СЕМИНАР

„АЛГЕБРА И ЛОГИКА”

Драги колеги,
Следващото заседание на семинара ще се проведе
на 26 януари 2018 г. (петък) от 13:00 часа
в зала 578 на ИМИ – БАН.
Доклад на тема

n-Периодични чисти пръстени

ще изнесе Петър Данчев.

Поканват се всички желаещи.

От секция „Алгебра и логика” на ИМИ – БАН

http://www.math.bas.bg/algebra/seminarAiL/

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Abstract

Recall that an arbitrary ring is called *clean* by Nicholson (TAMS, 1977) if each its element is the sum of a unit and an idempotent. In addition, if these two elements commute, the clean ring is said to be *strongly clean*.

In the present talk we define and characterize the following two proper subclasses:

Definition. Let n be an arbitrary natural. We shall say that a ring R is *n-torsion clean* if, for every $r \in R$, there exist a unit u with $u^n = 1$ and an idempotent e such that $r = u + e$ and n is the smallest possible natural number with the above property. If, in addition, $ue = eu$, R is called *strongly n-torsion clean*.

The main results are as follows:

Theorem 1. If R is a strongly n-torsion clean ring, then the following hold:

(1) R is a PI-ring;

(2) R has finite characteristic;

(3) $J(R)$ is a nil-ideal;

(4) R is either abelian or char(F) divides n, provided R is an algebra over a field F.

Theorem 2. For a ring R, the following conditions are equivalent:

(1) There exists $n \in \mathbb{N}$ such that R is an n-torsion clean abelian ring.

(2) char(R) is finite, $J(R)$ is nil of bounded index, idempotents lift uniquely modulo $J(R)$ and $R/J(R)$ is a subdirect product of finite fields F_i, where i ranges over some index set I, such that $LCM(|F_i| - 1| \, i \in I)$ exists.

(3) R is abelian clean such that $U(R)$ is of finite exponent.

In parallel to the last assertion, the following is true:

Theorem 3. For a ring R, the following statements are equivalent:

(1) R is strongly n-torsion clean for some $n \in \mathbb{N}$;

(2) R is strongly clean and $U(R)$ is of finite exponent.

In case that n is odd, we have the following satisfactory structural description:

Theorem 4. Suppose $n \in \mathbb{N}$ is odd. For a ring R, the following points are equivalent:

(1) R is strongly n-torsion clean.

(2) There exist integers $k_1,\dots,k_t \geq 1$ such that $n = LCM(2^{k_1} - 1,\dots, 2^{k_t} - 1)$ and R is a subdirect product of copies of fields $F_{2^{k_i}}$, $1 \leq i \leq t$.

(3) R is clean in which orders of all units are odd, bounded by n, and there is a unit of order n.

This gives a new recent advantage on the general classification problem for clean rings.

The paper is a joint project with J. Matczuk from Math. Inst. of the Polish Acad. Sci. (Univ. of Warsaw) and will be published by the AMS in a subsequent issue of the Contemp. Math. Series (2017). These results are reported of the conference "Noncommutative Rings and their Applications - V", held in Lens, France, on 12-15 June, 2017.