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Central polynomials and exponential growth of the codimensions

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Let F be a field of characteristic zero, A an associative F-algebra and $F\langle X\rangle$ the free associative algebra, freely generated over F by the set X of variables. A non-zero polynomial $f(x_1,\ldots,x_n)\in F\langle X\rangle$ is a central polynomial for A if for all $a_1,\ldots,a_n\in A$, $f(a_1,\ldots,a_n)\in Z(A)$, the center of A. Clearly, the polynomial identities are in particular central polynomials, thus if a polynomial is central but it is not an identity, then we call it a proper central polynomial.

If we consider for any $n\geq 1$ the space P_n of multilinear polynomials of degree n, then we attach to it three numerical sequences: $c_n(A)$, the dimension of P_n modulo the polynomial identities of A; $c_n^z(A)$, the dimension of P_n modulo the central polynomials of A; $\delta_n(A)$, the dimension of the space of multilinear central polynomials of degree n modulo the identities of A. They are called the ordinary, central and proper central codimension sequence, respectively, and for all $n\geq 1$,

$$c_n(A)=c_n^z(A)+\delta_n(A).$$

If the algebra A satisfies a non-trivial polynomial identity, then it is well-known that $c_n(A)$ is exponentially bounded. Therefore a similar result holds also for the central and the proper central codimension sequences.

In this talk we shall present some recent results about the exponential growth of such sequences in the setting of ordinary algebras, algebras with involution and algebras graded by a finite abelian group. In particular, we give a positive answer to the Amitsur conjecture concerning the existence and the integrability of the so-called PI-central and PI-proper central exponent. Finally, we compare the PI-exponent with the central one.