions AMS arXiv preprint arXiv:1905.11281 (2019).
- [2] Tatiana Gateva-Ivanova, Gunnar Floystad, *Monomial algebras defined by Lyndon words*, Journal of Algebra **403** (2014), 470{496.
- [3] Tatiana Gateva-Ivanova, *Quadratic algebras, Yang-Baxter equation, and Artin-Schelter regularity*, Advances in Mathematics **230** (2012), 2152{2175.
- [4] Tatiana Gateva-Ivanova, *Global dimension of associative algebras*, Applied Algebra, Algebraic Algorithms and Error-Correcting Codes, Lecture Notes in Computer Science, **357** (1989), 213-229.

Йорг Копиц

The generators of the semigroup of all transformations preserving a crown

In this presentation, we will give a survey about the status of the study of monoids of transformations preserving a fence and a crown, respectively. In particular, we will consider the monoid of automorphisms preserving fence and crown, respectively. Finally we will give an idea of the current status of study of the rank of the monoid of all partial transformations preserving a finite crown.

Димитринка Владева

Диференцирания в полупръстени от асиметрични полиноми на Оре

Derivations of skew Ore polynomial semirings

В това представяне са изследвани диференцирания в полупръстен от асиметрични полиноми на Оре над адитивно идемпотентен полупръстен  $S$ . Показано е, че като се умножи произволен полином отляво с  $x$  се дефинира диференциране. Построен е комутативен идемпотентен полупръстен състоящ се от диференцирания на полупръстена от полиномите на Оре. Разглеждани са наследствени диференцирания и обобщени наследствени диференцирания, които променят само коефициентите на полинома. Построено е  $S$ -диференциране в класическия смисъл на Джекобсън. В края е дадено описание на диференциранията на полупръстена  $S[x]$  от асиметричните полиноми на Оре при предположение, че  $S$  е адитивно идемпотентен полупръстен, като е показано, че произволно диференциране може да се изрази чрез обобщено наследствено диференциране и  $S$ -диференциране.

In this project we investigate derivations in the semiring of skew Ore polynomials over an additively idempotent semiring. We show that multiplying each polynomial by  $x$  on left is a derivation and construct commutative idempotent semiring consisting of derivations of a skew polynomial semiring. We introduce hereditary derivations and generalized hereditary derivations defined as derivations

acting only over the coefficients of the polynomial and also construct an  $S$ -derivation in the classical sense of Jacobson. Finally we give a description of the derivations in a skew polynomial semiring  $S[x]$ , assuming that  $S$  is an additively idempotent semiring and show that an arbitrary derivation can be represented by a generalized hereditary derivation and an  $S$ -derivation.

Владимир С. Герджиков

Рекурсионни оператори и йерархии от мКдВ уравнения свързани с алгебрите на Кац-Мууди  $D_4^{(1)}$ ,  $D_4^{(2)}$  и  $D_4^{(3)}$

Recursion operators and the hierarchies of MKdV equations related to  $D_4^{(1)}$ ,  $D_4^{(2)}$  and  $D_4^{(3)}$  Kac-Moody algebras

Reference:

V. S. Gerdjikov, A.A. Stefanov, I. D. Iliev, G. P. Boyadjiev et al. Recursion operators and the hierarchies of MKdV equations related to  $D_4^{(1)}$ ,  $D_4^{(2)}$  and  $D_4^{(3)}$  Kac-Moody algebras. Theoretical and Mathematical Physics, **204** (3): 1110–1129 (2020), ArXiv:2006.16323 [nlin.SI]

Елица Христова

Регулярност на алгебри от  $O(n)$ -инварианти чрез използване на редове на Хилберт

Regularity of algebras of  $O(n)$ -invariants using Hilbert series

Let  $W$  be a polynomial representation of the complex general linear group  $GL(n)$ . In this talk, we discuss the question when the algebra of invariants  $\mathbb{C}[W]^{O(n)}$  is regular, i.e. isomorphic to a polynomial algebra. For  $n=2$ , we give a list of polynomial  $GL(2)$ -representations, so that if  $\mathbb{C}[W]^{O(2)}$  is regular, then up to an  $O(2)$ -isomorphism  $W$  is in this list. For general  $n$ , we prove regularity in particular cases. The talk is based on a joint work with Vesselin Drensky.

Веселин Филев

Холографски матричен модел на Брекуз-Дъглас при крайна температура

Holographic Berkooz-Douglas Matrix Model at Finite Temperature

I will report on ongoing work to construct the holographic dual supergravity background of the Berkooz-Douglas matrix model at finite temperature.

Ще докладвам текуща работа по конструирането на супергравитационно решение холографски дуално на матричния модел на Беркуз-Дъглас при крайна температура.

