(Strategy Profiles and a Vocabulary for Solving Infinite Concurrent Games with Temporary Coalitions in QCTL^*)

Dimitar P. Guelev

IMI/BAS

http://www.math.bas.bg/~gelevdp

Research supported by Bulgarian NSF Grant DN02/15/19.12.2016

The Geography of Applied Systems of Logic

Many applied systems of logic are embeddable into Second Order Predicate Logic,

and are extensions of (Classical) Propositional Logic

Second Order Predicate Logic is computationally hard, unsolvable on the whole.

Wrt complexity, the images of applied logics vary.

For efficiency, applied logics have application-specific constructs in their languages.

Systems of applied logic are embeddable in each other too.

In case the target logic is tractable, the embedded logic is automatically supplied some tractability, depending on the possibility of blowups upon the embedding.

This talk is on an example of that.

Plan of Talk

Our application is strategic reasoning of finitely many players.

We discuss an extension of QCTL^* (a target logic for embedding ATLs) for reasoning about temporary coalitions as opposed to ATLs' inherent permanent ones.

Our scope is complete information concurrent games with LTL-definable objectives.

We include preference between finitely many objectives, for each player.

To this end, we extend strategy profiles to account of temporary coalitions and augment the established embedding technique of ATL into QCTL^* by adding

(1) A propositional vocabulary for naming both decisions and evolving coalition structure.

(2) A temporal form of an established binary preference operator.

We give the part of axiomatisations for (1) and (2) that is specific to time.

We illustrate the vocabulary upon presenting some derived constructs it enables.

Preliminaries: Concurrent Game Models (CGM)

$Ag = \{1, \ldots, N\}$ - players, $AP \neq \emptyset$ - atomic propositions

$\langle W, w^0, \langle Act_i : i \in Ag \rangle, o, V \rangle$ - CGM for Ag and AP

$W \neq \emptyset$ - the statespace; w^0 - the initial state;

$Act_i \neq \emptyset$ - the actions of player $i \in Ag$; $Act_\Gamma \hat{=} \prod\limits_{i \in \Gamma} Act_i$;

$o : W \times Act_{Ag} \to W$ - outcome function.

$V \subseteq W \times AP$ - valuation.

$\mathbf{w} \hat{=} w^0 w^1 \ldots \in W^+ \cup W^\omega$

The set of the infinite continuations of $\mathbf{w} \in W^+$:

$R_M^{\mathrm{inf}}(\mathbf{w}) \hat{=} \{\mathbf{v} \in W^\omega : \mathbf{v}^0 \ldots \mathbf{v}^{|\mathbf{w}|-1} = \mathbf{w}, (\forall k < \omega)(\exists a \in Act_{Ag})(\mathbf{v}^{k+1} = o(\mathbf{v}^k, a))\}$

Strategies $s : W^+ \to Act_i$ (for player $i \in Ag$)

$\mathbf{s} = \langle \mathbf{s}_i : i \in \Gamma \rangle$ - a strategy profile for $\Gamma \subseteq Ag$; S_Γ - the set of all SPs for Γ.

Preliminaries: CTL^* and ATL^* with Past on CGMs

$\mathrm{CTL}: \; M = \langle W, w^0, R, V\rangle$, $\mathrm{ATL}: \; M = \langle W, w^0, \langle Act_i : i \in Ag\rangle, o, V\rangle$

$$\varphi ::= \bot \mid p \mid \varphi \Rightarrow \varphi \mid \underbrace{\exists\psi}_{\mathrm{CTL}} \mid \underbrace{\langle\!\langle\Gamma\rangle\!\rangle\psi}_{\mathrm{ATL}}$$

$$\psi ::= \varphi \mid \psi \Rightarrow \psi \mid \bigcirc\psi \mid (\psi \mathsf{U} \psi) \mid \ominus\psi \mid (\psi \mathsf{S} \psi)$$

