Type-Theory of Parametric Algorithms

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A glimpse at developments and applications, in my focus:

- Moschovakis [4], 1989: new theory of the mathematical notion of algorithm: untyped, full recursion
  - computations, by saving the algorithmic steps in memory locations
- Moschovakis [5], 2006, Type-Theory of Acyclic Recursion, $\mathrm{L}^{\lambda}_{\mathrm{ar}}$

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- extended type systems
  variously, in Syn, Sem, SynSem; since 2007 in $\mathrm{L}^{\lambda}_{\mathrm{ar}}$ & before in SitT
- extended reduction calculi in $\mathrm{L}^{\lambda}_{\mathrm{ar}}$, Loukanova, since 2011
- parametric / underspecified algorithms
- restrictor operator Loukanova [3], new 2020 & differently before
  - constrained computations
  - restricted memory locations as generalised, restricted parameters
  - restricted objects
- Dependent-Type Theory of
  Full Recursion & Situated Information (DTTSitInfo)

This overview is about:

- Type-Theory of (Acyclic) Algorithms, $\mathrm{L}^{\lambda}_{\mathrm{r}}$ ($\mathrm{L}^{\lambda}_{\mathrm{ar}}$): a new approach to
  - the math notion of algorithm
  - Computational Semantics — CompSem (AL / FL /NL / HL)

Placement of $\mathrm{L}^{\lambda}_{\mathrm{ar}}$ in a class of type theories

$$\text{Montague IL} \subsetneq \text{Gallin } \mathrm{TY}_2 \subsetneq \text{Moschovakis } \mathrm{L}^{\lambda}_{\mathrm{ar}} \subsetneq \text{Moschovakis } \mathrm{L}^{\lambda}_{\mathrm{r}} \tag{1}$$

$$\stackrel{?}{\equiv} \text{DTTSitInfo} \tag{2}$$

- Montague IL for PTQ (1970-73)
- Gallin TY_2 (1975)
- Acyclic (closing-off) Recursion, $\mathrm{L}^{\lambda}_{\mathrm{ar}}$
  Moschovakis [5] (2006), Loukanova (since 2007)
- $\mathrm{L}^{\lambda}_{\mathrm{ar}}$ is a higher-order type theory with terms for acyclic recursion:
  - the algorithmic meanings of the meaningful $\mathrm{L}^{\lambda}_{\mathrm{ar}}$ terms are acyclic algorithms that close-off
- $\mathrm{L}^{\lambda}_{\mathrm{r}}$ is a higher-order type theory with full recursion
- Dependent-Type Theory of
  Full Recursion & Situated Information (DTTSitInfo)

Some Classes of Type Theories (TTs)

$$\text{IL} \subsetneq \text{TY}_2 \subsetneq \cdots \subsetneq \mathrm{L}^{\lambda}_{ar} \subsetneq \mathrm{L}^{\lambda}_{ra} \subsetneq \mathrm{L}^{ta}_{ar} \subsetneq \mathrm{L}^{\lambda}_{r} \subsetneq \mathrm{L}^{ta}_{r} \tag{3}$$

$$\mathrm{L}^{\lambda}_{ar} \subsetneq \mathrm{L}^{\lambda}_{ra} \subsetneq \mathrm{L}^{ta}_{ar} \subsetneq \mathrm{L}^{ta}_{r} \subsetneq \text{DTTSitInfo} \overset{?}{\subsetneq} \mathrm{L}^{st}_{gp} \tag{4}$$

- Montague IL for PTQ (1970-73) | Gallin TY_2 (1975)
- $\mathrm{L}^{\lambda}_{ar}$, $\mathrm{L}^{\lambda}_{ra}$, L^{λ}_{r}, and L^{ta}_{ar}, L^{ta}_{r} are TTs of Algorithms (2006–ongoing)
    1. Recursion Operator: Recursion Terms, Moschovakis, 1989

Reduction Calculi in the $\mathrm{L}^{\lambda}_{ar}$ Classes via Reduction Rules

- Reducing every term A to a canonical form cf(A)
- cf(A) of an algorithmically meaningful A determines the algorithm alg(A) for computing den(A), in a step-by-step mode

2. Restrictor Operator: Restrictor Terms, Loukanova (2020; earlier vars)
3. L^{ta}_{ar}, L^{ta}_{r} — polymorphic types
4. Reduction Calculi, for each extended $\mathrm{L}^{\lambda}_{ar}$
   η-reduction, η^*-reduction (in prog.), γ-reduction, γ^*-reduction

Gallin Types: $\sigma :\equiv \mathsf{e} \mid \mathsf{t} \mid \mathsf{s} \mid (\tau_1 \to \tau_2)$ (Gallin, 1975)

For all $\tau \in$ Types:

Constants: $\mathsf{Consts}_\tau = \{\mathsf{c}_0^\tau, \mathsf{c}_1^\tau, \ldots, \mathsf{c}_{k_\tau}^\tau\}$

