Associative-admissible operad

Askar S. Dzhumadil'daev

Institute of Mathematica, Almaty, Kazakhstan

dzhuma@hotmail.com

An algebra is called associative-admissible, if this algebra under anti-commutator $\{a, b\} = ab + ba$ becomes associative. For example, Zinbiel algebra, i.e., algebra with identity $a(bc) - (ab+ba)c = 0$ is associative-admissible. Associative algebra is associative-admissible iff it satisfes the identity $[[a, b], c] = 0$. Let $\mathcal{A}sAdm$ and $\mathcal{L}eib$ be operads generated by associative-admissible algebras and two-sided Leibniz algebras respectively.

Associative-admissible operad has the following properties.

- Operads $\mathcal{A}sAdm$ and $\mathcal{L}eib$ are Koszul
- $\mathcal{A}sAdm^! = \mathcal{L}eib$
- $\mathcal{A}sAdm = \mathcal{A}sCom \star \mathcal{A}com$
- Dimensions of multi-linear parts $d_n = \dim \mathcal{A}sAdm(n)$ satisfy the following recurrence relations

$$d_n = \sum_{k=1}^{n-1} k! F_{k+2} B_{n-1,k}(d_1, d_2, \ldots, d_{n-k}), \quad n > 1,$$

$$d_1 = 1,$$

  where F_n are Fibonacci numbers and $B_{n,k}(x_1, \ldots, x_{n-k+1})$ are Bell polynomials.

- If p is prime, then

$$d_{p-1} \equiv \begin{cases} 1(mod\, p), & \text{if } p \neq 3, \\ -1 & \text{if } p = 3, \end{cases}$$

$$d_p \equiv \begin{cases} 1(mod\, p), & \text{if } p \neq 2, \\ 0 & \text{if } p = 2, \end{cases}$$

$$d_{p+1} \equiv 2(mod\, p),$$

$$d_{p+2} \equiv 10(mod\, p).$$