The outcome of following $\mathbf{s} \in S_\Gamma$ after finite play $\mathbf{w}$:

$$\mathrm{out}(\mathbf{w}, \mathbf{s}) \hat{=} \{\mathbf{v} \in R_M^{\inf}(\mathbf{w}) : (\forall k \geq |\mathbf{w}|)(\exists \mathbf{b} \in Act_{Ag\setminus\Gamma})(\mathbf{v}^k = o(\mathbf{v}^{k-1}, \mathbf{s}(\mathbf{v}^0 \ldots \mathbf{v}^{k-1}) \cup \mathbf{b}))\}$$

$M, \mathbf{w} \models p$ iff $V(\mathbf{w}^{|\mathbf{w}|-1}, p)$ for $p \in AP$

$M, \mathbf{w} \models \bot, (\varphi \Rightarrow \psi)$ and $\top, \neg, \vee, \wedge, \Leftrightarrow$ as in classical propositional logic

$M, \mathbf{w} \models_{\mathrm{CTL}^*} \exists\varphi$ iff $M, \mathbf{v}, |\mathbf{w}| - 1 \models \varphi$ for some $\mathbf{v} \in R_M^{\inf}(\mathbf{w})$

$M, \mathbf{w} \models_{\mathrm{ATL}^*} \langle\!\langle\Gamma\rangle\!\rangle\varphi$ iff there exists an $\mathbf{s} \in S_\Gamma$ s. t. $M, \mathbf{v}, |\mathbf{w}| - 1 \models \varphi$ for all $\mathbf{v} \in \mathrm{out}(\mathbf{w}, \mathbf{s})$

$M, \mathbf{w}, k \models \bigcirc\varphi, \ominus\varphi, (\varphi \mathsf{U} \psi), (\varphi \mathsf{S} \psi)$, and $\Diamond, \Box, \diamondminus, \boxminus$ – as in LTL

$\forall \hat{=} \neg\exists\neg$, $[\![.]\!] \hat{=} \neg\langle\!\langle.\rangle\!\rangle\neg$.

The (genetic) Relationship between ATL and CTL

$\mathrm{CTL}: \; M = \langle W, w^0, R, V \rangle$, $\mathrm{ATL}: \; M = \langle W, w^0, \langle Act_i : i \in Ag \rangle, o, V \rangle$

$M, \mathbf{w} \models \exists\varphi$	iff	$M, \mathbf{v}, \lvert\mathbf{w}\rvert - 1 \models \varphi$ for some $\mathbf{v} \in R_M^{\mathrm{inf}}(\mathbf{w})$
$M, \mathbf{w} \models \langle\langle\Gamma\rangle\rangle\varphi$	iff	there exists an $\mathbf{s} \in S_\Gamma$ s. t. $M, \mathbf{v}, \lvert\mathbf{w}\rvert - 1 \models \varphi$ for all $\mathbf{v} \in \mathrm{out}(\mathbf{w}, \mathbf{s})$

For $R(w,v) \leftrightarrow (\exists \mathbf{a} \in Act_{Ag})(v = o^T(w, \mathbf{a}))$, the path quantifier is a special case of $\langle\langle . \rangle\rangle$: $\forall = \langle\langle \emptyset \rangle\rangle$, $\exists = [\![\emptyset]\!] = \neg\forall\neg$

CTL^* (with past) can be viewed as the sublanguage of ATL^* with state formulas of the form

$\varphi ::= \bot \mid p \mid \varphi \Rightarrow \varphi \mid \forall\psi$ where $\forall \mathrel{\hat{=}} \langle\langle \emptyset \rangle\rangle$, $\exists \mathrel{\hat{=}} [\![\emptyset]\!] = \neg\forall\neg$

Unwindings CGMs and the Tree Semantics of CTL^*

The unwinding $M^T \hat{=} \langle W^T, w_I^T, \langle Act_i : i \in Ag \rangle, o^T, V^T \rangle$ of M is defined as follows:

$$\begin{array}{rclrcl} W^T & \hat{=} & W(Act_{Ag}\, W)^* & o^T(w^0\mathbf{a}^1 \cdots \mathbf{a}^n w^n, \mathbf{b}) & \hat{=} & w^0\mathbf{a}^1 \cdots \mathbf{a}^n w^n \mathbf{b} o(w^n, \mathbf{b}) \\ w_I^T & \hat{=} & w_I & V^T(w^0\mathbf{a}^1 \cdots \mathbf{a}^n w^n, p) & \hat{=} & V(w^n, p) \end{array}$$

M^T and M are bisimilar and $R_M^{\inf}(w_I)$ and $R_{M^T}^{\inf}(w_I^T)$ are isomorphic. Importantly, o^T is invertible.

