

# СЕМИНАР

## „АЛГЕБРА И ЛОГИКА”

Драги колеги,

Следващото заседание на семинара ще се проведе на 13 септември 2019 г. (петък) от 13:00 часа в зала 578 на ИМИ – БАН.

Доклад на тема

### **On commutator Krylov transitive and commutator weakly transitive Abelian $p$ -groups**

ще изнесе Петър Данчев.

Поканват се всички желаещи.

От секция „Алгебра и логика” на ИМИ – БАН

<http://www.math.bas.bg/algebra/seminarAiL/>

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## Abstract

The reported paper deals with a question, posed a few years ago by Laszlo Fuchs and other international experts in the contemporary theory of Abelian groups, having a special impact on the modern aspects of non-commutative ring theory. Specifically, the problem asks for the structural description of those Abelian  $p$ -groups whose endomorphisms are (additively) generated by commutators. In other words, all endomorphism are representable as a finite sum of products of commutators. We will be trying to give in the current study some satisfactory affirmative answer in this subject. Our solution will depend heavily on the structure of the first Ulm subgroup of the whole group, determined by the action of the full endomorphism ring on this subgroup.

In our terminology, three new classes of Abelian  $p$ -groups are defined, namely the classes of **commutator transitive**, **commutator Krylov transitive** and **commutator weakly transitive groups**. These are common extensions of the well-known concept of commutator fully transitive groups as well as of the classical notions of transitivity and full transitivity, the latter ones being defined by Irving Kaplansky in his famous red-book published by the Michigan University Press.

The most important results in this subject, showing some pathology of the commutator endomorphisms and so the difficulty in their study, are the following ones:

- (1) *A separable  $p$ -group is commutator Krylov transitive if, and only if, it is commutator fully transitive.*
- (2) *If  $G$  is a 2-group of length  $\leq \omega \cdot n$  for some natural  $n \geq 1$ , then  $G$  is commutator Krylov transitive if, and only if,  $G$  is commutator fully transitive.*
- (3) *Commutator (Krylov, weakly) transitive  $p$ -groups are not closed under taking direct summands and with respect to the formation of (finite or infinite) direct sums.*

The methods we have developed in order to establish these results are certain innovations in the representation of commutator endomorphisms in terms of matrices, by strengthening the methodology utilized in some classical papers in the present topic.

***This is a joint paper with A.R. Chekhlov from Tomsk State University (Russia) and it will be published in the journal FORUM MATHEMATICUM (Walter de Gruyter) on year 2020.***