Вилислав Бучакчиев

Използване на макроикономически индикатори при прогнозирането на нивата на индекса на цените на жилищата

Forecasting of the Bulgarian House price index using some macroeconomic indicators

With the introduction of IFRS9 accounting standard in 2018 many banks were required to use statistical models for forecasting the liquidation values of houses used as collateral for mortgages. The nature of estimation of the expected credit loss requires the evaluation of levels of House Price Index from available statistical data which is, usually, one year old. Several specifications of the models were studied to confirm that HPI is correlated with various indicators, including RE market demand, construction industry business cycle and general macroeconomic environment. The general conclusion was, however, that the two most prominent drivers of HPI remain the interest rates and the internal inertia of the RE market.

Димитър Добрев

Език за описание на светове

Language for Description of Worlds

Ще сведем задачата за създаването на ИИ към задачата за намирането на подходящия език за описание на света. Този език няма да е език за програмиране, защото езиците за програмиране описват само изчислими функции, докато този език ще опише малко по-широк клас от функции. Друга особеност на този език ще е, че при него описанието ще може да бъде разбито на отделни модули. Това ще ни позволи да търсим описанието на света автоматично, като го откриваме модул по модул. Подходът ни за създаването на този нов език ще бъде да започнем от един конкретен свят и да напишем описанието на тази конкретен свят. Идеята ни е, че езикът, който може да опише този конкретен свят, ще е подходящ за описанието на произволен свят.

We will reduce the task of creating AI to the task of finding an appropriate language for description of the world. This will not be a programming language because programming languages describe only computable functions, while our language will describe a somewhat broader class of functions. Another specificity of this language will be that the description will consist of separate modules. This will enable us look for the description of the world automatically such that we discover it module after module. Our approach to the creation of this new language will be to start with a particular world and write the description of that particular world. The point is that the language which can describe this particular world will be appropriate for describing any world.

Валентин Илиев

Принципът за неопределеност при скрининг тестовете

The Uncertainty Principle for Screening Tests

The aim of this elementary note is to describe the relation between the conditional probabilities of a false positive and a false negative screening test. Non-formally, we can state the main result of the paper as an Uncertainty Principle: In general, if one has better knowledge that the test is really positive (the probability  $F_{+}$  of false positive test is small), then for one is hard to know that the test is really negative (the probability  $F_{-}$  of false negative test is large). And the better one knows that the test is really negative ( $F_{-}$  is small), the harder it is to know that the test is really positive ( $F_{+}$  is large).

Русанка Луканова

Type-Theory of Parametric Algorithms

I shall present a class of Moschovakis type-theories of recursion. My focus is on an overview from the perspective of existing and potential applications. I shall point to some of my contributions on these topics.

Валентин Горанко

Рационални принципи на координация в чисто координационни игри

Rational coordination in Pure Win-Lose Coordination Games

The main question I will address is: when and how can rational agents coordinate without any prior communication or conventions? I will consider this question in the abstract framework of multi-player pure coordination games, where each player has a number of possible choices, every choice profile determines a unique outcome, and in every outcome all players have identical payoffs, 'win' or 'lose'.

I will formally introduce pure win/lose coordination games and will present and discuss a hierarchy of 'rationality principles' that can be applied by rational players in such games to determine their choices of action. Then I will compare the strength of some of these principles in terms of the classes of coordination games that can be solved by them by using only pure reasoning, without any preplay communication and conventions. I will argue that the boundaries between pure rationality principles and other rational decision methods used for solving coordination games are quite debatable and there is apparently no clear distinction between these.

Lastly, time permitting I will discuss briefly how pure coordination games can be solved with the use of 'structural' conventions (only based on structural properties of the games), agreed in a preplay communication, and will describe precisely the scope of purely rational coordination.

The talk is based on this recent joint paper with Antti Kuusisto and Raine Rönholm:

<https://academic.oup.com/logcom/article/30/6/1183/5869758?guestAccessKey=374b9c38-2900-4302-91c6-8c8c1eac6ac4>.

Димитър Гелев

Набори от стратегии и означения за решаването на безкрайни синхронни игри с временно коалиране в темпоралната логика QCTL\*

Strategy Profiles and a Vocabulary for Solving Infinite Concurrent Games with Temporary Coalitions in QCTL\*

This is a short version of my seminar talk from October 30. It highlights the key notion and notations proposed in that talk. The key notion is a straightforward extension of the notion of strategy profile for registering varying partitionings of the totality of the players into disjoint coalitions. The notation is a vocabulary for the propositionally quantified branching time temporal logic QCTL\* which augments the encoding of strategy profiles into this logic as known from the literature, including my previous work, with symbols for specifying shifting coalition structure. In this short presentation, we focus on the notation and sideline the key technical results of the work, which show that complete information concurrent multiplayer infinite games with LTL-definable partially ordered objectives are solvable wrt whatever solution concepts happen to be expressible in the proposed vocabulary. That includes temporary coalition generalisations of some established solution concepts. The work is available from the 8th International Workshop on Strategic Reasoning and arXiv.