Propositional Quantification (on CGMs)

Propositional Quantification wrt the state semantics:

$M_p^X \hat{=} \langle W, w_I, \langle Act_i : i \in Ag \rangle, o, V_p^X \rangle$ where

$V_p^X(w, p) \leftrightarrow w \in X$ and $V_q^X(w, q) \leftrightarrow V(w, q)$ for $q \in AP \setminus \{p\}$.

$M, \mathbf{w} \models_s \exists p \varphi$ iff there exists an $X \subseteq W$ s. t. $M_p^X \models \varphi$

The tree semantics allows quantified variables to have different values in the various occurrences of the same state along paths. Equivalently

$M, \mathbf{w} \models_t \exists p \varphi$ iff $M^T, \mathbf{w}^T \models_s \exists p \varphi$

The Kripke model $\langle W^T, R^T, V^T \rangle$ where

$R^T(w, v) \leftrightarrow (\exists \mathbf{a} \in Act_{Ag})(v = o^T(w, \mathbf{a}))$

is a tree one. Crucially, propositionally quantified CTL^* is decidable wrt the tree semantics (Tim French, 2001).

Some Limitations of ATL's Game-theoretic Construct

The straightforward use of $\langle\langle . \rangle\rangle$ refers to permanent coalitions.

$\langle\langle \Gamma \rangle\rangle \ldots$, is about Γ playing against $Ag \setminus \Gamma$.

The Setting in This Talk

Objectives are individual to each player.

Players' objectives are ordered by preference.

These orderings are known to all.

Players revise their alliances at every move.

Naming Strategies in Propositionally Quantified CTL^* as Known

Given a strategy profile $\mathbf{s} : W^+ \to Act_\Gamma$, in M^T, let

$$\mathbf{finitePlay}(\mathbf{s}) \hat{=} \{o^T(\mathbf{w}^{|\mathbf{w}|-1}, \underline{\underline{\mathbf{s}(\mathbf{w}^{|\mathbf{w}|-1})}} \cup \mathbf{b}) : \mathbf{w} \in R^{\mathrm{fin}}_{M^T}(w_I^T), \mathbf{b} \in Act_{-\Gamma}\}. \quad (1)$$

Then, because of the invertibility of o^T, given a $\mathbf{v} \in R^{\mathrm{fin}}_M(w_I) \subseteq W^T$,

$\mathbf{w} \in \mathbf{finitePlay}(\mathbf{s})$ iff $\mathbf{w} \geq 2$ and $\mathbf{w}^{|\mathbf{w}|-1} = o(\mathbf{w}^0 \cdots \mathbf{w}^{|\mathbf{w}|-2}, \underline{\underline{\mathbf{s}(\mathbf{w}^{|\mathbf{v}|-2})}} \cup \mathbf{b})$

for some $\mathbf{b} \in Act_{-\Gamma}$. For $s \in AP$ such that $[\![s]\!]_{M^T} = \{\mathbf{w}^{|\mathbf{w}|-1} : \mathbf{w} \in \mathbf{finitePlay}(\mathbf{s})\}$, $\mathbf{s}$ can be recovered from $[\![s]\!]_{M^T}$ and $s \in AP$ can be used to name $\mathbf{s}$.

Assuming that $\delta_\Gamma(s)$ restricts s to range over $X \subseteq W^T$ which have the form (1),

$(M^T)^{\mathbf{finitePlay}(s)}_s, \mathbf{w} \models \langle\!\langle\Gamma\rangle\!\rangle\varphi \Leftrightarrow \exists s(\delta_\Gamma(\mathbf{s}) \wedge \forall(\Box\bigcirc s \Rightarrow \varphi)$

This is how strategies are named using a propositional vocabulary in CTL^* in tree models. If only some finite set of objectives is relevant, then this works in corresponding finite quotients of the tree model too.

