СЕМИНАР

„АЛГЕБРА И ЛОГИКА”

Драги колеги,
Следващото заседание на семинара ще се проведе
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Доклад на тема

Enumerating geometric configurations of conics and lines

ще изнесе Gheorghe Pupazan
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Поканват се всички желаещи.

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Enumerating geometric configurations of conics and lines

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An enumerative geometry problem is one that asks to find how many algebra-geometric objects of a specified type satisfy certain given conditions. One of the earliest such problems is the famous Apollonius' problem: *How many circles are tangent to three given plane circles?* Apollonius found that generically there are 8 such circles, but enumerative geometry did not develop its tools until the middle of the 19th century. In 1848 Steiner posed the following generalization of the Apollonius' problem: *How many conics are generically tangent to 5 given conics in the complex projective plane?* Steiner's original answer was $6^5 = 7776$, but first de Jonquieres in 1859 and then Chasles in 1864 found that the correct number is 3264. This achievement marked the birth of "modern" enumerative geometry. Through the works of Grassmann, Pluecker and Schubert, the Schubert calculus was developed during the second half of the 19th century. Schubert introduced locally closed sets in a Grassmannian, nowadays called Schubert cells, defined by conditions of incidence of a linear subspace in projective space with a given flag. The intersection theory of these cells, which can be considered as the product structure in the Chow ring of the Grassmannian of associated Chern classes, allows one to predict when the intersections of these cells gives a finite set of points. More precisely, Schubert considered a family of algebraic varieties that are identified with points of an irreducible algebraic variety $X \subset \boldsymbol{P}^N$. The conditions imposed on these varieties amount to conditions imposed on the points identified with them. This defines some algebraic subvarieties $Y_j \subset X, \ \forall\, 1 \le j \le t$. If the product cycle $[Y_1] \cdot [Y_2] \cdot\cdot\cdot\ [Y_t]$ turns of dimension 0, then its degree gives the number of varieties satisfying all the required conditions simultaneously. To find this number, Schubert calculated, the relations between the classes $[Y_j]$ in the Chow ring of X. This allowed him to find in 1877 that: *"A general quintic threefold in P4 contains exactly* $5^3 \cdot 23 = 2875$ *lines"*. Schubert's result can be considered a generalization of the Cayley-Salmon theorem (1869), which states that: *"A smooth cubic surface in* $\boldsymbol{P}^3$ *contains 27 straight lines"*.

In this expository talk we will present an explicit geometric calculation of the Chow ring of the Grassmannian of lines in $\boldsymbol{P}^3$ and in $\boldsymbol{P}^4$, which allows to deduce the Cayley-Salmon and Schubert's theorems. The technique also allows the calculation of the number of lines on a general hypersurface of degree $2n - 3$ in $\boldsymbol{P}^n$.

The work is part of the author's BSc thesis defended at the American University in Bulgaria on May 4, 2015.