Fine gradings on classical simple real Lie algebras

Mikhail Kotchetov

Memorial University of Newfoundland, St. Johns', NF, Canada

mikhail@mun.ca

If G is a group and A is an algebra with any number of multilinear operations over a field $\mathbb{F}$, then a G-*grading* on A is a family of subspaces $\{A_g\}_{g\in G}$ such that $A = \bigoplus_{g\in G} A_g$ and, for any operation φ defined on A, we have $\varphi(A_{g_1}, \dots, A_{g_n}) \subset A_{g_1\cdots g_n}$ for all $g_1, \dots, g_n \in G$, where n is the number of variables taken by φ.

In the past two decades there has been considerable interest in classifying group gradings on algebras of different varieties including associative, associative with involution, Lie and Jordan. Of particular importance are the so-called *fine gradings* (that is, those that do not admit a proper refinement), because any grading on a finite-dimensional algebra A can be obtained from them via a group homomorphism, although not in a unique way. If the ground field $\mathbb{F}$ is algebraically closed and of characteristic 0, then the classification of fine abelian group gradings on A up to equivalence is the same as the classification of maximal quasitori in the algebraic group $\mathrm{Aut}(A)$ up to conjugation. In particular, it is known for all finite-dimensional simple complex Lie algebras.

In this talk I will present a recent joint work with A. Elduque and A. Rodrigo-Escudero in which we classify fine gradings on simple associative algebras with involution over the field of real numbers and, as a consequence, on classical simple real Lie algebras.