Strategy Profiles for Temporary Coalitions

A Strategy Profile with Temporary Coalitions (SPTC) s is a mapping of type $R^{\mathrm{inf}}(w_I) \rightarrow Act_{Ag} \times$ part(Ag).

SPTCs are prescriptions both strategies and coalition structure.

Upon $\mathbf{w}$, $s(\mathbf{w}) = \langle \mathbf{a}, C \rangle$ prescribes coalition structure C and $\mathbf{a}|_\Gamma$ as the local decision of Γ, $\Gamma \in C$.

Strategy Profiles for Temporary Coalitions

Naming an SPTC s takes a system of propositions $\mathbf{s} \hat{=} \langle \mathbf{s}_\Gamma : \Gamma \subseteq Ag \rangle$. Let

$\mathbf{finitePlay}_\Gamma(s) \hat{=} \{o^T(\mathbf{w}^{|\mathbf{w}|-1}, \mathbf{a}|_\Gamma \cup \mathbf{b}) : \mathbf{w} \in R^{\mathrm{fin}}_{M^T}(w_I^T), \langle \mathbf{a}, C \rangle = s(\mathbf{w}), \Gamma \in C, \mathbf{b} \in Act_{-\Gamma}\}$.

Then

$$(M^T)^{\langle \mathbf{finitePlay}_\Gamma(s) : \Gamma \subseteq Ag \rangle}_{\langle \mathbf{s}_\Gamma : \Gamma \subseteq Ag \rangle}, \mathbf{v} \models \mathbf{s}_\Gamma \text{ indicates that}$$

$\mathbf{v}^{|\mathbf{v}|-1}$ can be reached from $\mathbf{v}^{|\mathbf{v}|-2}$ by Γ's part of the C-partitioned decision for $\mathbf{v}^0 \cdots \mathbf{v}^{|\mathbf{v}|-2}$ with the express condition that Γ acts as a coalition.

Assuming that $\delta_\Gamma(z)$ constrains z to denote any decision by Γ,

$$\exists z(\delta_\Gamma(z) \wedge \forall \bigcirc (z \wedge \mathbf{s}_{-\Gamma} \Rightarrow \varphi))$$

means that Γ can enforce φ (in one step) provided that $-\Gamma$ go for an $\mathbf{s}_{-\Gamma}$-move as a coalition.

$\mathbf{s} \hat{=} \langle \mathbf{s}_\Gamma : \Gamma \subseteq Ag \rangle$ can be axiomatically constrained to express SPTC.

Preference and LTL-definable Ordered Objectives

Ag, AP, $\langle W, w^0, \langle Act_i : i \in Ag\rangle, o, V, \langle <_i : i \in Ag\rangle\rangle$

$<_i$ - partial orders on $R_M^{\inf}(w_I) \subseteq W^\omega$, expressing i's preference, $i \in Ag$

$[\mathbf{w}]_i \hat{=} \{\mathbf{v} \in R_M^{\inf}(w_I) : (\forall \mathbf{x} \in R_M^{\inf}(w_I))(\mathbf{v} <_i \mathbf{x} \leftrightarrow \mathbf{w} <_i \mathbf{x} \wedge \mathbf{x} <_i \mathbf{v} \leftrightarrow \mathbf{x} <_i \mathbf{v})\}$

We require $O_i \hat{=} \{[\mathbf{w}]_i : \mathbf{w} \in W^\omega\}$ to be finite partitions of $R_M^{\inf}(w_I)$ into LTL-definable classes, i.e., for every $o \in O_i$ there exists a LTL formula θ s. t.

$o = [\![\theta]\!]_M \hat{=} \{\mathbf{w} : M, \mathbf{w}, 0 \models \theta\}$ are i's objectives.