Variables: $\mathsf{PureV}_\tau = \{v_0^\tau, v_1^\tau, \ldots\}$

$\mathsf{MemoryV}_\tau = \mathsf{RecV}_\tau = \{p_0^\tau, p_1^\tau, \ldots\}$

Terms of $\mathrm{L}_{\mathrm{ar}}^\lambda$ ($\mathrm{L}_{\mathrm{r}}^\lambda$):

$$\mathsf{A} :\equiv \mathsf{c}^\tau : \tau \mid x^\tau : \tau \quad (\text{for } \mathsf{c}^\tau \in \mathsf{Consts}_\tau,\ x^\tau \in \mathsf{PureV}_\tau \cup \mathsf{RecV}_\tau) \tag{5a}$$

$$\mid \mathsf{B}^{(\sigma\to\tau)}(\mathsf{C}^\sigma) : \tau \tag{5b}$$

$$\mid \lambda(v^\sigma)(\mathsf{B}^\tau) : (\sigma \to \tau) \quad (\text{for } v^\sigma \in \mathsf{PureV}_\sigma) \tag{5c}$$

$$\mid \left[\mathsf{A}_0^{\sigma_0} \text{ where } \{ p_1^{\sigma_1} := \mathsf{A}_1^{\sigma_1}, \ldots, p_i^{\sigma_i} := \mathsf{A}_i^{\sigma_i}, \ldots, p_n^{\sigma_n} := \mathsf{A}_n^{\sigma_n} \}\right] : \sigma_0 \tag{5d}$$

$$\mid \left[\mathsf{A}_0^{\sigma_0} \text{ such that } \{ \mathsf{C}_1^{\tau_1}, \ldots, \mathsf{C}_m^{\tau_m} \}\right] : \sigma_0' \tag{5e}$$

- $\mathsf{B}, \mathsf{C} \in$ Terms, $p_i^{\sigma_i} \in \mathsf{RecV}_{\sigma_i}$, $A_i^{\sigma_i} \in \mathsf{Terms}_{\sigma_i}$
  $\mathsf{C}_j^{\tau_j} \in \mathsf{Terms}_{\tau_j}$ (for propositions): $\tau_j \equiv \mathsf{t}$ or $\tau_j \equiv \widetilde{\mathsf{t}} \equiv (\mathsf{s} \to \mathsf{t})$
- Acyclicity Constraint, for $\mathrm{L}_{\mathrm{ar}}^\lambda$; without it, $\mathrm{L}_{\mathrm{r}}^\lambda$
  $\{ p_1^{\sigma_1} := \mathsf{A}_1^{\sigma_1}, \ldots, p_i^{\sigma_i} := \mathsf{A}_i^{\sigma_i}, \ldots, p_n^{\sigma_n} := \mathsf{A}_n^{\sigma_n} \}$ is acyclic iff:
  - there is a rank: $\{p_1, \ldots, p_n\} \to \mathbb{N}$ such that:
    if $p_j \in \mathsf{FreeVars}(A_i)$ then $\mathsf{rank}(p_i) > \mathsf{rank}(p_j)$

- Generalised Computational Grammar:
  via faithfull representations of CompSynSem interfaces in HL
  - Hierarchical lexicon with morphological structure and lexical rules
  - Lexicon that propagates into the phrasal structure of sentences
  - Syntax of HL expressions (phrasal and grammatical dependences)
  - Syntax-semantics inter-relations in lexicon and phrases
  - Abstract and Parametric Grs:
    across syn categories and semantic types
    vs.
    Specific Instantiations of lexemes and phrases:
    in and across languages
- A Big Picture — simplified and approximated, but realistic:

Algorithmic CompSynSem of Human Language (HL)

$$\underbrace{\text{HL Syn} \Longleftrightarrow \mathrm{L}^{\lambda}_{\mathrm{ar}} \,/\, \mathrm{L}^{\lambda}_{\mathrm{r}} \,/\, SitI \Longleftrightarrow \underbrace{\text{Algorithmic Semantics}}_{\text{Canonical Computations}} \Longrightarrow \text{Denotations}}_{\text{Algorithmic CompSynSem Interfaces}}$$

(I've done quite a lot of it, but still a lot to do!)

Some Current Tasks (among many others) and Future Work

- My focus is on:
  - Development of $\mathrm{L}^{\lambda}_{\mathrm{ar}}$ and $\mathrm{L}^{\lambda}_{\mathrm{r}}$
  - Dependent-Type Theory of Situated Information and Algorithms
  - Applications to formal and natural languages
    - Extending the Coverage of Computational Semantics
    - Computational Syntax-Semantics Interfaces
    - Semantics of programming and specification languages
    - Theoretical foundations of (parts of) compilers
- More to come

Thank You!

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