СЕМИНАР

„АЛГЕБРА И ЛОГИКА”

Драги колеги,
Следващото заседание на семинара ще се проведе
на 22 февруари 2019 г. (петък) от 13:00 часа
в зала 578 на ИМИ – БАН.
Доклад на тема

Nilpotent and unipotent matrices over a field with applications to von Neumann regular rings and Functional Analysis

ще изнесе Петър Данчев.

Поканват се всички желаещи.

От секция „Алгебра и логика” на ИМИ – БАН

http://www.math.bas.bg/algebra/seminarAiL/

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Abstract

Introduction: The question of whether or not the nilpotent square matrix is a linear combination of idempotent matrices is too long-standing and left-open yet. So, further results of this type would be of some interest and eventual importance. The goal of the present note is to promote such a kind of results with a possible application to the classical von Neumann regular rings and to the functional analysis as well.

Results: It is proved that any square nilpotent matrix over a field is the difference of two idempotent matrices. The proof involves a non-standard technique by putting the nilpotent matrices in Jordan normal form, which is possible because all their eigenvalues lie in the field, and by using some special tricks with diagonal idempotent matrices. Moreover, a classical result by Beidar-Meara-Raphael from Comm. Algebra (2004) is, furthermore, in use to deduce that the general considerations can be restricted to the aforementioned special nilpotent matrix's forms.

Applications: It is shown that in a von Neumann regular ring every unipotent is the sum of two idempotents. Some examples are also given illustrating that the assertion fails beyond this class of rings. Likewise, pertaining to the functional analysis and in regard to a nice result of Rudin concerning homomorphisms of group algebras, and specifically that the group algebra of a locally compact abelian group G is isomorphic to that of the circle group T if and only if G = T + F, where F is a finite abelian group, which result is published in *C. R. Acad. Sci. Paris* **274**(1958), 773–775, it is shown that this result can be strengthened to a larger class than that of locally compact abelian groups. Actually, this question of homomorphism of group algebras is linked with an apparently different question, namely with the characterization of idempotent measures. Here again Helson and Rudin paved the way, and the final result was obtained by Paul Cohen, proving a conjecture of Rudin (Cambridge meeting, 1958): The supports of the Fourier transforms of idempotent measures (these Fourier transforms take values 0 and 1) are the members of the “coset ring” of the group G^∗, defined as generated by all cosets of subgroups of G^∗ by means of complementation and finite intersection (see also Walter Rudin, *Nonanalytic functions of absolutely convergent Fourier series*, Proc. Nat. Acad. Sci. USA **41** (1955), 238–240; *A strong converse of the Wiener-Levy theorem*, Canad. J. Math. **14** (1962), 694–701).

Publication: The results obtained by the reported author will be subsequently published in an **ISI** journal, specialized in linear algebra and/or functional analysis.