Coalitions have finer systems of objectives:

$$O_\Gamma \hat{=} \{\bigwedge_{i\in\Gamma} o_i : o_i \in O_i\}, \qquad o' <_\Gamma o'' \hat{=} \bigwedge_{i\in\Gamma} o' <_i o'', \qquad \Gamma \subseteq Ag.$$

Preference in the Temporal Language

Given path formulas φ_1, φ_2,

$$M, \mathbf{v} \models \varphi_1 <_i \varphi_2 \quad \text{iff} \quad \mathbf{w}_1 <_i \mathbf{w}_2 \text{ for all } \mathbf{w}_k \in R_M^{\inf}(\mathbf{v}) \text{ s. t. } M, \mathbf{w}_k, |\mathbf{v}| \models \varphi_k,\ k = 1, 2.$$

$$M, \mathbf{v} \models \varphi_1 \not\lessdot_i \varphi_2 \quad \text{iff} \quad \mathbf{w}_1 \not<_i \mathbf{w}_2 \text{ for all } \mathbf{w}_k \in R_M^{\inf}(\mathbf{v}) \text{ s. t. } M, \mathbf{w}_k, |\mathbf{v}| \models \varphi_k,\ k = 1, 2.$$

Reference to Objectives as Stated wrt the Beginning of Time

$[\theta] \hat{=} \Diamond\!\!\!\!-(\mathsf{I} \wedge \theta)$, then, e.g.,

$M, \mathbf{w} \models [\theta]$ iff $\mathbf{w} \in R_M^{\mathrm{fin}}(w_I)$ can be extended to a play from $[\![\theta]\!]_M$.

Example: Comparing Strategy Profiles with Temporary Coalitions

$\mathbf{s} = \langle \mathbf{s}_\Gamma : \Gamma \subseteq Ag \rangle$, $\mathbf{t} = \langle \mathbf{t}_\Gamma : \Gamma \subseteq Ag \rangle$ - vocabularies for strategy profiles

Forming alliances as in s is no worse for i than as in $\mathbf{t}$:

$$\not<_i (\mathbf{s}, \mathbf{t}) \hat{=} \bigwedge_{\theta' \in \Theta_i} \left(\forall (\Box \bigcirc \widetilde{\mathbf{t}}_{Ag} \Rightarrow \theta') \Rightarrow \bigvee_{\theta'' \in \Theta_i} \forall (\Box \bigcirc \widetilde{\mathbf{s}}_{Ag} \Rightarrow \theta'') \wedge \theta'' \not<_i \theta' \right)$$

$\mathbf{s} \sim_i \mathbf{t} \hat{=} \forall \Box \bigcirc \left(\bigwedge_{i \in \Gamma \subseteq Ag} \mathbf{s}_\Gamma \Leftrightarrow \mathbf{t}_\Gamma \right)$ - s and $\mathbf{t}$ agree on i's alliances and agendas

$\{\Gamma_0, \ldots, \Gamma_{2^{|Ag|}-1}\} \hat{=} \mathcal{P}(Ag)$, $\forall \mathbf{s} \hat{=} \forall \mathbf{t}_{\Gamma_0} \ldots \forall \mathbf{t}_{\Gamma_{2^{|Ag|}-1}}$n

$\delta_{Ag}(\mathbf{s}) \wedge \forall \mathbf{t}(\delta_{Ag}(\mathbf{t}) \wedge \mathbf{s} \sim_i \mathbf{t} \Rightarrow \not<_i (\mathbf{s}, \mathbf{t}))$ - teaming as in s is optimal for i

$\mathbf{s} \sqsupseteq_{-i} \mathbf{t} \hat{=} \forall \Box \bigcirc \left(\bigwedge_{\Gamma \subseteq Ag \setminus \{i\}} \mathbf{s}_\Gamma \Rightarrow \mathbf{t}_\Gamma \right)$ - s and $\mathbf{t}$ agree, except on i's alliances and agendas

$\delta_{Ag}(\mathbf{s}) \wedge \forall \mathbf{t}(\delta_{Ag}(\mathbf{t}) \wedge \mathbf{s} \sqsupseteq_{-i} \mathbf{t} \Rightarrow \not<_i (\mathbf{s}, \mathbf{t}))$ - teaming as in s is optimal for i, if everyone else acts as in s.

Axioms for Preference

The non-temporal archetype of this preference operator: Von Wright, 1963. Let $\sigma \in \{< . \not\ll\}$. Then:

$$\varphi_1 \sigma_\Gamma \psi_1 \wedge \forall\bigcirc(\varphi_2 \Rightarrow \varphi_1) \wedge \forall\bigcirc(\psi_2 \Rightarrow \psi_1) \Rightarrow \varphi_2 \sigma_\Gamma \psi_2 \tag{P1}$$

$$\varphi_1 \sigma_\Gamma \psi \wedge \varphi_2 \sigma_\Gamma \psi \Leftrightarrow (\varphi_1 \vee \varphi_2) \sigma_\Gamma \psi \tag{P2}$$

$$\varphi \sigma_\Gamma \psi_1 \wedge \varphi \sigma_\Gamma \psi_2 \Leftrightarrow \varphi \sigma_\Gamma (\psi_1 \vee \psi_2)$$

$$\varphi <_\Gamma \psi \Rightarrow \forall\bigcirc\neg(\varphi \wedge \psi) \tag{P3}$$

$$\varphi <_\Gamma \psi \Rightarrow \psi \not\ll_\Gamma \varphi$$

$$\varphi \not\ll_\Gamma \psi \Rightarrow \neg(\varphi <_\Gamma \psi)$$

$$\varphi <_\Gamma \psi \wedge \psi <_\Gamma \chi \wedge \exists\bigcirc\psi \Rightarrow \varphi <_\Gamma \chi \tag{P4}$$

$$\bot \sigma_\Gamma \varphi, \qquad \varphi \sigma_\Gamma \bot \tag{P5}$$

$$[\varphi] \sigma_\Gamma [\psi] \Leftrightarrow \forall\Box([\varphi] \sigma_\Gamma [\psi]) \tag{P6}$$

Axioms for a Given System of Objectives $\langle \Theta_{I,i}, <_i \rangle$, $i \in Ag$

$$\exists \bigcirc(\varphi \wedge [\theta]) \Rightarrow (\varphi \sigma_\Gamma \psi \Leftrightarrow (\varphi \vee [\theta])\sigma_\Gamma \psi) \qquad \text{(O1)}$$

$$\exists \bigcirc(\psi \wedge [\theta]) \Rightarrow (\varphi \sigma_\Gamma \psi \Leftrightarrow \varphi \sigma_\Gamma (\psi \vee [\theta]))$$

$$\bigvee_{\theta \in \Theta_{I,\Gamma}} [\theta], \qquad \bigwedge_{\theta_1, \theta_2 \in \Theta_{I,\Gamma}} \forall \Box([\theta_1] \Leftrightarrow [\theta_2]) \vee \forall \Box \neg([\theta_1] \wedge [\theta_2]) \qquad \text{(O2)}$$

$[\theta_1] <_\Gamma [\theta_2]$, resp. $[\theta_1] \not\#_\Gamma [\theta_2]$, if $\theta_1 <_\Gamma \theta_2$, resp. $\theta_1 \not<_\Gamma \theta_2$, is given. (O3)

$\sigma \in \{<, \not\#\}$.

Axioms with No Prespecified Objectives

Two axioms about the interaction of σ_i, $\sigma \in \{<, \not<\}$, with the separated normal form [?] of PLTL formulas and the guarded normal form in (future) LTL.

$$\{g_0, \ldots, g_{2^{|AP|}-1}\} \mathrel{\hat{=}} \left\{ \bigwedge_{p \in AP} \varepsilon_p p : \varepsilon_p \text{is either } \neg \text{ or nothing, } p \in AP \right\}.$$

$$\left(\bigwedge_{k'} \ominus \pi'_{k'} \Rightarrow \varphi'_{k'}\right)\sigma_\Gamma\left(\bigwedge_{k''} \ominus \pi''_{k''} \Rightarrow \varphi''_{k''}\right) \Leftrightarrow \bigwedge_{k',k''} (\pi'_{k'} \wedge \pi''_{k''} \Rightarrow \varphi'_{k'}\sigma_\Gamma\varphi''_{k''}) \tag{P7}$$

$$\left(\bigvee_{k<2^{|AP|}} g_k \wedge \bigcirc\varphi^t_k\right)\sigma_\Gamma\left(\bigvee_{k<2^{|AP|}} g_k \wedge \bigcirc\psi^t_k\right) \Leftrightarrow \forall\bigcirc\left(\bigvee_{k<2^{|AP|}} g_k \wedge \varphi^t_k\sigma_\Gamma\psi^t_k\right) \tag{P8}$$

The End